INSTRUCTORS MANUAL
Velocity and Acceleration in Polar Coordinates

Goals:
Students should be able to:

- sketch and label the position vectors and vector components for a particle in 2D space,

- apply the chain and product rules from differential calculus to vector expressions,

- use trigonometry to determine vector components, and

- derive expression for velocity and acceleration in plane polar coordinates.

This tutorial is based on:

- Original work written by Corinne A Manogue (`http://www.physics.oregonstate.edu/~corinne/`).

Materials needed:

- (optional) whiteboards, dry-erase markers, erasers

Special Instructions:

- None.

Homework Connections: Homework 4, Problem X

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Reflections on this Tutorial:

This tutorial is based on one originally written by Corrine Manogue (Oregon State). The original tutorial gave vector formulas to students (which seemed to confuse them). This version has been rewritten so that students must construct the vector formulas from a sketch which they must generate.

In prior uses, students struggle with applying the chain and product rule to this derivation. We have offered a compromise asking students to defend the product rule and emily the chain rule. Also in prior uses, most students only complete the derivation for the velocity in the time allotted (25 minutes). This is probably OK, since many of the same calculations are done for the acceleration. The instructor should sketch this derivation out after completing the tutorial.

In the wrap-up, students should be asked to make sense of the derived formulae for velocity and acceleration in plan polar coordinates. Few do this spontaneously.

Part 1 – Getting Oriented

A particle moves in the plane. We could describe its motion in two different ways:

CARTESIAN: I tell you $x(t)$ and $y(t)$.
POLAR: I tell you $r(t)$ and $\phi(t)$. (Here $r(t) = |\vec{r}(t)|$, it's the "distance to the origin")

(a) Draw a picture showing the location of the point at some arbitrary time, labeling x, y, r, ϕ and also showing the unit vectors $\hat{i}, \hat{j}, \hat{r}$, and $\hat{\phi}$, all at this one time.

(b) Using this picture, determine the formula for $\hat{r}(t)$ in terms of the Cartesian unit vectors. You answer should contain $\phi(t)$.

(c) Write down the analogous expression for $\hat{\phi}(t)$.

(d) I claim the position vector in Cartesian coordinates is $\vec{r}(t) = x(t)\hat{I} + y(t)\hat{j}$. Do you agree? Is this consistent with your picture above?

(e) I claim the position vector in polar coordinates is just $\vec{r}(t) = r(t)\hat{r}$. Again, do you agree? Why isn't there a $+\phi(t)\hat{\phi}$ term?

PAUSE and check with an instructor or another group

Part 2 – Getting Kinetic

(a) Now let's find the velocity, $\vec{v}(t) = d\vec{r}/dt$.
In Cartesian coordinates, it's just $\vec{v}(t) = \dot{x}(t)\hat{i} + \dot{y}(t)\hat{j}$. Explain why, in polar coordinates, the velocity can be written as $d\vec{r}/dt = r(t)\, d\hat{r}/dt + dr(t)/dt\, \hat{r}$.

(b) It appears we need to figure out what $d\hat{r}/dt$ is. Use the formula your determined in question 1b to get started – first in terms of $\hat{i}$ and $\hat{j}$, and then converting to pure polar.

(c) Write an expression for $\vec{v}(t)$ in polar coordinates.

(d) Finally, determine the acceleration $\vec{a} = d\vec{v}(t)/dt$. In Cartesian coordinates, it's just $\vec{a}(t) = \ddot{x}(t)\hat{i} + \ddot{y}(t)\hat{j}$. Work it on in polar coordinates.