

BASIC EQUATIONS OF ELECTRODYNAMICS

Maxwell's Equations

In General:

$$\nabla \cdot \bar{\mathbf{E}} = \frac{1}{\epsilon_0} \rho$$

$$\nabla \times \bar{\mathbf{E}} = -\frac{\partial \bar{\mathbf{B}}}{\partial t}$$

$$\nabla \cdot \bar{\mathbf{B}} = 0$$

$$\nabla \times \bar{\mathbf{B}} = \mu_0 \bar{\mathbf{J}} + \mu_0 \epsilon_0 \frac{\partial \bar{\mathbf{E}}}{\partial t}$$

In Matter:

$$\nabla \cdot \bar{\mathbf{D}} = \rho_f$$

$$\nabla \times \bar{\mathbf{E}} = -\frac{\partial \bar{\mathbf{B}}}{\partial t}$$

$$\nabla \cdot \bar{\mathbf{B}} = 0$$

$$\nabla \times \bar{\mathbf{H}} = \bar{\mathbf{J}}_f + \frac{\partial \bar{\mathbf{D}}}{\partial t}$$

Auxiliary Fields

Definitions:

$$\bar{\mathbf{D}} = \epsilon_0 \bar{\mathbf{E}} + \bar{\mathbf{P}}$$

$$\bar{\mathbf{H}} = \frac{1}{\mu_0} \bar{\mathbf{B}} - \bar{\mathbf{M}}$$

Linear Media:

$$\bar{\mathbf{P}} = \epsilon_0 \chi_e \bar{\mathbf{E}}, \quad \bar{\mathbf{D}} = \epsilon \bar{\mathbf{E}}$$

$$\bar{\mathbf{M}} = \chi_m \bar{\mathbf{H}}, \quad \bar{\mathbf{H}} = \frac{1}{\mu} \bar{\mathbf{B}}$$

Potentials

$$\bar{\mathbf{E}} = -\nabla V - \frac{\partial \bar{\mathbf{A}}}{\partial t}, \quad \bar{\mathbf{B}} = \nabla \times \bar{\mathbf{A}}$$

Lorentz Force Law

$$\bar{\mathbf{F}} = q(\bar{\mathbf{E}} + \bar{\mathbf{v}} \times \bar{\mathbf{B}})$$

Energy

$$\text{Energy: } U = \frac{1}{2} \int \left(\epsilon_0 \mathbf{E}^2 + \frac{1}{\mu_0} \mathbf{B}^2 \right) d\tau$$

FUNDAMENTAL CONSTANTS

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{Nm}^2$$

(permittivity of free space)

$$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$$

(permeability of free space)

$$e = 1.60 \times 10^{-19} \text{ C}$$

(charge of the electron)

$$m = 9.11 \times 10^{-31} \text{ kg}$$

(mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

$$\hat{x} = \sin \theta \cos \phi \hat{r} + \cos \theta \cos \phi \hat{\theta} - \sin \phi \hat{\phi}$$

$$\hat{y} = \sin \theta \sin \phi \hat{r} + \cos \theta \sin \phi \hat{\theta} + \cos \phi \hat{\phi}$$

$$\hat{z} = \cos \theta \hat{r} - \sin \theta \hat{\theta}$$

Cylindrical

$$x = s \cos \phi$$

$$y = s \sin \phi$$

$$z = z$$

$$\hat{x} = \cos \phi \hat{s} - \sin \phi \hat{\phi}$$

$$\hat{y} = \sin \phi \hat{s} + \cos \phi \hat{\phi}$$

$$\hat{z} = \hat{z}$$

FUNDAMENTAL THEOREMS

$$\text{Gradient Theorem: } \int_a^b (\nabla f) \cdot d\bar{l} = f(\bar{b}) - f(\bar{a})$$

$$\text{Divergence Theorem: } \int (\nabla \cdot \bar{\mathbf{A}}) d\tau = \oint \bar{\mathbf{A}} \cdot d\bar{a}$$

$$\text{Curl Theorem: } \int (\nabla \times \bar{\mathbf{A}}) \cdot d\bar{a} = \oint \bar{\mathbf{A}} \cdot d\bar{l}$$

