

Phys 2210 Lecture Notes (1)
spring 2011

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Texts, grading, schedule, notes, office hrs, lots more
(2) - See web! ↳ Tues & Wed.

Homework due every Thurs, start of class
(1st one due this week)

Online HW due every Tues AM (10 AM!)

- See web!

Exams: Feb 10, Mar 17 + final
↳ in-class.

Bring clickers to class

Math co-reg is APPM 2360 (or MATH 3130)

CLASSICAL MECHANICS & MATHEMATICAL METHODS I

2 sides to this course!

Flesh out 1110 concepts & build math toolbox

→ Problem-solving skills, & background for rest of physics.

Big topics for this term (subject to change...)

	<u>Physics</u>	<u>Math</u>
- Motion + Energy	<ul style="list-style-type: none"> • Newton's Laws with friction/drag • Potential Energy • Stability • Non-inertial frames 	<ul style="list-style-type: none"> • Ordinary diff eq's, <u>ODE'S</u> • Curvilinear coords • gradient operator
- Harmonic Motion	<ul style="list-style-type: none"> • With damping, + outside forces • <u>Model</u> for lots of physics 	<ul style="list-style-type: none"> • Fourier series + transform • Delta functions
- Heat flow & Gravity	<ul style="list-style-type: none"> • <u>model</u> " " " " 	<ul style="list-style-type: none"> • Partial diff eq's <u>PDE'S</u> • Special functions

Overview of CLASSICAL MECHANICS: (Taylor 1.3-1.5)KinematicsDescription of
MOTION

- Coordinate systems
- Vector nature
- (+ Rotational Kinematics)

DynamicsExplanation of
MOTIONNEWTON'S
LAWS

- 1) Object at rest [uniform motion]
remains " " "
if $\vec{F}_{\text{net, external}} = 0$
[Defines inertial reference frame]

$$2) \vec{F}_{\text{net, external}} = d\vec{P}_{\text{system}}/dt$$

$$= m \vec{a}_{\text{cm}} \quad (\text{if mass, } m, \text{ is constant})$$

$$3) \vec{F}_{AB} = -\vec{F}_{BA}$$

(+ Rotational extensions, e.g. $\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$)

Conservation Laws

1) If $\vec{F}_{\text{net, ext}} = 0$, then \vec{P}_{system} is conserved $\left(\begin{array}{l} \text{And similarly for} \\ \vec{L}, \text{ if } \vec{\tau}_{\text{net}} = 0 \end{array} \right)$

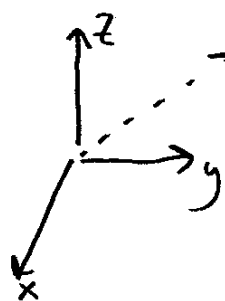
2) If Forces are all conservative, E_{Total} is conserved
(KE + PE)

Kinematics + coordinate systems (Taylor 1.2)

Laws of physics independent of choice of coord. system (!!)

(Vector notation helps make this clear)

• Cartesian ("rectangular") coordinates



$$\vec{r} = (x, y, z)$$

\vec{r} = "position vector"

$$= x \hat{i} + y \hat{j} + z \hat{k}$$

$$\text{or } x \hat{x} + y \hat{y} + z \hat{z}$$

$$\text{or } x \hat{e}_1 + y \hat{e}_2 + z \hat{e}_3$$

Unit vectors: $|\hat{x}| = 1$

(Unit vector has no units!)
no dimensions

It just "points".

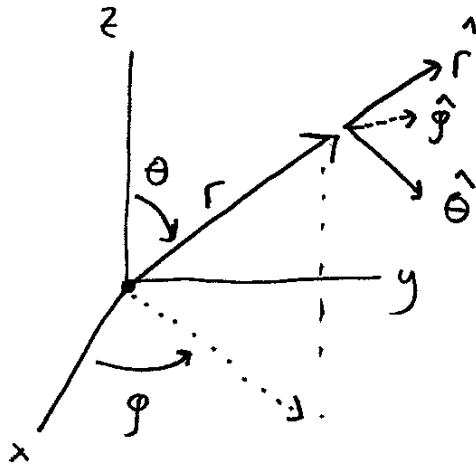
Generally, $\vec{A} = \sum_i A_i \hat{e}_i$

$$\vec{A} \cdot \vec{B} = \sum_i A_i B_i = |\vec{A}| |\vec{B}| \cos \theta_{\text{between } \vec{A} \text{ \& } \vec{B}}$$

$$= AB \cos \theta$$

A means $|\vec{A}| = \sqrt{\vec{A} \cdot \vec{A}}$

Spherical coords
(Taylor 1.7)



Note! θ, ϕ are opposite what math uses!
This is the physics convention.
(Taylor p. 134)

\hat{r} , $\hat{\theta}$, and $\hat{\phi}$ depend on where the point is (!)

