

Transformed E&M I homework

Conductors + Capacitors (Griffiths Chapter 2)

Conductors

Question 1. Questions about conductors

DEEPER QUESTIONS: Questions about Conductors (see P435_Lect_05_QA.pdf in Univ of Illinois Questions for Lecture). May be some good extension questions for HW here.

Question 2. Gauss's law and cavities

Griffiths 2.36 (p. 101)

Please be sure to explain your reasoning on all parts, and add to this the problem the following:

In part a, be sure to *sketch* the charge distribution.

In part c, *sketch* the E fields (everywhere in the problem, i.e. in the cavities and also outside the big sphere)

f) Lastly - if someone moved q_a a little off to one side, so it was no longer at the *center* of its little cavity, which of your answers would change? Please explain.

I really like this problem, lots of good physics in it! Try to think it through carefully, make physical sense of all your answers, don't just take someone else's word for it! "

Assigned in SP08 (average score: 9.5)

Assigned in FA08

Instructor notes: Many people can get the right answer but not explain why. Especially problematic was the "force of q_1 on q_2 " question.

Question 3. Coax capacitors

In HW 3 you found the E field everywhere in and around a coaxial cable. We can slightly modify that problem, making it more realistic by letting the inner cylinder be a conductor (a wire). The inner conducting cylinder has radius a and the outer conducting cylindrical shell has inner radius b . It is physically easy to set up any fixed potential difference ΔV between the inner and outer conductors. In practice, the cable is always electrically neutral.

A) Assuming charge per length $+\lambda$ and $-\lambda$ on the inner and out cylinders, derive a formula for the voltage difference ΔV between the cylinders.

B) Assuming infinite cylinders, find the energy stored per length (W/L) inside this capacitor. Once again, let's do it two ways so we can check:

i) Integrate the energy density stored in the E field. Write your answer in terms of ΔV

ii) Find the capacitance per length (C/L) of this system, and then use stored energy

$$W = \frac{1}{2} C (\Delta V)^2$$

Based on your answers above, *where* in space would you say this energy is physically stored?

C) Estimate the capacitance per meter of the coaxial cable that the cable company uses to send TV signals into homes.

D) This is also an excellent model for "axons", which are long cylindrical cells (basically coax cables) carrying nerve impulses in your body and brain. Estimate the *capacitance* (in SI metric units, Farads) of your sciatic nerve.

Assumptions - the sciatic nerve is the longest in your body, it has a diameter of roughly 1 micron, and a length of perhaps 1 m. Note that axons generally have a value of b which is very close to a (i.e. the gap is extremely tiny, $b-a$ is about 1 nanometer.) so you can simplify your expression using $\ln(1+\epsilon) \approx \epsilon$

Assigned in SP08 (average score: 8.25)

Assigned in FA08

Instructor notes: Some students didn't notice that the question was "energy/length" and were concerned about the infinite amount of energy in an infinitely long cylinder.

Question 4. Static charge shock

Suppose you walk across your rug on a dry winter's day, and see a centimeter-long blue spark jump from your finger to the door-knob. Let's estimate the energy involved and the number of electrons that passed between you and the knob.

A) Make a very rough estimate of the capacitance between you and the door knob, just before the spark jumped. State your assumptions clearly. HINT: *very rough*.

B) Knowing the breakdown voltage of air, estimate the energy stored in the charged human/door knob capacitor just before the spark jumped.

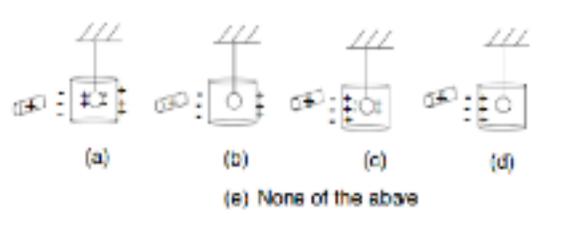
C) Estimate the charge on you, just before the spark. Give your answer in coulombs and in number of electrons.

Assigned in FA08

Question 5. Charge distribution on ball near metal cylinder

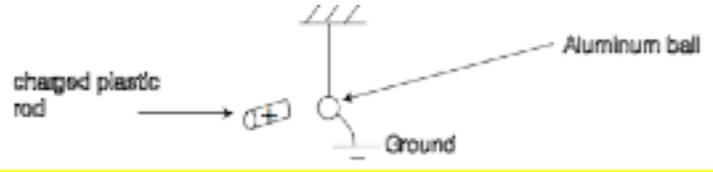
CONDUCTOR (C. Singh) – CONCEPT TEST?

A small aluminum ball hanging from a thread is placed at the center of a tall metal cylinder and a positively charged plastic rod is brought near the aluminum ball in such a way that the metal wall is in between the rod and the ball. Which of the following is the correct qualitative picture of the charge distribution in this situation?



