Transformed E&M I homework

Fieldo f Magnetized Object (Bound Currents)
(Griffiths Chapter 6)

Field of magnetized object, bound currents

Question 1. Bound current always a surface current in uniform material?
Pollack and Stump, 9-22 pg. 353
Is the following statement true or false? In a uniform material with magnetic susceptibility X_m and electric conductivity 0, and bound current distribution can only be a surface current (assume no time dependence).

Question 2. B due to conducting slab
Pollack and Stump, 9-12 pg. 351
A conducting slab, parallel to the xy plane and extending from z = -a to z = a, carries a uniform free current density $J_f = J_0\hat{i}$. The magnetic susceptibility is 0 in the slab and X_m outside. Determine the magnetic field and the bound current distribution.

Question 3. B near nonmagnetic wire in water
Pollack and Stump, 9-14 pg. 352
A long straight wire of circular cross section with radius r_0 carries current I and is immersed in a large volume of water. The wire is nonmagnetic. Calculate the magnetization **M**(**x**) in the water. (The susceptibility is -9.0 x 10^{-6}.) What is the bound current density $\mathbf{J}_b(\mathbf{x})$ for r> r0? Calculate the total current (free plus bound). [Answer: $I(1+X_m)$]

Question 4. Bound currents I
A) Consider a long magnetic rod (cylinder) of radius a. Imagine that we have set up a permanent magnetization inside, $\mathbf{M}(s,\phi,z) = k\ \hat{\mathbf{z}}$, with k=constant. *Neglect end effects, i.e., assume the cylinder is infinitely long.* Calculate the bound currents $\mathbf{K}_b$ and $\mathbf{J}_b$ (on the surface, and interior of the rod respectively). What are the units of "k"? Use these bound currents to find the magnetic field **B** inside and outside the cylinder (direction and magnitude). Find the **H** field inside and outside the cylinder, and verify that Griffiths' Eq. 6.20 (p. 269) works. Explain briefly in words why your answers for **B** and **H** are reasonable.

B) Now relax the assumption that the rod is infinitely long; consider a cylinder of *finite* length.

Sketch the magnetic field **B** (inside and out) for *two* cases: one for the case that the length *L* is a few times bigger than *a* (a "long-ish" rod), and second for the case $L<<a$ (which is more like a magnetic *disk* than a rod, really). Briefly but clearly explain your reasoning.

C) Consider again the "long-ish" magnetized rod. Sketch **H**. Are there any free currents in this problem? Do you see any inconsistency in your answers?
*(Hint: It is probably useful to also sketch **B** and **M** in separate diagrams.)*

Assigned in SP08 (average score: a) 9.63, b) 9.7)
Assigned in FA08
Instructor notes: Student difficulties with Ampere's Law resurface here as they struggle to use the bound current to obtain the B field, thus reconstructing the same Amperian arguments to get the B field for a solenoid. Some reverted to Biot-Savart. In part (b) students are concerned, but then reason through, the changes resulting when the solenoid is finite.

Question 5. Bound currents II

Like the last question, consider a long magnetic rod, radius a. This time imagine that we can set up a permanent *azimuthal* magnetization $\mathbf{M}(s,\phi,z) = c\, s\, \hat{\varphi}$, with c=constant, and s is the usual cylindrical radial coordinate. *Neglect end effects, assume the cylinder is infinitely long.*
Calculate the bound currents $\mathbf{K}_b$ and $\mathbf{J}_b$ (on the surface, and interior of the rod respectively). What are the units of "c"? Use these bound currents to find the magnetic field **B**, and also the **H** field, inside and outside. (Direction and magnitude) Also, please verify that the *total bound* current flowing "up the cylinder" is still zero.

Assigned in SP08 (average score: 93.25%)
Assigned in FA08
Instructor notes: As in "Bound Currents I", students struggle to get the B-field from K and J. There was some discussion on how to get the direction of B given the bound current.

Question 6. B in cylinder

Reitz, Milford, Christy, 9-11 pg. 253
A long cylinder of radius a and permeability μ is placed in a uniform magnetic field B_0 such that the cylinder axis is at right angles to B_0. (a) Calculate the magnetic induction inside the cylinder. (b) Make a semiquantitative sketch showing typical lines of induction through the cylinder. (Assume from the beginning that φ^* can be completely specified in terms of the $\cos\theta$ cylindrical harmonics. This assumption is justified, since all boundary conditions can be satisfied in terms of the $\cos\theta$ harmonics.)

Question 7. B in wire – extension of Question 6 above

Reitz, Milford, Christy, 9-12 pg. 253
A long straight copper wire and a long straight iron wire each carry the same current *I* in a uniform B-field, B_0. Show that the force on the iron wire is nearly twice the force on the copper wire. [Hint: Use the result of Problem 9-11.]