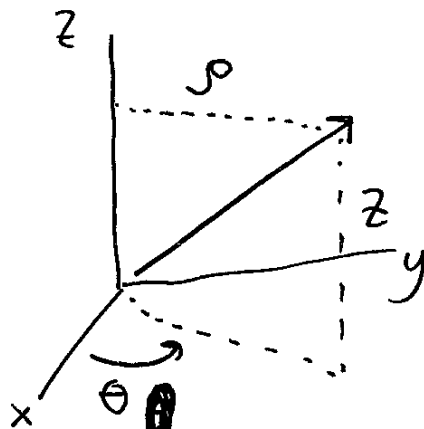
Each points in direction of increase in r , (or θ , or ϕ)

- \vec{r} = position vector (**Bold** in textbook)
- $r = |\vec{r}|$ is the distance from the origin.
- \hat{r} = unit vector pointing along \vec{r} , so $\hat{r} = \frac{\vec{r}}{r}$

cylindrical coordinates:

ρ = distance from
z-axis

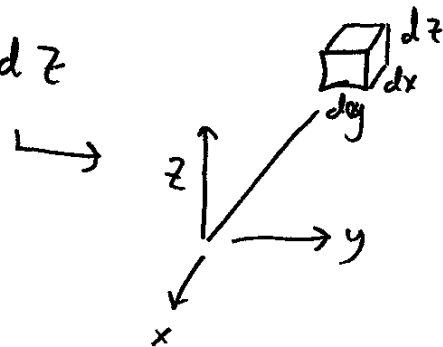
Now it is θ , not ϕ



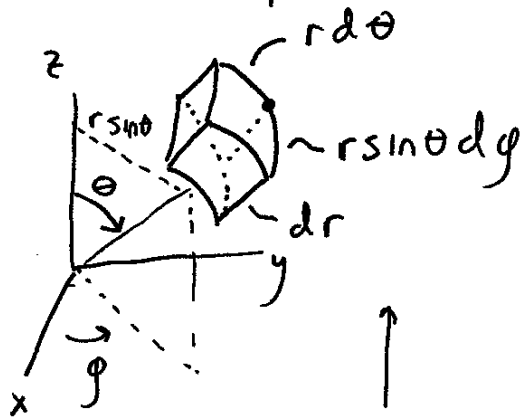
(Except sometimes we stick with ϕ here too, ack!)

When integrating (over surfaces or volumes), draw a picture! (See. Boas ch 5.4)

e.g. $d(\text{Volume}) = dx dy dz$



But, in spherical coords:



$$dV = r^2 \sin\theta d\theta d\phi dr$$

Note, $\theta \leftrightarrow \phi$ swap from math texts.

convince yourself!

Life is 3-D! $v = dx/dt$ is too simple, really

position $\vec{r} = x \hat{x} + y \hat{y} + z \hat{z}$ ↗ $\hat{x}, \hat{y}, \hat{z}$ are all constants in magnitude + direction, so

velocity $\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt} \hat{x} + \frac{dy}{dt} \hat{y} + \frac{dz}{dt} \hat{z}$ ↘

$$= v_x \hat{x} + v_y \hat{y} + v_z \hat{z}$$

Could write this as $\vec{v} = \dot{\vec{r}} = \text{"Time derivative of } \vec{r}\text{"}$

$$= (\dot{x}, \dot{y}, \dot{z})$$

[Boas will use ' (PRIME) for derivatives, so e.g.
 • if $f(x)$ is a function of x , then f' means $\frac{df(x)}{dx}$]

acceleration $\vec{a} = \frac{d\vec{v}}{dt} = \dot{\vec{v}}$

so in cartesian coords, $\vec{a} = (\ddot{x}, \ddot{y}, \ddot{z})$

Comments on Newton's Laws, p. 3 of these notes
(Taylor 1.4)

- They apply in inertial reference frames, non-accelerating.
(We'll talk about physics in non-inertial frames later!)
- They apply only if $v \ll c$ (non-relativistic speeds)
and for "macroscopic objects" (non-quantum mechanical)
- Newton's 3rd law even has issues in E&M with \vec{v} dependent forces.
It is saved if you consider EM fields too, as part of your system. This is upper-division E&M stuff!

Newton's Laws are empirical, experimental facts

They are meaningless without operational definitions of the kinematic variables, force, and mass

Force is a measure of interactions, the "push or pull" on an object.

$\vec{a} = \vec{F}/m$ is the "equation of motion" of an object.

Given \vec{x} & \vec{v} of all particles, \vec{F} can be computed, + thus \vec{a} which gives \vec{x} + \vec{v} at all future times.

Cornerstone of mechanics: predictive power!

