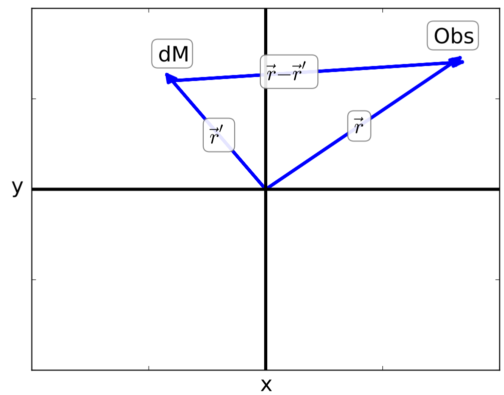
⋆ TUTORIAL: GRAVITATION ⋆

Part 1 – The Gravitational Potential and dM

It can be shown that the potential contributed $d\Phi$ by an infinitesimal mass dM at a location $\mathbf{r} - \mathbf{r}'$ is,

$$d\Phi = -G\frac{dM}{|\mathbf{r} - \mathbf{r}'|}$$

where dM is the mass contained in an infinitesimal space (volume dV' with density $\rho(\mathbf{r}')$, or area dA' with density $\sigma(\mathbf{r}')$, or path ds with density $\lambda(\mathbf{r}')$).
Notice that dM is located at $\mathbf{r}'$; this is not $\mathbf{r}$.



i. Consider a sphere of mass M and radius R with uniform density. One of your classmates was asked to determine dM in terms of the variables given and an appropriate choice of integration variables. This is what she's written on the paper.

$$dM = \rho(\mathbf{r}')\, dV'$$

$$dM = \frac{3M}{4\pi R^3}\, dV'$$

$$dM = \frac{3M}{4\pi R^3}\, dr' r' d\theta' r' \sin\theta' d\phi'$$

a) Carefully explain each step that she's taken. Where did the extra r' come from? What about the $r' \sin\theta'$? Sketching the top and side view of the sphere might help.

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b) Is $dM = 3M/(4\pi R^3)\, dx'dy'dz'$ a valid expression? Why or why not? Discuss with your group members.

c) Why are these variables (dV', dA', or ds') "primed" variables? It might help to look at the figure on the first page and compare $\mathbf{r}'$ to $\mathbf{r}$.

Now, you can get some practice.

ii. For the following mass distribution, determine dM in terms of the variables given and the proper integration variables
(That is, dV', dA', or ds' in an appropriate choice of coordinate system).

A thin circular ring of mass M and radius a with uniform density.
(Hint: the mass is confined to a 1D space.)

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Part 2 - Setting Up Gravitational Potential Problems

i. Your classmate has determined $|\mathbf{r} - \mathbf{r}'|$ for an observation location at a distance Z above the center of a sphere of mass M and radius R with uniform density ($Z > R$). This is what she's written on the page.

$$d\Phi = -G\frac{dM}{|\mathbf{r} - \mathbf{r}'|}$$

$$|\mathbf{r} - \mathbf{r}'| = |\langle 0, 0, Z\rangle - \langle x', y'z'\rangle|$$

$$|\mathbf{r} - \mathbf{r}'| = |\langle 0, 0, Z\rangle - \langle r'\sin\theta'\cos\phi', r'\sin\theta'\sin\phi', r'\cos\theta'\rangle|$$

$$|\mathbf{r} - \mathbf{r}'| = |\langle -r'\sin\theta'\cos\phi', -r'\sin\theta'\sin\phi', Z - r'\cos\theta'\rangle|$$

$$|\mathbf{r} - \mathbf{r}'| = \sqrt{Z^2 + r'^2 - 2Zr'\cos\theta'}$$

Where is the sphere located in her (undrawn) sketch? Sketch the figure she would have used. Clearly label the distances that she used.

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Let's get some practice doing this.

ii. Consider an observation location at a distance Z above the center of a thin circular ring of mass M and radius a with uniform density. (Hint: the mass is confined to a 1D space.)

Sketch a diagram for this situation. Label the location of dM with respect to the origin ($\mathbf{r}'$), the observation location with respect to the origin ($\mathbf{r}$), and the relative location of the observation point with respect to dM ($\mathbf{r} - \mathbf{r}'$).

Determine $|\mathbf{r} - \mathbf{r}'|$ in terms of the integration variables. Set up, **but do not evaluate**, the integrals necessary to determine the gravitational potential, Φ. Your integral should include all limits of integration.

a.) Your sketch:

b.) $dM =$

c.) $|\mathbf{r} - \mathbf{r}'| =$

d.) $\Phi = \int d\Phi =$

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