

Transformed E&M I homework

Current Density (Griffiths Chapter 5)

Question 1. Current densities

- A) A solid cylindrical straight wire of radius a has a current I flowing down it. If that current is uniformly distributed over the outer surface of the wire (none is flowing through the "volume" of the wire; it's all surface charge), what is the surface current density \mathbf{K} ?
- B) Suppose that current *does* flow throughout the volume of the wire, in such a way that the volume current density \mathbf{J} grows quadratically with distance from the central axis, what then is the formula for \mathbf{J} everywhere in the wire?
- C) A CD has been rubbed so that it has a fixed, constant, uniform surface electric charge density σ everywhere on its top surface. It is spinning at angular velocity ω about its center (which is at the origin). What is the surface current density \mathbf{K} at a distance r from the center?
- D) A sphere (radius R , total charge Q uniformly distributed throughout the volume) is spinning at angular velocity ω about its center (which is at the origin) What is the volume current density \mathbf{J} at any point (r, θ, ϕ) in the sphere?
- E) A very thin plastic ring has a constant linear charge density, and total charge Q . The ring has radius R and it is spinning at angular velocity ω about an axis thru its center (which is at the origin) and perpendicular to the plane of the ring. What is the current I , in terms of given quantities?

Assigned in SP08 (average score: a) 1.94, b) 1.53, c) 1.71, d) 3.18, e) 2.76)

Assigned in FA08

Instructor notes: Many students don't know how to think about the vector direction of \mathbf{J} or \mathbf{K} . Some thought that \mathbf{v} was in the direction of ω ; few had thought of $\mathbf{v} = \omega \times \mathbf{r}$, and NONE spontaneously knew even what the direction of ω for a spinning disk or sphere or ring was, and thought that ω pointed in the ϕ direction. Several were confused about how to deal with the last part, the one with delta functions in r and z .

Question 2. Magnetic field of spinning charged sphere

Lorrain, Dorson, Lorrain, 14-14 pg. 261

A conducting sphere of radius R is charged to a potential V and spun about a diameter at an angular velocity ω .

(a) Show that the surface current density is $\alpha = \epsilon_0 \omega V \sin \theta = M \sin \theta$, where M is $\epsilon_0 \omega V$.

(b) Find the magnetic flux density B_0 at the center.

(c) What is the numerical value of B_0 for a sphere 100 millimeters in radius, charged to 10.0 kilovolts, and spinning at 10,000 turns per minute?

(d) Show that the dipole moment is $\frac{4}{3} \pi R^3 M \hat{z}$ where \hat{z} is a unit vector along the axis, related to the direction of rotation by the right-hand screw rule.

(e) What current flowing through a loop 100 millimeters in diameter would have the same dipole moment?

Question 3. Estimate drift speed

Sanjoy Mahajan

(a) Estimate the drift speed of electrons in a wire that feeds a typical house lamp. (b) How long does it take for electrons to flow from the light switch to the bulb? (c) So how come the lamp turns on so fast when you flip the switch?

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E) A very thin plastic ring has a constant linear charge density, and total charge Q . The ring has radius R and it is spinning at angular velocity ω about an axis thru its center (which is at the origin) and perpendicular to the plane of the ring. What is the current I , in terms of given quantities?

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