Transformed E&M I materials

Separation of Variations in Cartesian and Spherical
(Griffiths Chapter 3)

TIMELINE

Prof A covers this in lectures 14-18.
Prof B. covers this in lecture 13-16.
Transformed course covered in lectures 14-19.

LEARNING GOALS

1. Students should be able to state the appropriate boundary conditions on V in electrostatics and be able to derive them from Maxwell's equations.
2. Students should recognize where separation of variables is applicable and what coordinate system is appropriate to separate in.
3. Students should be able to outline the general steps necessary for solving a problem using separation of variables.
4. Students should be able to state what the basis sets are for separation of variables in Cartesian and spherical coordinates (ie., exponentials, sin/cos, and Legendre polynomials.)
5. Students should be able to apply the physics and symmetry of a problem to state appropriate boundary conditions.
6. Students should be able to solve for the coefficients in the series solution for V, by expanding the potential or charge distribution in terms of special functions and using the completeness/orthogonality of the special functions, and express the final answer as a sum over these coefficients.

CLASS ACTIVITIES

DIscussion

Questions for Lecture (from UIUC)

1.) What are the physical reasons/motivation for wanting to solve the Laplace equation or the Poisson equation?

2.) What is the general form of the 1-D solution to Laplaces equation?
3.) The nature of Laplace's equation is such that it tolerates/allows NO local maxima or minima for V(r); all extrema of V(r) must occur at endpoints/boundaries. Why?

Whiteboards

Sinh and Cosh

Draw sinh and cosh, is it even/odd, what's the curvature, behaviour as x=0, infinity, what's cosh(pi), etc...
Show my mathematica solution in powerpoint (I did *not* show the 2D "0-V0-0-V0" boundary value problem, called "alternative_separation_variables.nb")

Whiteboards

Boundary conditions on E

One persistent difficulty that students have is an inability to recreate the mathematical steps to determine the boundary conditions on the parallel and perpendicular components of E. After watching 3 continuous semesters of this course, I strongly recommend having a whiteboard or worksheet activity where students are asked to derive those boundary conditions given a surface charge.

Simulation

Fourier

Show the PhET Fourier sim. Also show the mathematic notebook I made where we can *look* at the Legendre Polynomials. See http://phet.colorado.edu/index.php

Discussion

Separability

Make up a function f(x) and a function g(y) and come up with a pair that satisfies f(x)+g(y)=0. (You can't do it unless they're constants).

Tutorial

Electric Field Continuity across a Boundary
Oregon State University

Students working in small groups use Maxwell's equations to determine the continuity of the electric field across a charged surface

Discussion

Cartesian separation of variables.

Taking the first several terms in the series will give an approximation of the solution, much like in a Taylor expansion. How many terms should we keep to have a reasonable approximation of the potential? *This discussion question, along with mathematica plots, is contained in the Chapter 3 clicker question document.*

Discussion

Cartesian separation of variables
Rectangular Pipe 2 is twice as wide as rectangular Pipe 1 and the shaded face is at a potential V(x,y). Before applying the boundary conditions how will the solutions to Laplace's Equation be qualitatively different or similar? *This is included in Chapter 3 clicker question document.*

Worksheet
Identifying separable differential equations
Dawn Meredith "Meaning in Mathematics"
http://pubpages.unh.edu/~dawnm/connectm&m.html
This sheet is practice (not guided inquiry) on recognizing separable diff eqs. The second page gives students a chance to see why second order diff eqs are never separable.

Tutorial
Laplace's Equation
Paul van Kampen – Dublin University (Tutorials 9-16, page 10)
Tutorial on Laplace's equation. Conducting cylinder in E field. First describe what happens to the cylinder when placed in the E field. Do separation of variables in cylindrical coordinates.

Griffiths by Inquiry (Lab 2): Laplace's Equation and Boundary Value problems

Griffiths by Inquiry (Lab 3): 2D Boundary Value problems in Cartesian coordinates

Griffiths by Inquiry (Lab 4): 2D Boundary Value problems in Cylindrical coordinates

Griffiths by Inquiry (Lab 5): 3D Boundary Value problems in electrostatics