

# Transformed E&M I homework

## Electric Field, Coulomb's Law (Griffiths Chapter 2)

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### Coulomb's Law, E field

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#### Question 1. Plotting field of electric dipole

COMPUTATION (Munsat)

*Plotting the field of an electrostatic dipole*

[http://plasma.colorado.edu/mathcad/ES\\_Dipole\\_plot.mcd](http://plasma.colorado.edu/mathcad/ES_Dipole_plot.mcd)

This exercise begins by using the vector field plot to display the field of a single charge and of a pair of charges. For the dipole, the lines of force are followed by integrating a short differential distance  $ds$  that is parallel to the  $E$  field. Euler's method is used for the integration. Field line plots are also made in the Magnetostatics Exercises below.

#### Question 2. Sketch of different vector fields

VISUALIZATION, FIELDS (Berkeley, Stamper-Kurn)

Here we will practice the use of field differential operators and visualize some fields.

- Find the gradient of the scalar potential  $\phi(x, y, z) = \alpha xy$ . Provide a clear sketch of the contour lines of  $\phi$  in the  $\hat{x}$ - $\hat{y}$  plane and a representation of its gradient field. Such a field is known as a radial quadrupole field, and is used in the focusing of charged particle beams and of dipolar (electric or magnetic) particles.
- Provide a clear sketch (in the  $\hat{x}$ - $\hat{y}$  plane) of the vector field expressed in cylindrical coordinates as  $E = \alpha \rho \hat{\rho}$  where  $\alpha$  is a constant. Calculate its divergence. If  $E$  is an electric field, what charge distribution generates it?
- Consider a large sheet placed on a turntable that is set rotating at angular velocity  $\Omega$ . What vector field  $v$  describes the local velocity on this sheet? Provide a clear sketch of this field. What is the curl of this vector field (now expressed as a vector in a three-dimensional space, i.e. including a coordinate for the  $z$  axis)? If a magnetic vector potential  $A$  is defined with the functional form of  $v$  (just a different constant out front), what magnetic field  $B = \nabla \times A$  does it represent?
- Now generate new vector fields by taking your vector field  $v$  obtained in part (c) above, and adding (1) a constant field, and (2) a radial quadrupole field (from part (a)). Provide clear sketches of these two fields and compare with that of  $v$ . Show explicitly that the curl of all these fields is the same. Note that these three fields all represent forms of the magnetic vector potential that describe the same magnetic field.

#### Question 3. Charge density and E field of atmosphere

REALWORLD (U WASHINGTON)

6.\* The electric field in the atmosphere at the earth's surface is approximately 200 V/m, directed downward. At 1400m above the earth's surface, the electric field in the atmosphere is only 20 V/m, again directed downward.

- What is the average charge density in the atmosphere below 1400 m?
- Does this consist predominantly of positive or negative ions?

#### Question 4. Electric field of finite rod

CALCULATION, LIMITING CASE (U WASHINGTON)

5\* Consider a positively charged rod (charge per unit length  $\lambda$ ) in the  $x y$  plane which lies between the coordinates  $x = 1, y = 1$  and  $x = -1, y = 1$ . A negatively charged rod (charge per unit length  $-\lambda$ ) lies in the same plane between coordinates  $x = 1, y = -1$  and  $x = -1, y = -1$ . Calculate the magnitude and direction of the electric field along the  $z$  axis. Show that for  $|z| \gg 1$ , the field reduces to that of a dipole with the appropriate dipole moment. For a collection of charges, the dipole moment is

$$\mathbf{p} = \sum_i q_i \mathbf{r}_i.$$

For a continuous (and linear) distribution of charge, this becomes

$$\mathbf{p} = \int \mathbf{r} \lambda(\mathbf{r}) d\ell.$$

#### Question 5. Fermi estimation

ESTIMATION

Fermi-estimate problems – estimate voltage knowing distance.... Estimate charge or current.

#### Question 6. Nonuniform surface charge

We typically consider cases in class where charge is distributed completely uniformly, but it's not always that way - let's consider an insulating sphere (radius  $R$ ) with surface charge density  $\sigma = \sigma_0 \sin^2(\theta) \cos^2(\phi)$ .

- Find the total charge on this sphere.
- Describe briefly in words and pictures what this charge distribution "looks like".
- Then, briefly but clearly, describe a *procedure* to find  $\mathbf{E}(x,0,0)$  (for  $x > R$ )?

**Note:** you do not need to come up with a final closed-form answer! We just want a discussion, with formulas, of how you would proceed. Get as far as you reasonably can, but stop when the going gets too nasty, and discuss what you would do next if you really *needed* to know this  $\mathbf{E}$  field in, say, a laboratory/experimental situation.

