✯ Fourier Series ✯

A common way to write a function as a Fourier series is:

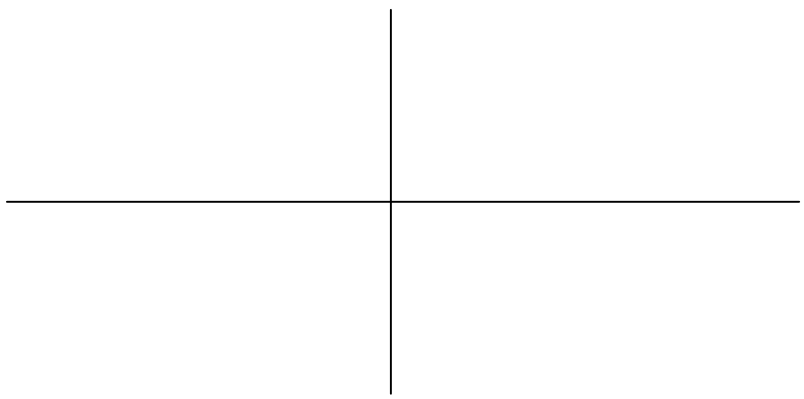
$$f(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t) \qquad (1)$$

Part 1

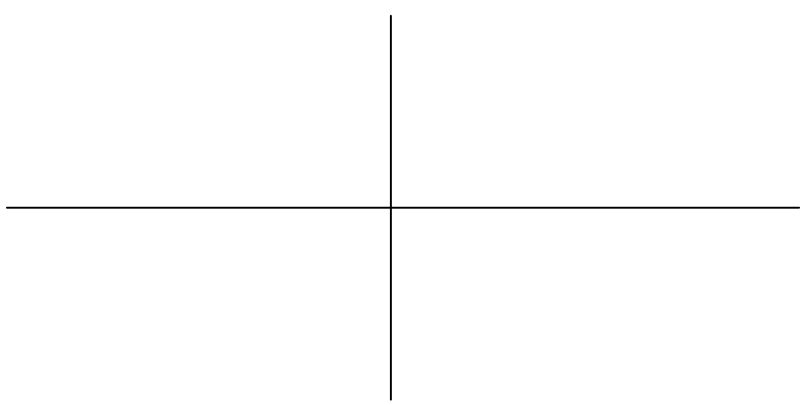
Sketch f(t) for the following cases:

(Please label the axis and include labeled tick marks)

a) $a_0 = 2$, all the other a_n's and b_n's = 0.



b) $a_0 = 1$, $b_1 = 2$, all the other a_n's and b_n's = 0. Assume $\omega = 1$ in Equation (1).



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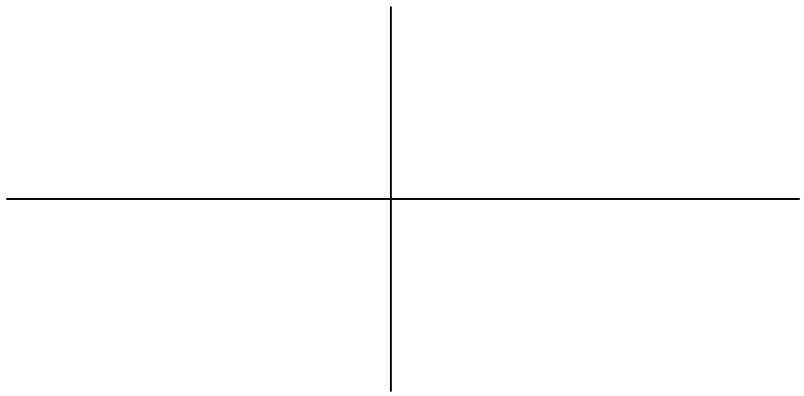
c) $a_2 = 3$, all the other a_n's and b_n's = 0. Assume $\omega = 1$ in Equation (1).