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In 3-D Cartesian coordinates,

$$\vec{r} = x \hat{x} + y \hat{y} + z \hat{z}$$

$$\ddot{\vec{r}} = \ddot{x} \hat{x} + \ddot{y} \hat{y} + \ddot{z} \hat{z}$$

so $F_x = m \ddot{x}$

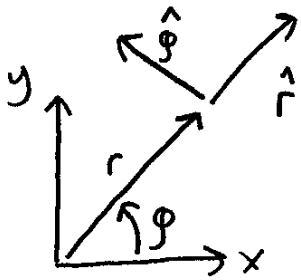
$$F_y = m \ddot{y}$$

$$F_z = m \ddot{z}$$

Separate eq'ns! Handy, and easy enough if you know \vec{F} . But sometimes we need to work in other coord systems!

Newton's Law in Polar - coordinates

(so, this is 2-D, basically same as 3-D cylindrical)



$$x = r \cos \phi$$

$$y = r \sin \phi$$

Look at figure + convince yourself:

$$\hat{r} = \cos \phi \hat{x} + \sin \phi \hat{y}$$

$$\hat{\phi} = -\sin \phi \hat{x} + \cos \phi \hat{y}$$

(Taylor 1.7, Boas 6.4)

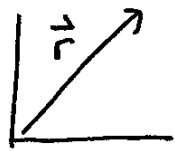
$$\text{Force } \vec{F} = F_r \hat{r} + F_\phi \hat{\phi}$$

So... does $F_r = m \ddot{r}$? does $F_\phi = m \ddot{\phi}$? No!!

Why not? Because \vec{r} is not $r \hat{r} + \phi \hat{\phi}$!!

(This can't be right, units don't even make sense.)

Look again



$$\vec{r} = \underbrace{r}_{\text{distance}} \hat{r}_{\text{direction}}$$

distance direction

now $\vec{v} = \dot{\vec{r}}$ is not $\dot{r} \hat{r}$! Why not? Because \hat{r} is itself a function of where the object is, so it has a derivative.

Taylor gives a "graphical" derivation, but I prefer a direct calculation:

Chain rule says $\frac{d}{dt} (f \cdot \vec{g}) = f \frac{d\vec{g}}{dt} + \frac{df}{dt} \vec{g}$

\uparrow scalar function \uparrow vector function

so $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt} (r \hat{r}) = \dot{r} \hat{r} + \underbrace{r \frac{d\hat{r}}{dt}}_{\text{something extra!}}$

what's $d\hat{r}/dt$? Go back to prev. page:

$$\hat{r} = \cos \phi \hat{x} + \sin \phi \hat{y} \quad \leftarrow \hat{x}, \hat{y} \text{ have no time dep}$$

$$\begin{aligned} \text{so } d\hat{r}/dt &= -(\sin \phi) \dot{\phi} \hat{x} + (\cos \phi) \dot{\phi} \hat{y} \\ &= \dot{\phi} (-\sin \phi \hat{x} + \cos \phi \hat{y}) \end{aligned}$$

$$= \dot{\phi} \hat{\phi} \quad \text{Cool! } (\hat{r} \text{ never changes length})$$

but direction can change (if ϕ varies with time)

Put it together, using our new identity $\hat{r} = \dot{\phi} \hat{\phi}$

$$\vec{v} = \dot{\vec{r}} = \frac{d}{dt} (r \hat{r}) \stackrel{\text{chain rule}}{=} \underbrace{\dot{r} \hat{r}} + \underbrace{r \dot{\phi} \hat{\phi}}$$

The 1st term is just "radial velocity"

The 2nd term is maybe familiar from 1110, where we used to call angular velocity " ω ", (now it's $\dot{\phi}$).

Remember $V_{\text{tangential}} = \omega r$?

That's our new term! $V_{\phi} = \dot{\phi} r$

$$\vec{a} = \dot{\vec{v}} = \frac{d}{dt} (\underbrace{\dot{r} \hat{r}} + \underbrace{r \dot{\phi} \hat{\phi}}) \rightarrow \text{chain rule gives 3 terms!}$$

$$= \ddot{r} \hat{r} + \dot{r} \dot{\hat{r}} + \dot{r} \dot{\phi} \hat{\phi} + r \ddot{\phi} \hat{\phi} + r \dot{\phi} \dot{\hat{\phi}}$$

we need $\dot{\hat{\phi}} = \frac{d}{dt} (-\sin\phi \hat{x} + \cos\phi \hat{y}) = \dot{\phi} (-\cos\phi \hat{x} - \sin\phi \hat{y})$

$$= -\dot{\phi} \hat{r} \leftarrow (\text{Look back at p. 9})$$

$$\text{so } \vec{a} = \ddot{r} \hat{r} + \dot{r} \dot{\phi} \hat{\phi} + \dot{r} \dot{\phi} \hat{\phi} + r \ddot{\phi} \hat{\phi} - r \dot{\phi}^2 \hat{r}$$

$$\vec{a} = \hat{r} (\ddot{r} - r \dot{\phi}^2) + \hat{\phi} (r \ddot{\phi} + 2 \dot{r} \dot{\phi})$$

Let's make sense of all these pieces!

$$\vec{a} = \hat{r} (\ddot{r} - r \dot{\phi}^2) + \hat{\phi} (r \ddot{\phi} + 2 \dot{r} \dot{\phi})$$

Pure "radial accel" \uparrow
 this is $-r\omega^2$ in 1110 language,
 $= -\frac{v^2}{r}$, centripetal accel!

