03 – Faraday's Law

Topics: Faraday's Law, fields of a solenoid with time-varying current.

Summary: Students first sketch the B-field for a long solenoid, and then consider whether there is a non-zero electric field anywhere in space when the current in the solenoid is changing with time. They then use Faraday's law in integral form to compute the electric field inside and outside the solenoid, and sketch the induced field as a function of distance from the center.

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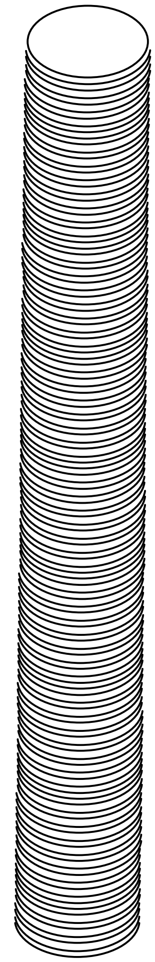
Comments: Most students should be able to finish these activities in less than 30 minutes. This is a shortened version of a tutorial on *EMF* from a series created by the University of Colorado for the first semester of this course. [#12, originally written by Mike Dubson and Ed Kinney, with contributions from Rachel Pepper.] The initial task assumes that students are familiar with the magnetic field of a long solenoid with constant current, and instructors may want to review this sometime prior to implementation; results from our pre-instruction assessment have shown that many students will need to re-familiarize themselves with this, particularly following a long break between semesters. The biggest conceptual difficulty for students has been with the idea that there is a non-zero electric field in a region of space where the magnetic field is zero (outside the solenoid). This can lead to good discussions on the difference between the differential and integral forms of Faraday's law. We have found it helpful to draw attention to the fact that a $1/s$ tangential field may look like a "curly" field, but in fact has zero curl, which can be verified by looking at the divergence in cylindrical coordinates. This is analogous to the divergence of a $1/r^2$ field being zero everywhere except at the origin (it looks like the field lines are "diverging" as they spread out away from the source). There have been a few students who were concentrating only on the electric field driving the current in the coil, and weren't thinking there could be an electric field anywhere but inside the wire. This can lead to interesting discussions about the relative magnitudes of the induced electric field and the field driving the current, and how this could depend on the dimensions of the solenoid or the rate of change in the current.



Consider a very long solenoid of radius ***R***, with *n* turns per unit length and carrying a current *I*.

(A) Make a sketch of the *magnitude* of the B-field as a function of distance from the center, both inside and outside the solenoid.

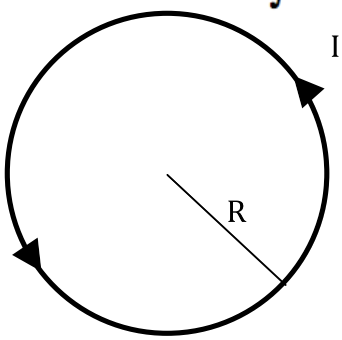
(B) From Ampere's Law, one can show that the B-field inside the solenoid is $B = \mu_0 nI$. [You can prove this later, if you have extra time.]

Suppose the current *I* in the coil of wire initially starts at a large value and is decreasing with time, so that the *B-field decreases* inside the solenoid according to the equation $B = B_0 - Ct$ (where C is a positive constant with appropriate units). Where in space is the **electric** field *zero*, and where is it *non-zero*? [Inside the solenoid? Outside? Everywhere?]

Make a quick sketch of what you think this induced E-field would look like. Just use your intuition for now, and we'll check with calculations later.

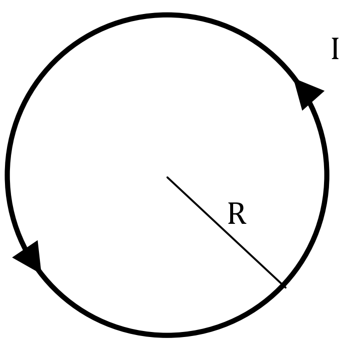
(C) Use Faraday's law in integral form to compute the electric field *inside* the solenoid. Be sure to specify in the diagram the surface you're using for the integrals, and the direction of the induced E-field.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\ell} = -\frac{d}{dt}\iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}}$$



Top View of Solenoid

(D) Use Mr. Faraday to compute the E-field *outside* the solenoid.



Top View of Solenoid

(E) Make a sketch of the magnitude of the *E-field* as a function of distance from the center, both inside and outside the solenoid.