Question 8. Right-circular cylinder magnet

Reitz, Milford, Christy, 91 pg. 252

A permanent magnet has the shape of a right circular cylinder of length L. The magnetization **M** is uniform and has the direction of the cylinder axis. (a) Find the magnetization current densities, J_m and j_m. (b) Compare the current distribution with that of a solenoid.

Question 9. Magnetic field of coaxial cylinders with insulating material

CALCULATION; ANSWER CHECK (deGrand)

Two coaxial cylinders of inner radius a and outer radius b are separated by an insulating material of susceptibility χ_M. A current I flows down the inner conductor and returns through the outer one; in both cases the current distributes itself uniformly over the surface. Find the magnetic field between the two conductors. Check your answer by finding all the bound currents and the magnetization and show that they and the free current generate the correct field.

Question 10. Current required to magnetize iron

Reitz, Milford, Christy, 9-9 pg. 253

A toroid of annealed soft iron has a mean length of 0.1 m and is wound with a coil of 100 turns. Calculate the current required to magnetize the specimen to a field strength of (a) B=1.0 T, (b) B = 1.5 T.

Question 11. Correcting the trajectory of a satellite in orbit

Lorrain, Dorson, Lorrain, 16-16 pg. 291

This problem concerns both magnetic materials and magnetic forces, which are discussed in Chapter 17.

Someone suggests the following device for exerting a force on a satellite in orbit. A loop of wire fixed to the satellite carries an electric current. Say the loop is rectangular. Since the earth's magnetic field in the satellite is essentially uniform, the net magnetic force exerted on the loop is zero. But if you shield one side with a high-permeability tube, there will be a net magnetic force exerted on the coil.

What is your opinion? Refer to problem 16-13.

See Problem 18-3 on the magnetic braking of satellites.

Question 12. The crossed-field photomultiplier

Lorrain, Dorson, Lorrain, 17-4 pg. 317

Figure 17-16 shows the principle of operation of a crossed-filed photomultiplier. A sealed and evacuated enclosure contains two parallel plates called *cynodes*. They provide the electric field **E**. An external permanent magnet superimposes the magnetic field **B**.

A photon ejects a low-energy photoelectron. The electron accelerates upward, but the magnetic field deflects it back to the negative dynode. At this point t ejects a few secondary electrons, and the process repeats itself. Eventually, the electrons impinge on the collector C.

Let us find the value of a.

(a) Find the differential equations for v_x and v_y. The trajectory is not circular. You can simplify the calculation by setting Be/m=ω_c, the cyclotron frequency.

(b) Find v_x as the function of y.

(c) You can now find y, and then x, as functions of t. Set t = 0, dx/dt = 0, and dy/dt = 0 at x = 0, y = 0. You should find that the trajectory is a cycloid.

(d) What is the maximum value of y?

(e) What is the value of a?

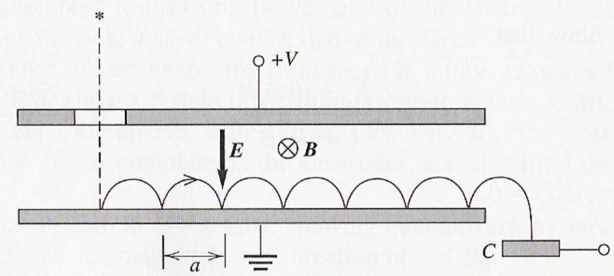

Figure 17-16

Question 13. B of magnetic dipole

SKETCH, CALCULATION (deGrand)

Suppose the dipole moment $\vec{m}$ is oriented in the $\hat{z}$ direction. Evaluate $\vec{B}$ explicitly in spherical coordinates and sketch it.

Question 14. Vector potential / magnetic field from dipole

MATH PROOF (deGrand)

Starting with the formula for the vector potential from a magnetic dipole

$$A(r) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

Show that you can write the magnetic field as

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\left(-\frac{\vec{m}}{r^3} + 3\frac{(\vec{m}\cdot\vec{r})\vec{r}}{r^5}\right)$$

Question 15. Magnetic field of magnetized cylinder

CALCULATION; ALTERNATE METHOD OF SOLVING PROBLEM (deGrand)

An infinitely long cylinder of radius R oriented along the z-axis has an intrinsic magnetization $M = \hat{z}kr$. (a) [15 points] Find all the bound currents and compute B directly from them. (b) [5 points] Repeat the calculation using H and Ampere's law.

Question 16. Magnetic scalar potential of magnetized sphere

CONNECTIONS WITH OTHER AREAS OF PHYSICS; LIMITING CASES (deGrand)

Return again to the uniformly magnetized sphere of radius *R*, with $M = \hat{z}M_0$. Compute the magnetic scalar potential *U* for points on the z-axis. Consider separately the cases z > *R* and z < *R*. Use the fact that *U* is a solution of Poisson's equation to write down the solution for *U* for arbitrary radius *r* and polar angle θ (with respect to the z-axis).

