Classical Mechanics and Math Methods SPRING, 2012 **PHYS 2210**

Homework 9

(Due Date: Start of class on Thurs. March 15)

1. Check out http://www.colorado.edu/physics/phys2210/phys2210_sp12/project_info.html (also available directly from our course website) For *this homework*, please simply indicate the names of the people you plan to work with. (Groups of two or three are suggested, please no more than that.)

2. Assuming that asteroids have roughly the same mass density as the moon, make an estimate of the largest asteroid that an astronaut could be standing on, and still have a chance of throwing a small object (with their arms, no machinery!) so that it completely escapes the asteroid's gravitational field. (This minimum speed is called "escape velocity") Is the size you computed typical for asteroids in our solar system?

3. *Determining the gravitational force between two rods:*

(a) Consider a thin, uniform rod of mass m and length L (and negligible other dimensions) lying on the x axis (from x=-L to 0), as shown in fig 1a. Derive a formula for the gravitational field "g" at any arbitrary point x to the right of the origin (but still on the x-axis!) due to this rod.

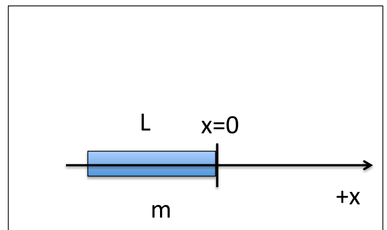


Fig 1a

(b) Now suppose a second rod of length L and mass m sits on the x axis as shown in fig 1b, with the left edge a distance "d" away. Calculate the mutual gravitational force between these rods.

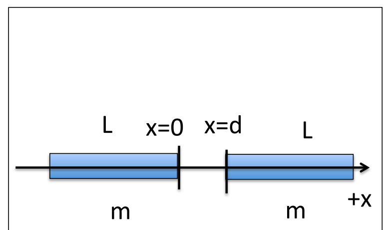


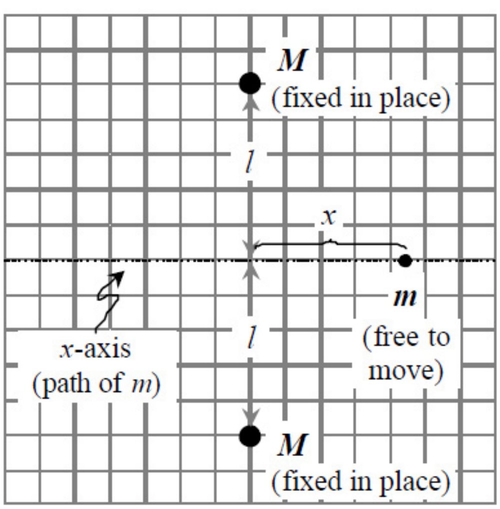
Fig 1b

(c) Let's do some checks! Show that the units work out in parts a and b.

Find the magnitude of the force in part a, in the limit $x >> L$. What do you expect? Briefly, discuss!

Finally, verify that your answer to part b gives what you expect in the limit $d >> L$. *(Hint: This is a bit harder! You need to consistently expand everything to second order, not just first, because of some interesting cancellations)*

4. Two large solid spheres, each with mass M, are fixed in place a distance 2l apart, as shown. A small ball of mass m is constrained to move along the x-axis, shown as a dashed line. (Let x = 0 represent the point on the axis directly between the spheres, with +x to the right.)



(a) Compute the net force on the ball by both large spheres, and write down a differential equation that governs the motion of the small ball.

(b) Is the net force exerted on the small ball a restoring force? Explain! Under what limiting conditions for x can we say the motion of m can be *approximated* as simple harmonic motion? (For large x? For small x? Large or small compared to what?) In this limiting case, find the period of motion in terms of the given parameters. Explain your reasoning.

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5. Last week you showed that a pendulum's potential energy (measured from the equilibrium level) is $U(\phi) = mgL(1 - \cos\phi)$ (where L is the pendulum length) and that under the small angle assumption the motion is periodic with period $\tau_0 = 2\pi\sqrt{L/g}$. In this problem we will find the period for large oscillations as well:

(a) Using conservation of energy, determine $\dot{\phi}$ as a function of ϕ, given that the pendulum starts at rest at a starting angle of Φ_0. Now use this ODE to find an expression for the time the pendulum takes to travel from $\phi = 0$ to its maximum value Φ_0. (You will have a formal integral to do that you cannot solve analytically! That's ok, just write down the integral, being very explicit about the limits of integration) Because this time is a quarter of the period, write an expression for the full period, as a multiple of τ_0.

(b) Use your favorite computational environment to evaluate the integral and plot τ/τ_0 for $0 \leq \Phi_0 \leq 3$ rad. For small Φ_0, does your graph look like that you expect? (Discuss - what value DO you expect?) What is τ/τ_0 for $\Phi_0 = \pi/2$ rad? (Give us several significant figures)

(c) What happens to τ as the amplitude of the oscillation approaches π? Discuss the physics here! What can you say about how well simple harmonic motion approximates the behavior of a real pendulum?

Note: You may see some curious numerical pathologies if you use MMA. If your plot has little gaps, it's because MMA is generating tiny imaginary terms added to the result. You might just try plotting the real part of the integral, using Re[ ]. If it gives you symbolic results, you might fix that by using N[ ], which forces MMA to evaluate the expression as a number, if it can! By the way, this integral is called the "complete elliptic integral of the first kind".

(d) A real grandfather clock has a length of about half a meter, and you pull the pendulum, oh, a few cm to the side (let's say $\sim$ 10 cm, for a decent sized clock). Use your code from the previous part to compute τ/τ_0 for this clock. When designing the clock for practical use, would the clockmaker be safe in making the "small angle" approximation? Don't just look at the number you got and say "well, that looks pretty close to 1, so we should be ok". What we mean here is to *justify your answer with a calculation* that tells you whether this clock (calibrated assuming the small angle approximation) would be annoying or useful to have in your house! Can you be concrete about what you would consider "annoying" about a clock? (I was surprised by the result - are you?)

6. *Practicing with complex numbers. (2 pt each). See Boas Ch 2.4-12 for more review, or further practice!)*

(a) If $z_1 = 1 - \sqrt{3}i$, draw z_1 in the complex plane. Compute its real and imaginary parts and magnitude. Write z_1 as $Ae^{i\theta}$ and determine A and θ.

(b) If $z_2 = 1/(1 - i)$, draw z_2 in the complex plane. Compute its real and imaginary parts and magnitude. Write z as $Ae^{i\theta}$ and determine A and θ.

(c) If $z_3 = 0.5e^{-i\pi/6}$, what are the real and imaginary parts of $1/z_3^2$? *(Note that this is just a simple square, not an absolute value squared, so the answer does not need to come out purely real!)*

(d) Compute $Z = z_1 * z_3$ with z_1 and z_3 given in parts (a) and (c). Draw Z in the complex plane. Compute its real and imaginary parts and magnitude. Write Z as $Ae^{i\theta}$ and determine A and θ.

(e) Pick either one of the following two items to prove - whichever you prefer!

- Use Euler's theorem to rewrite $\sin x$ in terms of complex exponentials, and then use that to show that $\int_0^{2\pi} \sin(n\theta)\sin(m\theta)d\theta$ vanishes if n and m are unequal integers. *(We'll make good use of this, soon!)*
- Euler's theorem says that $e^{i\theta} = \cos\theta + i\sin\theta$. Call $z = e^{i\theta}$ and evaluate z^2 two ways to show the (very handy!) trig identities $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$.