

Transformed E&M I homework

Electric Displacement (Griffiths Chapter 4)

Electric displacement

Question 1. Field patterns from bound charges

Let's go back to the "solid" teflon rod (radius a), but let's change the polarization once again - now assume it is uniform and parallel to the z -axis, i.e. $\mathbf{P} = k \hat{\mathbf{z}}$. This time, let's also keep it finite, and consider realistic "end effects"

a) Sketch the electric field lines, inside and outside, in two different limits: First, when the cylinder is very LONG compared to its diameter, and second, when it is very SHORT compared to its diameter. Explain in words and formulas your physical reasoning behind the sketches.

b) In both of the above two limiting cases, sketch (and explain in words) \mathbf{P} everywhere in space, and also \mathbf{D} everywhere (inside and outside the rods)

c) In both of the above limits, explain how Griffiths' "boundary condition" (page 178) Eq 4.26 and 4.28 work out near the top end of the rod.

Also (just briefly, *very* qualitatively, e.g. simply in terms of signs or directions of these fields) explain how boundary conditions 4.27 and 4.29 makes sense/agree with your sketches in part b. (Here, you only need to really consider the outer edge of the middle of the cylinder at $z=0$ i.e. at $(s=a, z=0)$ if the rod runs from $z=-L$ to $z=+L$)

Such an object is referred to as a "bar electret", it is an electrical analogue of a permanent bar magnet. Do you see why, from your sketches? Griffiths points out that this is quite unusual, most materials cannot maintain such a permanent electric polarization in the absence of an external field, although he claims they do exist. But if you did have one, it would attract charged ions from the air which would negate the interesting fields you found above.

Assigned in SP08 (average score: a) 4.24, b) 7.09, c) 4.90)

Instructor notes: This is a hard problem for students and many leave it blank. Some students struggle with how to find the \mathbf{E} field or \mathbf{D} field. This was especially troublesome for the long thin rod, where they hadn't realized they could estimate \mathbf{E} in various "limiting regions" and thence get \mathbf{D} . Many think that \mathbf{E} field is uniform inside the rod in the limiting case of a long rod.

Question 2. \mathbf{D} and \mathbf{E} of dielectric slab at an angle to external \mathbf{E}

GAUSS' LAW; CALCULATION (U. Nauenberg; solutions available)

Consider an uncharged dielectric slab of material with dielectric constant ϵ , so placed that the normal to the slab is at an angle of 30 degrees to the direction of an incident electric field " \mathbf{E} ". Obtain mathematical expressions describing

the magnitude and direction of the “E” and “D” fields, and of the dipole moment “P” inside the dielectric.

Question 3. Varying dielectric constant

GAUSS’ LAW; CALCULATION (U. Nauenberg, solutions available)

A dielectric material separates two conducting plates. The area of the plates is “A” and a charge “+Q” is placed on one plate and “-Q” on the other. The separation between the plates is “t”. Assume that the area of the plates is much larger than the separation so that you can neglect edge effects. The dielectric constant varies linearly between the plates as follows:

$$\epsilon = \epsilon_0 + kx \quad 0 \leq x \leq t$$

- Find expressions for the Displacement field “D” and the dipole moment “P” between the plates.
- Find expressions for the “E field” everywhere, between the plates and on the outside of the plates.
- Obtain an expression for the potential difference between the plates.

Question 4. Dielectric sphere

GAUSS’ LAW; CALCULATION (U. Nauenberg, solutions available)

A sphere of radius “R” is made up of a dielectric material with dielectric constant ϵ and contains a uniform free charge density per unit volume ρ_f . Show that the potential at the center is given by:

$$\phi(r = 0) = \frac{4\pi}{3} \frac{2\epsilon + 1}{2\epsilon} \rho_f R^2$$

Question 5. Concentric spherical shells with dielectric liquid

BOUNDARY CONDITIONS; CALCULATION (U. Nauenberg, solutions available)

Consider two concentric conducting spherical shells of radii a and b as shown in the figure. The space between them is half filled with a liquid having a dielectric constant “ ϵ ”. A total charge of “+Q” is placed in the inner shell and “-Q” in the outer shell.

This is a beautiful example of the power of the uniqueness theorem. Be careful to satisfy the boundary conditions everywhere, including the boundary between the dielectric and the air in between the spherical shells. If you think a bit about satisfying the boundary conditions you will notice that the solution for the “E” field is very simple.

- Find the magnitude and direction of the fields E, D, P everywhere; namely for $r > b$, $a < r < b$, and $r < a$.
- Determine expressions for the free surface charge on each conducting shell everywhere (namely, on the dielectric side, and the air side).
- Determine expressions for the bound charge at $r = a$ and $r = b$.
- Determine the potential everywhere.

Question 6. Charge density and E field of charged dielectric cylindrical shell

GAUSS’ LAW, ESTIMATIONS (OSU Paradigms, Vector Fields HW9)

We have solutions to this in OSU folder

2. A positively charged dielectric cylindrical shell of inner radius a and outer radius b has a cylindrically symmetric internal charge density

$$\rho = 3\alpha \exp(kr^2)$$

where α and k are constants with appropriate units.

- (a) Write the volume charge density everywhere in space as a single function, sketch the charge density, and find the total charge.

Assume b is large enough (what does this mean?) that there are charges of BOTH signs of charge in your shell.

- (b) Use Gauss's Law and symmetry arguments to find the electric field in each of the regions given below:

- (i) $r < a$
- (ii) $a < r < b$
- (iii) $r > b$

- (c) Pick sensible values of α and k (what values do you choose and what units?) and sketch the r -component of the electric field as a function of r .

3. Referring to the charge distribution in the previous problem, take the limit as $a \rightarrow b$ so that the shell becomes infinitely thin, but keeping the total charge Q constant.

- (a) Give a formula for the charge density everywhere in space.

Be careful: Integrate your charge density to get the total charge as a check.

- (b) Use Gauss's Law and symmetry arguments to find the electric field at each region given below:

- (i) $r < b$
- (ii) $r > b$

- (c) Using your previous values of α and k , sketch the r -component of the electric field as a function of r .

CHALLENGE:

4. Take the limits of the shell in the previous problem as $a \rightarrow b$ and then $b \rightarrow 0$, so that the shell becomes a line charge, but keeping the total charge Q constant.

- (a) Give a formula for the charge density everywhere in space.

Be careful: Integrate your charge density to get the total charge as a check.

- (b) Use Gauss's Law and symmetry arguments to find the electric field for $r > 0$.