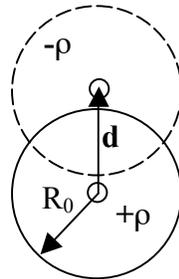
Assigned in SP08 (average score: 7)

Assigned in FA08

Instructor notes: This problem was given in the Gauss' Law section to see if they knew when they could use Gauss' Law (which you can't in this situation). About 40% of students used Gauss' Law for part (c). Students still struggled with setting up "dA" and "curly R". In part (a) several students still want to write  $Q = \sigma * A$ , not recognizing that you have to do an integral. This is also a good problem for connecting the physics to vector calculus. What does it mean to do an integral at one point in space  $(x, 0, 0)$ ? Several students wanted to set  $E_y$  and  $E_z$  equal to zero because at  $(x, 0, 0)$   $y=z=0$ . However,  $E_y=E_z=0$  because of this charge distribution, not because of the point of evaluation. Note that the problem as-worded does not indicate whether  $\sigma$  is + or -.

### Question 7. Field of overlapping charged spheres

CALCULATION (Griffiths, Introduction to Electrodynamics, 3rd edition, Problem 2.18)  
 You have two spheres. The first is centered at the origin, has uniform positive charge density  $\rho$ , and radius  $R_0$ . The second has uniform negative charge density  $-\rho$ , same radius  $R_0$ . It is shifted up by a distance  $d$ .



A) Show that the field in the region of overlap of the two spheres is constant, and find its value. (Please check that the units are correct)

[Hint: You will first want to figure out what the E field is in a *single* sphere with uniform charge density  $\rho$ . It is definitely not zero, nor is it uniform. Another possible hint:

$$\vec{r}_1 - \vec{r}_2 = \vec{d} \text{ for suitable } \vec{r}_1 \text{ and } \vec{r}_2.]$$

B) In the limit that  $|d|$  becomes small compared to  $R_0$ , discuss in words and make a sketch of what the resulting (total, physical) charge distribution in space really *looks* like (so that later in the course when we encounter such a charge distribution, we will know where it came from and what the E field looks like inside!)

Assigned in SP08 (average score: a) 7.8, b) 8.08)

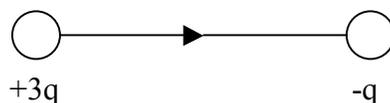
Assigned in FA08

Instructor notes: Many students forget that a constant E field has both constant magnitude and *direction*. On part (b), most students indicate that the configuration gradually becomes a neutral ball.

### Question 8. Sketch of E field of two charges

SKETCH; FIELD LINES (Downloaded from Reed)

a) Sketch the field lines for the configuration of charges shown below. I drew one to get you started. Make sure that you are consistent with the number of field lines that you attach to each charge. Think about what the field should look like (how many field lines there should be) far away from the charges.



**b)** In the previous question, assume the  $+3q$  charge is located at  $x = -D$  and the  $-q$  is located at  $x = +D$ . What is a *fourth order* approximation to the electric field  $E(0,y,0)$  for points far away, but on the  $y$  axis, i.e.  $|y| \gg D$ ? ("fourth order" means expanding in some small quantity, then in the end keeping only terms out to  $(\text{small thing})^4$ )

For what values of  $y$  does your series converge?

For what values of  $y$  is your approximation a good one?

Check (show us!) that the units of your answer are correct, that the direction of your  $E$  field makes sense, and that the limiting behavior ( $y \rightarrow$  infinity) makes sense.

*Note: Approximating (in a problem when we have the exact answer!) may seem a little odd, but we'll find as we move on that this is often the only way to practically solve ever more complex problems!*

**Assigned in SP08 (average score a) 7.78 b) 6.35)**

**Assigned in FA08**

Instructor comments (a): The most common error on student work is that the field line density is not shown as 3:1.

Instructor comments (b): Most students know how to write the expression, but some of them do not expand the final result, just giving a complex expression.

### **Question 9. Electron in annulus of charge**

COULOMB; INTEGRATION, SUPERPOSITION

If an electron is placed near an annulus of charge (a disk of charge with a hole in the middle), describe what will happen to the electron's motion. Be specific in terms of its behavior at different locations around the disk and explain your reasoning with respect to electric potential. *It will oscillate if it's placed on axis because of symmetry, but anywhere else it will get stuck to the disk.*

### **Question 10. Angle between two suspended charges**

COULOMB REPULSION (deGrand)

Two charges of mass  $m$  and charge  $q$  and  $2q$  hang from strings of length  $l$  from a common point. Find the expression for the angle  $\theta$  each makes with respect to the vertical.

