Tutorial (part II): Forced harmonic oscillator

Phys 2210

A harmonic oscillator with a restoring force $25\alpha^2x$ is subject to a damping force $3\alpha v$ and a sinusoidal driving force $F_0\cos(10\alpha t)$.

(a) Write down the differential equation that governs the motion of this oscillator.

(b) Show that if the driving force were removed the oscillator would become underdamped, and express the frequency of the oscillator in terms of the given quantities. Explain your reasoning.

(c) For any damped oscillator that is driven by a sinusoidal external force, we know that the eventual (steady-state) motion is sinusoidal in nature. However, before the oscillator reaches steady state, its motion can be thought of as the algebraic sum of the steady-state motion plus a transient oscillatory motion whose amplitude dies exponentially with time.

i. Each $x$ vs. $t$ graph below illustrates the actual motion (transient plus steady state) of a damped, driven oscillator starting at $t = 0$. For each case, is the frequency of the steady-state motion greater than, less than, or equal to that of the transient motion? Explain.

j. Identify which graph (1 or 2) would better correspond to the damped, driven oscillator described in parts a) of this problem. Explain your reasoning.