Question 17. Magnetic scalar potential of loop of wire

EXPANSION, CONNECTION TO OTHER AREAS OF PHYSICS (deGrand)

Find the magnetic scalar potential for a single circular loop of wire of radius *a* carrying a current I. The easiest way to do this is to begin by finding the field on the symmetry axis, then moving off axis by using the Legendre expansion. Consider the cases *r* < *a* and *r* > *a* separately.

Question 18. Refrigerator magnets and atom mirrors

CALCULATION, REAL-WORLD, REASONING. (Berkeley; Stamper-Kurn)

This is an excellent problem, though hard to read. You can see the original PDF in the Berkeley folder in "other universities."

Refrigerator magnets and atom mirrors:

Most of you have probably seen large, flexible refrigerator magnets. These are flat magnetic materials, up to a couple of inches on a side, that stick to an iron refrigerator door by inducing magnetization in the door with the magnetic fields emanating from the surface of the magnet.

a. Suppose you have such a magnet with a circular shape, and that magnet is uniformly magnetized, with a magnetization out of the plane. What is the magnetic field on the surface of the magnet at its center, as a function of the radius of the magnet? On the basis of this observation, explain why this is not how refrigerator magnets are magnetized.

The magnetization in such a refrigerator magnet is actually a periodic linear array of oppositely oriented magnetization. Let us consider the field produced by such a magnetization. Specifically, consider a magnet that fills the space −d < z < 0, being infinite in extent in the ˆx and ˆy directions. Let the magnetization M be oriented in the plane of the magnet, varying as

M = M sin kx ˆx (1)

where M gives the maximum value of the magnetization and k the spatial wavevector of its modulation.

b. Show that this magnetization can be represented by bound surface currents at z = 0 (top layer) and z = −d (bottom layer).

c. Consider just the field due to the top layer of bound currents. In the region z no currents, and therefore ∇ · B = 0 and ∇ × B = 0. Thus, we can define a scalar magnetic potential Vm for that obeys ∇ 2 Vm = 0 and B = −∇Vm. Considering these relations, the boundary condition at z → ∞, and the y-independence of this problem, show that Vm can be written as

$$V_m(\mathbf{r}) = \pm V_0 \cos(kx + \phi) e^{-|k|y}$$

where the ± refer to solutions for Vm either above or below the layer of currents. The values of V0 and φ are still to be determined. Hint: recall solutions to the Laplace equation that are separated by Cartesian coordinates.

d. Now obtain an expression for the magnetic fields from the top layer of currents, still containing unknown quantities V0 and φ. Use Ampere's law to determine those quantities and thus to fully determine the fields.
e. So far you have determined the field from the top layer. Now add the fields from the bound currents on both layers of the magnet. What is the magnitude of the field on the surface of the magnet (z = 0)?
Aside from binding to refrigerators, such periodic magnetizations can also be used as mirrors for beams of atoms with magnetic moments. See, for instance, Roach et al., Physical Review Letters 75, 629 (1995).

Question 19. Magnetic scalar potential

CALCULATION, REAL-WORLD, REASONING. (Berkeley; Stamper-Kurn)
The magnetic scalar potential
Maxwell's equations for the magnetic field (Ampere's law, in particular) clearly demonstrate that the magnetic field cannot be defined via a scalar potential.
Further, there are no magnetic charges, and, hence, no magnetic charge density.
Nevertheless, such quantities are useful even though they are solely fictional. Here we consider the use of the magnetic charge density ρm and the magnetic scalar potential Vm as devices to determine the fields from magnetized materials.
a. We start with the field of the ideal magnetic dipole. In class, we found out that the electric field of an ideal electric dipole and the magnetic field of an ideal magnetic dipole are quite similar in form, the only difference being the singular field at the position of the dipole. Considering the relation between these two fields, argue why one can write the field from a magnetic dipole as

$$B(r) = -\nabla Vm(r) + \mu 0\ M\ (r)\ (3)$$

and determine the magnetic scalar potential Vm in this case.
b. Now consider the field produced by a magnetization M. Using Eq. 3, and following our
treatment of the electric field from polarized ob jects, show that the magnetic scalar potential in this case would be defined as

$$V_m(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_S \frac{\sigma_M(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}da + \frac{\mu_0}{4\pi}\int_V \frac{\rho_M(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3r'$$

Here V denotes the volume of the magnetized ob ject, S its surface (with normal vector ˆn), and ρM = −∇ · M and σM = M · ˆn. The reason why this analogy works so far is that there are, as of yet, no free currents in the systems we're considering. Now we add those currents back in, and we find, of course, that the magnetic field can no longer be given just as the gradient of a potential.
c. Give expressions for the B and H fields in the presence of both a magnetization M and a free current density J, still using the magnetic scalar potentials described in parts (a) and (b).