Part 2

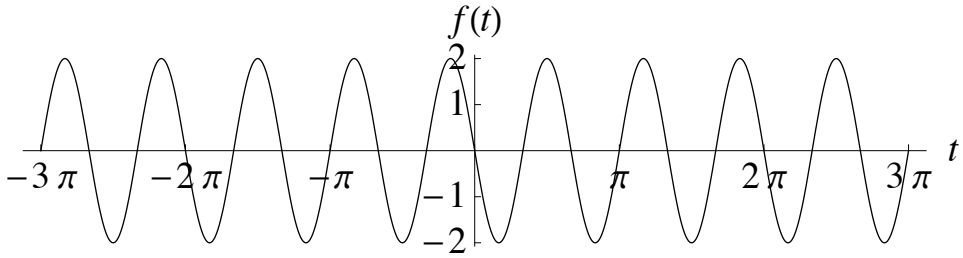
$$f(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t) \qquad (1)$$

Given f(t), as written or drawn below, tell us, **without calculations**, anything you know about the a_n's and b_n's. You can just answer qualitatively – are some definitely zero, positive, negative? Assume $\omega = 1$ in Equation (1).

a) $f(t) = 3\cos(17t)$

What do you know about the a_n's and b_n's?

b)



What do you know about the a_n's and b_n's?

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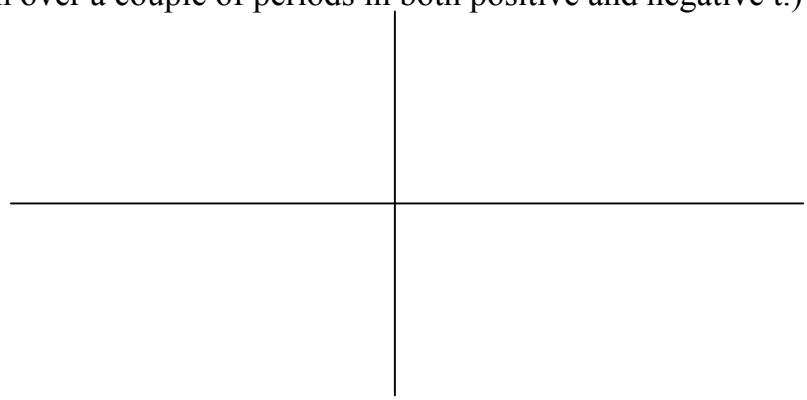
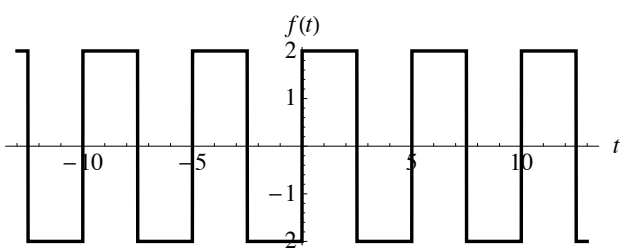
c) $f(t) = \begin{cases} 1, & (|t| < \frac{\pi}{2}) \\ -1, & (\pi > |t| > \frac{\pi}{2}) \end{cases}$, outside this region it repeats, so $f(t+2\pi) = f(t)$.

What do you know about the a_n's and b_n's? (Hint: it might help to draw this function over a couple of periods in both positive and negative t.)



Part 3



Given the function above for f(t), what would you choose for ω in the general form for the Fourier series: $f(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$?

Please explain your answer:

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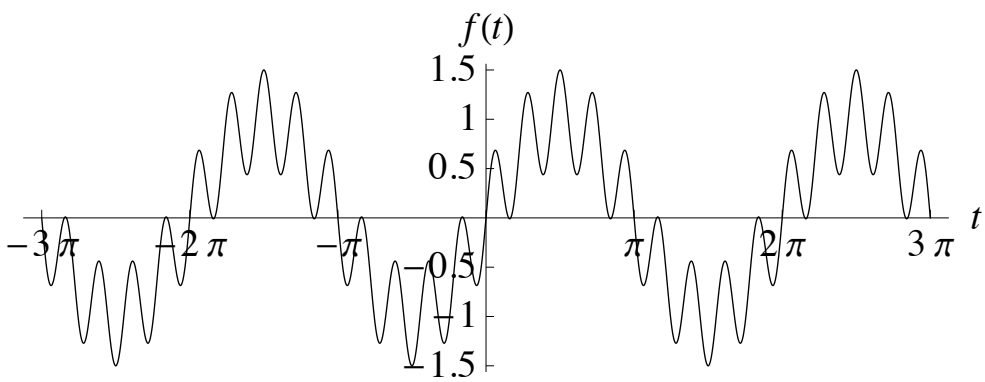
Bonus – (Do this if you finish the other parts and still have time)

$$f(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

1) For $f(t) = 2\sin(15t)$, what are some possible values of ω that would work in formula above? For some of these ω's, what do you know about the a_n's and b_n's?

2) What is the period, T, of $f(x,t) = 3\cos\left(\frac{\pi b}{3}x + \frac{2}{\pi a}t\right)$?

3)



Without calculation, what do you know about the a_n's and b_n's for f(t)?

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