Question 6. Force between ball and charged rod

CONDUCTOR (Singh) – CONCEPT TEST? A positively charged plastic rod is taken close to (but not touching) a neutral aluminum ball hanging from a thread. The aluminum ball is grounded by connecting it with a wire that touches the ground. Which of the following statements is true about the force between the ball and the rod?

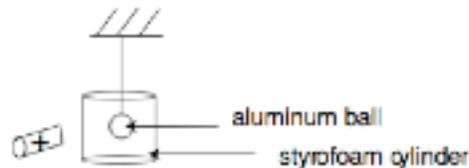


- A. The aluminum ball will not feel any force due to the charged rod
- B. The aluminum ball will be attracted to the charged rod and the force of attraction is the SAME as that without the grounding wire
- C. The aluminum ball will be attracted to the charged rod and the force of attraction is MORE than that without the grounding wire
- D. The aluminum ball will be attracted to the charged rod and the force of attraction is LESS than that without the grounding wire
- E. The aluminum ball will be attracted or repelled depending upon the magnitude of the charge on the rod.

Question 7. Force between insulator and charged rod

CONDUCTOR – CONCEPT TEST? (COPYRIGHT C. Singh)

A small aluminum ball hanging from a thread is placed at the center of a tall cylinder made with a good insulator (Styrofoam) and a positively charged plastic rod is brought near the ball in such a way that the Styrofoam wall is in between the rod and the ball. Which one of the following statements is true about this situation?



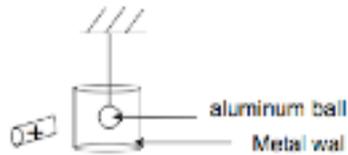
- A. The aluminum ball will not feel any force due to the charged rod
- B. The aluminum ball will be attracted to the charged rod and the force of attraction is the SAME as that without the Styrofoam cylinder
- C. The aluminum ball will be attracted to the charged rod but the force of attraction is MORE than that without the Styrofoam cylinder
- D. The aluminum ball will be attracted to the charged rod and the force of attraction is LESS than that without the Styrofoam cylinder
- E. None of the above

Question 8. Force between conductor and charged rod

CONDUCTOR – CONCEPT TEST? (COPYRIGHT C. Singh)

A small aluminum ball hanging from a thread is placed at the center of a tall METAL cylinder and a positively charged plastic rod is brought near the ball in such a way that

the metal wall is in between the rod and the ball. The distance between the rod and the ball is the same as in the previous problem when the metal wall was not there. Which one of the following statements is true about this situation?

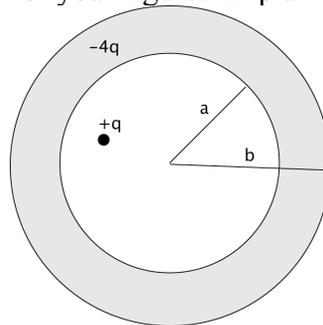


- F. The aluminum ball will not feel any force due to the charged rod
- G. The aluminum ball will be attracted to the charged rod and the force of attraction is the SAME as that without the metal cylinder
- H. The aluminum ball will be attracted to the charged rod but the force of attraction is MORE than that without the metal cylinder
- I. The aluminum ball will be attracted to the charged rod and the force of attraction is LESS than that without the metal cylinder
- J. None of the above

Question 9. Charge on conducting shell

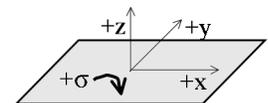
CONDUCTOR (Singh)

A spherical conducting shell has a and b as the inner and outer radii. A charge $+q$ is placed off-center in the cavity, and a charge $-4q$ is placed on the conductor. (a) Show the equilibrium charge configuration by drawing on the figure below. Explain your reasoning. (b) Write an expression for the electric field in each region and sketch the electric field lines in each region of your figure. Explain your reasoning.

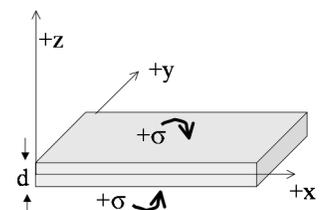


Question 10. Flat slabs and conductors

a) Griffiths (Ex 2.4) works out the E-field everywhere in space due to an infinite thin plane of charge, with surface charge density σ . Let's assume our sheet lies in the x-y plane. Sketch the z-component of the E field as a function of z (include both $+$ and $-z$)



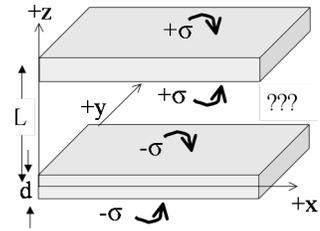
b) Now we have an infinite *conducting slab* of metal which has a charge density $+|\sigma|$ on each surface (both top *and* bottom) Write a mathematical function for $\mathbf{E}(z)$ (and again sketch the result) for the E-field everywhere in space. (This slab is oriented like the sheet in part a, parallel to the



xy plane, and is still infinite in the x-y directions, but *this* slab has a finite thickness d , in the z-direction).