### **Question 11. Electric field of charged disk**

COULOMB; INTEGRATION (deGrand and Griffiths 2.5 and 2.6)

*DISK OF CHARGE*

**a)** Find the electric field a distance  $R$  along the axis from a disc of radius  $R_0$  and uniform charge density  $\sigma$ . [Hint: a disk can be thought of as a bunch of concentric rings. Can you first find the  $E$  field a distance  $R$  along the axis up from a thin ring of charge  $dq$ ?]

**b)** Explicitly calculate the limiting forms of your solution at very small and at very large  $R$  (compared to  $R_0$ ) and discuss.

*Note: The disk of charge is an idealization of many physical devices: a capacitor plate, a small patch of any surface... Once you have solved this ideal problem, you will be able to apply it (many times this term!) to more realistic situations.*

Assigned in SP08 (average score a)9.04 b)6.65)

Assigned in FA08

Instructor comments: Most students can get the exact solution and they can see that for  $r \ll R$  it approximates an infinite plane. But for  $r \gg R$  they often don't keep the first non-zero term in the expansion (instead just stating that  $E=0$ ). Also, many struggle with the formal expansion here, not sure just what is "small."

### Question 12. Electric field of charged sphere 1

COULOMB; INTEGRATION (deGrand)

Find the electric field a distance  $R$  away from the center of a sphere of uniform charge density  $\sigma$  and radius  $R_0$  (with  $R > R_0$ ) WITHOUT using Gauss' law. (With Gauss' law this is trivial.) I found that the intermediate steps contained messy expressions. Explain the limiting form of your solution as  $R$  approaches  $R_0$  (including the 2).

### Question 13. Electric field of charged sphere 2

COULOMB; INTEGRATION; ESTIMATION; REAL WORLD (modified Griffiths 2.7, similar to deGrand above)

a) Find the electric field a distance  $z$  from the center of a spherical surface of radius  $R$  (Fig 2.11 in Griffiths) which carries a uniform surface charge density  $\sigma$ . Do this by explicit integration (i.e. starting from Griffiths Eq. 2.7), please.

You only need to treat the case  $z > R$  (*outside* the sphere). Express your answer in terms of the total charge  $q$  on the sphere.

[Hint: Use the law of cosines to write  $r$  in terms of  $R$  and  $\theta$ . I got an integral which I needed to look up - do you have access to tables, or Mathematica, or some other such tool? In the end, be careful if you get a square root to take the *positive* root:

$$\sqrt{R^2 + z^2 - 2Rz} = (R - z) \text{ if } R > z, \text{ but it's } (z - R) \text{ if } R < z].$$

b) Check your answer using knowledge from 1120 and intuition (briefly discuss - what *should* it be?) What would you expect the answer should be *inside* the sphere? Why?

c) Estimate the maximum charge you can put onto a child's balloon.

*Note that estimation problems give you a lot of freedom. How big is a balloon? Why should there be a maximum at all? That's key: if the balloon sparks, it can bear no more charge. Can you look up the breakdown electric voltage of air somewhere?*

*Your answer can be \*very\* rough, we only want an order of magnitude.*

Assigned in SP08 (average score a)8.43 b)7.17 c)4.04)

Assigned in FA08

Instructor comments: People were largely able to do the sphere of charge (part A), although at first they were all intimidated by the integral. Many people did not attempt the estimation problem (part C), left it blank, or gave a guess without a reason.

### Question 14. Table top physics

ESTIMATION, REAL-WORLD

"Table-top physics"

I started off the first day of class saying "electricity holds your table together". Let's make a crude order of magnitude estimate to think about this.

A table is built of a lattice of atoms, with typical atom-atom spacing of a few Angstroms (1 Angstrom = 0.1 nm) When you make crude order of magnitude estimates, you can make crude, simplistic models! Let's *model* this table in a very crude way - imagine the atoms are ionically bonded, with neighbors alternating +e, -e, +e, -e (thus, any given atom is strongly attracted to its nearest neighbor, holding the table together) What the heck, let's start by considering this to be one dimensional. (After all, what could considering the nastier 3 dimensional situation do for us? Change a few factors of 2?)

- What is the typical FORCE of attraction between two nearest-neighbor atoms, in N?
- Now you push on the table, applying pressure (units, F/m<sup>2</sup>) Assuming this spreads out evenly, estimate (crudely!) how much pressure must you apply to start breaking typical atomic bonds?
- Convert your pressure to lb/in<sup>2</sup>. Does this seem like a remotely realistic answer?
- Suppose instead you start rubbing the table, removing a few stray surface electrons.

