A common way to write a function as a Fourier series is:

\[ f(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t) \]  

(1)

Part 1
Sketch \( f(t) \) for the following cases:
(Please label the axis and include labeled tick marks)

a) \( a_0 = 2, \) all the other \( a_n \)'s and \( b_n \)'s = 0.

b) \( a_0 = 1, b_1 = 2, \) all the other \( a_n \)'s and \( b_n \)'s = 0. Assume \( \omega = 1 \) in Equation (1).
c) $a_2 = 3$, all the other $a_n$’s and $b_n$’s = 0. Assume $\omega = 1$ in Equation (1).

Part 2

$$f(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$  \hspace{1em} (1)

Given $f(t)$, as written or drawn below, tell us, without calculations, anything you know about the $a_n$’s and $b_n$’s. You can just answer qualitatively – are some definitely zero, positive, negative? Assume $\omega = 1$ in Equation (1).

a) $f(t) = 3\cos(17t)$

What do you know about the $a_n$’s and $b_n$’s?

b)

What do you know about the $a_n$’s and $b_n$’s?
c) \( f(t) = \begin{cases} 1, & (|t| < \frac{\pi}{2}) \\ -1, & (\pi > |t| > \frac{\pi}{2}) \end{cases} \), outside this region it repeats, so \( f(t + 2\pi) = f(t) \).

What do you know about the \( a_n \)'s and \( b_n \)'s? (Hint: it might help to draw this function over a couple of periods in both positive and negative \( t \).)

Part 3

Given the function above for \( f(t) \), what would you choose for \( \omega \) in the general form for the Fourier series: 
\[
 f(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t) 
\]?

Please explain your answer:
TUTORIAL: FOURIER SERIES

Bonus – (Do this if you finish the other parts and still have time)

\[ f(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t) \]

1) For \( f(t) = 2 \sin(15t) \), what are some possible values of \( \omega \) that would work in the formula above? For some of these \( \omega \)'s, what do you know about the \( a_n \)'s and \( b_n \)'s?

2) What is the period, \( T \), of \( f(x,t) = 3 \cos\left(\frac{\pi b}{3} x + \frac{2}{\pi a} t\right) \)?

3) Without calculation, what do you know about the \( a_n \)'s and \( b_n \)'s for \( f(t) \)?