Briefly state what would *change* if the charge density had been negative, i.e. $-|\sigma|$ on *each* surface, top and bottom.

c) Now imagine *two* such conductors, well separated by a distance $L \gg d$. Suppose, for the sake of this problem, that the upper one could still be exactly as described in part b (i.e. it has $+|\sigma|$ on each surface, top and bottom) and the lower one is like part b only with $-|\sigma|$ on each surface, top and bottom) Use the principle of superposition to find the E field everywhere in space. (That means between the slabs, outside either slab, but also *INSIDE* each slab).



As you may recall from Phys 1120, the E field inside of a slab of metal is supposed to be zero. So... what do you suppose might have gone wrong? Have you applied superposition right? Is Gauss' law invalid here? How does nature resolve this inconsistency?

Assigned in SP08 (average score: a) 7.28, b) 67.28, c) 6.24)

Instructor notes: Many students make the mistake of drawing vector fields instead of graphs, suggesting that the problem could be reworded. Several students also put the Gaussian surface entirely within the slab, thus wrongly concluding that the flux through the surface is zero, giving the wrong answer for part (c). Students require a lot of expert discussion on part (c). One student said nature's resolution is that there are never infinite slabs.

Capacitors

Question 11. Van de Graaf

REAL-WORLD, CAPACITOR

A Van de Graaf is a spherical conducting shell, with a rotating belt that carries charge to the inside. A wire connects the belt to the inside of the sphere, so that negative charge jumps out onto the wire (this needs to be written clearly). Thus, we have a uniformly charged sphere. What is the capacitance of a sphere 20 cm in diameter? If V is 30 kV (measured by breakdown of air), what's amount of charge on sphere? What is V or E outside surface of sphere. What's E inside. What's V inside sphere (30 KV). *Voltage constant, but E field zero.*

Question 12. Earth as spherical capacitor ; thunderstorms

CAPACITOR. REAL-WORLD. (Lorrain 3.15)

Earth's electric field maintained by thunderstorms. Earth as spherical capacitor. Surface of earth is conductor and so is ionosphere, so you have a concentric spherical capacitor. Thunderstorms are a battery that charges capacitor. The ground is negative and the ionosphere is positive, during course of day the V on capacitor varies sinusoidally. Peak of charge occurs at one time of day, 4pm in India, because most lightening storms over Indian ocean. Can do real calculation of capacitor with diameter of earth separated

by d of ionosphere. What's dielectric constant of air? See Feynman lectures on lightning.

Question 13. Earth as spherical capacitor; Farads

REAL-WORLD; UNITS (From Wangsness, Electromagnetic Fields, Problem 6-3)
The farad is actually an enormous unit of capacitance. To illustrate this, treat the Earth as a conducting sphere and find its capacitance.

Question 14. Faraday's ice pail experiment

CAPACITORS, POLARIZATION

Look up Faraday's ice pail experiment. They probably have this in demos. Glass jar, filled with conducting water, wrap in al foil and charge. Capacitor. Remove glass jar from Al foil and pour out water. Another one... the glass jar has an induced polarization and that's stored and it discharges into new one. Nice intro to polarization.

Question 15. Capacitance of non-parallel plate capacitor

LIMITING CASES, APPROXIMATION, CALCULATION (From Wangsness, Electromagnetic Fields, Problem 6-13)

Suppose that a parallel plate capacitor has rectangular plates but the plates are not exactly parallel. The separation at one edge is $d - a$ and $d + a$ at the other edge, where $a \ll d$. Show that the capacitance is given approximately by

$$C \approx \frac{\epsilon_0 A}{d} \left(1 + \frac{a^2}{3d^2} \right)$$

where A is the area of the plate.

Question 16. Capacitance of coaxial cylinders; nerve axons

ESTIMATION (U WASHINGTON)

4* Calculate the capacitance per unit length of coaxial cylinders of radius R_{inner} and R_{outer} which carry equal and opposite charge per unit length of magnitude λ . Define the distance $d \equiv R_{outer} - R_{inner}$. For an axon (a cable which carries nerve impulse in the body) d is much smaller than either R_{inner} or R_{outer} . Show that, in this case, the capacitance per unit area of cable is given to a good approximation by ϵ_0/d . Evaluate this (in F/m²) for a reasonable $d = 1 \text{ nm} = 1 \times 10^{-9} \text{ m}$.