### Question 15. Charge density of disk

CHARGE DENSITY; INTEGRATION. (OSU Paradigms, Symmetry, HW 4)

- You are part of the team building *Cosmic AC*, Asimov's ultimate, universe-sized computer. Your job is to fabricate a charged disk, 10 meters in radius and 1 cm thick. The charge density on the disk should be:

$$\rho = \alpha e^{-\beta r^2} \cos(\gamma z)$$

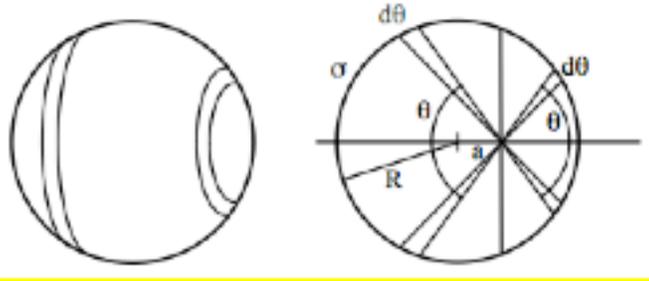
- What is the total charge on the disk, in terms of the parameters  $\alpha$ ,  $\beta$ , and  $\gamma$ ?
- What are the dimensions of  $\alpha$ ,  $\beta$ , and  $\gamma$ ?
- Design specifications indicate that: the maximum charge density should be  $27 \frac{\text{C}}{\text{cm}^3}$ , only one-half period of the  $\cos(\gamma z)$  term spans the whole width of the disk, the upper and lower circular surfaces are have zero charge density (note wording change), and the maximum values of the charge density on the ends of the disk should be 10 percent of the maximum in the center. Find values for  $\alpha$ ,  $\beta$ , and  $\gamma$ .
- What is the total charge on the disk?
- Estimate how much error you would make in your calculation of the total charge density if you assumed that the disk was infinitely wide.
- What is the surface charge density on the disk?

### Question 16. Electric field of charged sphere

SYMMETRY, PROOF (C. Gwinn, UC Santa Barbara)

One way to find the electric field inside a uniformly charged sphere is to divide the sphere into rings, and find the E-field from each ring. Of course, you know the answer: the net E-field is zero everywhere inside the sphere. A hollow spherical shell has radius R and surface charge  $\sigma$ . We wish to find the electric field at a "field point" inside the

sphere, displaced from the center by a distance  $a$ . Consider the pair of rings shown below. The 2 rings are at an equal angle  $\theta$  from the axis containing the field point and the center, when viewed from the field point. Their angular width, viewed from that point, is  $\delta$ .



a) Show (without using calculus) that the electric fields from the two rings cancel: they are equal and opposite. This is true for every pair of rings that make up the sphere's surface. Therefore, the net electric field anywhere inside the sphere is zero. The electric field is the product of the area of the ring, a cosine factor, and the inverse square of the ring's distance from the field point. The area of the ring is the product of the solid angle subtended by the ring, the square of its distance from the field point, and a tilt factor.

Interestingly, the solid angles subtended by the two rings are equal. Perhaps more surprisingly, the tilt factors are equal as well. Prove these facts, then complete your calculation of the net electric field.

Newton used this construction to find the gravitational field inside a mass shell in his Principia. Also without using calculus, he showed that the gravitational field outside a uniform spherical shell is the field of an equal point mass, located at the sphere's center. This proof is somewhat tricky and still excites debate: see R. Weinstock 1984, Am.J.Phys., 52, 10; H. Ehrlichson 1990, 58, 3.

<http://www.merlyn.demon.co.uk/gravity1.htm> has a nice exposition, but is unavailable as of this writing. Reputedly, Newton thought calculus should be kept secret, and held up publication of Principia until he could work out these non-calculus proofs. He published calculus much later, about the same time as Leibniz published his calculus.

### Question 17. Angle Between Suspended Charges

Two charges of identical mass  $m$ , one with charge  $q$ , the other with charge  $2q$ , hang from strings of length  $l$  from a common point. Assume  $q$  is sufficiently weak so that any angle you're looking for is very small, and find an approximate expression for the angle  $\theta$  each charge makes with respect to the vertical. Check (show us!) that the units work out, and that the limiting behavior for large mass, large length, and/or small  $q$  are at least sensible.

**Assigned in SP08 (average score: 8.09)**

**Assigned in FA08**

Instructor comments: We get few questions on this, except about the small angle approximation (and why we could ignore the non-leading correction to cosine).