\downarrow This looks like "r α " is
 1110 language, angular accel

Newton's 2nd law in Polar coords, then:

$$F_r = m (\ddot{r} - r \dot{\phi}^2)$$

This is N-2, or "eq'n of motion" for problems with

$$F_{\phi} = m (r \ddot{\phi} + 2 \dot{r} \dot{\phi})$$

"cylindrical symmetry."

Example Swing a rock on a string of length R

so $\dot{r} = \ddot{r} = 0$, and

$$F_r = -mR \dot{\phi}^2 \quad (\text{ah, that's just } "F_{\text{radial}} = \frac{-mv^2}{R} " !)$$

$$F_{\phi} = mR \ddot{\phi} \quad (\text{ah, that's just } "F_{\text{tangential}} = mR \alpha " !)$$

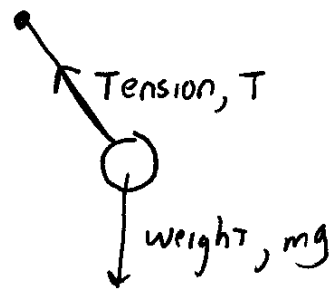
Example ϕ is fixed (no rotation)

$$F_r = m \ddot{r}$$

Basically just Newton's law in 1-D!

$$F_{\phi} = 0$$

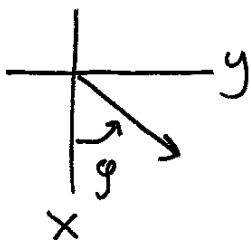
Example: Pendulum



We need to pick a coord. system

Polar seems natural!

But origin is down, I'd like that to be $\varphi = 0$, so let's tilt our head by 90° from previous pictures

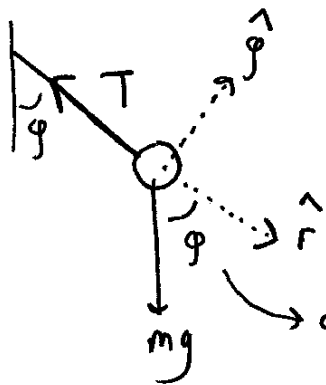


Now

$$F_r = m(\ddot{r} - r\dot{\varphi}^2)$$

$$F_\varphi = m(r\ddot{\varphi} + 2\dot{r}\dot{\varphi})$$

and r is fixed, it's R , so $\dot{r} = \ddot{r} = 0$



Look at picture: $\begin{cases} T_r = -T \\ T_\varphi = 0 \end{cases}$
convince yourself!

convince yourself this is also φ !

$$(Weight)_r = +mg \cos \varphi$$

$$(Weight)_\varphi = -mg \sin \varphi$$

N-2 says $\vec{F}_{net} = m\vec{a}$, or

$$(-T + mg \cos \varphi) = -mR\dot{\varphi}^2 \quad \leftarrow \text{this is } \sum F_r = m(\ddot{r} - r\dot{\varphi}^2)$$

$$-mg \sin \varphi = +mR\ddot{\varphi} \quad \leftarrow \text{this is } \sum F_\varphi = m(r\ddot{\varphi} + 2\dot{r}\dot{\varphi})$$

Phys 2a10 (14)

The eq'n of motion for φ is

$$\ddot{\varphi} = -\frac{g}{r} \sin \varphi$$

Familiar! If $\varphi > 0$, $\ddot{\varphi} < 0$, so

If $\varphi < 0$, $\ddot{\varphi} > 0$, so

Makes sense

If $\varphi = 0$, $\ddot{\varphi} = 0$. Stable equilibrium at bottom

If $\varphi = 180^\circ$, $\ddot{\varphi} = 0 \rightarrow$ (Unstable) equilibrium, (more later!)

For small φ , $\sin \varphi \approx \varphi$

\leftarrow more soon!

$$\text{so } \ddot{\varphi} = -\frac{g}{r} \varphi = -\omega^2 \varphi \quad \text{with } \omega = \sqrt{g/r}$$

This is an ordinary diff. eq with general sol'n

$$\varphi(t) = A \sin \omega t + B \cos \omega t. \quad \text{Simple Harmonic motion.}$$

More soon!