VECTOR DERIVATIVES

Cartesian. $d\vec{l} = dx \hat{x} + dy \hat{y} + dz \hat{z}$; $d\tau = dx dy dz$

$$\text{Gradient: } \vec{\nabla} t = \frac{\partial t}{\partial x} \hat{x} + \frac{\partial t}{\partial y} \hat{y} + \frac{\partial t}{\partial z} \hat{z}$$

$$\text{Divergence: } \vec{\nabla} \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\text{Curl: } \vec{\nabla} \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{x} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{y} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{z}$$

$$\text{Laplacian: } \nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\vec{l} = dr \hat{r} + r d\theta \hat{\theta} + r \sin \theta d\phi \hat{\phi}$; $d\tau = r^2 \sin \theta dr d\theta d\phi$

$$\text{Gradient: } \vec{\nabla} t = \frac{\partial t}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial t}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial t}{\partial \phi} \hat{\phi}$$

$$\text{Divergence: } \vec{\nabla} \cdot \mathbf{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta v_\theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi}$$

$$\text{Curl: } \vec{\nabla} \times \mathbf{v} = \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta v_\phi) - \frac{\partial v_\phi}{\partial \theta} \right] \hat{r} + \frac{1}{r} \left[\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} (r v_\phi) \right] \hat{\theta} + \frac{1}{r} \left[\frac{\partial}{\partial r} (r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\phi}$$

$$\text{Laplacian: } \nabla^2 t = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial t}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 t}{\partial \phi^2}$$

Cylindrical. $d\vec{l} = ds \hat{s} + s d\phi \hat{\phi} + dz \hat{z}$; $d\tau = s ds d\phi dz$

$$\text{Gradient: } \vec{\nabla} t = (\partial t / \partial s) \hat{s} + (1/s)(\partial t / \partial \phi) \hat{\phi} + (\partial t / \partial z) \hat{z}$$

$$\text{Divergence: } \vec{\nabla} \cdot \mathbf{v} = \frac{1}{s} \frac{\partial}{\partial s} (s v_s) + \frac{1}{s} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\text{Curl: } \vec{\nabla} \times \mathbf{v} = \left[\frac{1}{s} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right] \hat{s} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s} \right] \hat{\phi} + \frac{1}{s} \left[\frac{\partial}{\partial s} (s v_\phi) - \frac{\partial v_s}{\partial \phi} \right] \hat{z}$$

$$\text{Laplacian: } \nabla^2 t = \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial t}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

VECTOR IDENTITIES

Triple Products

$$(1) \quad \vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$$

$$(2) \quad \vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

Product Rules

$$(3) \quad \vec{\nabla}(fg) = f(\vec{\nabla}g) + g(\vec{\nabla}f)$$

$$(4) \quad \vec{\nabla}(\vec{A} \cdot \vec{B}) = \vec{A} \times (\vec{\nabla} \times \vec{B}) + \vec{B} \times (\vec{\nabla} \times \vec{A}) + (\vec{A} \cdot \vec{\nabla})\vec{B} + (\vec{B} \cdot \vec{\nabla})\vec{A}$$

$$(5) \quad \vec{\nabla} \cdot (f\vec{A}) = f(\vec{\nabla} \cdot \vec{A}) + \vec{A} \cdot (\vec{\nabla}f)$$

$$(6) \quad \vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{B})$$

$$(7) \quad \vec{\nabla} \times (f\vec{A}) = f(\vec{\nabla} \times \vec{A}) - \vec{A} \times (\vec{\nabla}f)$$

$$(8) \quad \vec{\nabla} \times (\vec{A} \times \vec{B}) = (\vec{B} \cdot \vec{\nabla})\vec{A} - (\vec{A} \cdot \vec{\nabla})\vec{B} + \vec{A}(\vec{\nabla} \cdot \vec{B}) - \vec{B}(\vec{\nabla} \cdot \vec{A})$$

Second Derivatives

$$(9) \quad \vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$$

$$(10) \quad \vec{\nabla} \times (\vec{\nabla}f) = 0$$

$$(11) \quad \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$$