Transformed E&M I homework

Magnetic Fields and Forces (Lorentz Force Law)

(Griffiths Chapter 5)

Lorentz Force Law

Question 1. Estimate drift speed

Sanjoy Mahajan

(a) Estimate the drift speed of electrons in a wire that feeds a typical house lamp. (b) How long does it take for electrons to flow from the light switch to the bulb? (c) So how come the lamp turns on so fast when you flip the switch?

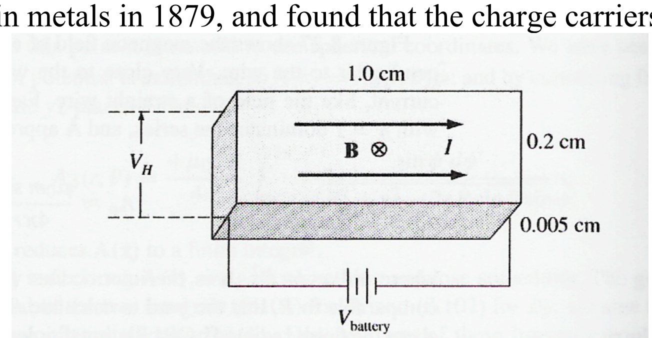
Question 2. Hall probe

Pollack and Stump, 8-1 pg. 296

A Hall probe is a device for measuring magnetic fields. In a typical device a current *I* flows in a ribbon of n-type doped semiconductor located in the magnetic field. In an n-type doped semiconductor the charge caries are electrons. In the steady state there is a voltage difference V_H between the edges of the ribbon, such that the net perpendicular force on a charge carrier $qE_\perp + q(v \times B)_\perp$ is 0. From V_H one can determine **B**.

(a) Assume the geometry shown in Fig. 8.28, with dimensions 1.0 cm by 0.2 cm by 0.005 cm. The material is As-doped Si, with 2 x 10^{15} conduction electrons per cm^3, and resistivity 1.6 Ω cm. The current is driven by a 3V battery across the 1.0 cm length. What Hall voltage will be measured across the 0.2 cm width if the field strength is 0.1 T? [Answer: V_H=11.7 mV]

(b) Show that the sign of V_H depends on the sign of the charge carriers. Edwin H. Hall measured V_H in metals in 1879, and found that the charge carriers are negative.



Question 3. Cycloid motion

LORENTZ, RELATIVITY, CONNECTIONS (Griffiths' Ex. 5.2 modified by Berkeley

Stamper-Kurn)
Cycloid motion: obtains a cycloid solution for motion of a charged particle in crossed electric and magnetic fields using the Lorentz force law in the laboratory frame. Read this example, and then solve the same problem using an alternate approach:

a. From relativity, but in the limit of velocities small compared to the speed of light, if frame S moves with velocity β c with respect to (laboratory) frame

S , the electric field is transformed as $\mathbf{E}' = \mathbf{E} + \vec{\beta} \times c\mathbf{B}$
Taking B along ˆx and E along ˆz, as in the text, and assuming E =cB in magnitude, choose the simplest β so that E vanishes.
b. The two frames coincide at t = 0. Using the Galilei transformation for velocities (valid for β << 1), if the particle is at rest at t = 0 in frame S , what is its initial velocity in frame S ?

c. The magnetic field transforms in this case as $c\mathbf{B}' = c\mathbf{B} - \vec{\beta} \times \mathbf{E}.$
Because E = cB and β << 1, approximate B to be equal to B. Now, in frame S solve for the motion of the particle. Using the Galilei transformation for positions, determine the motion of the particle in the lab frame.

Question 4. Circular coil oscillating in external B field

Pollack and Stump, 8-33 pg. 304
A circular coil (radius **R** and mass **M**) has **N** turns of wire carrying current I_0. The coil is free to rotate about the *z* axis, which lies in the plane of the coil and passes through its center, as shown in Fig. 8.36. There is a constant magnetic field $B = B_0\hat{i}$. Initially, the coil is in stable equilibrium with its magnetic moment parallel to the field.
(a) What is the frequency for small angle oscillations in ϕ about the equilibrium position?
(b) Now let **M** = 0.10 kg, **N** = 100 turns, I_0 = 0.1 A, and B_0=0.05 T. Evaluate the frequency of small angle oscillations.
(c) If the coil is released from rest at the small angle ϕ_0 = 15 degrees, what is its angular velocity as it swing through the equilibrium position?

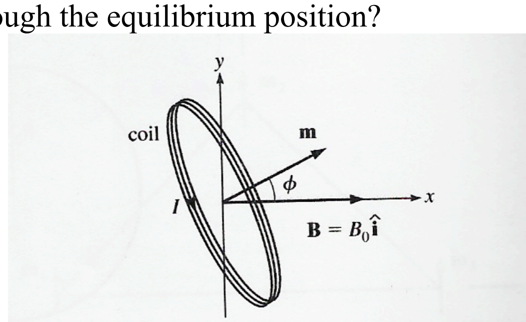


FIGURE 8.36 **Exercise 33.** Coil in a magnetic field.

Question 5. Oersted's experiment

Purcell, 6-9 pg. 246

Describing the experiment in which he discovered the influence of an electric current on a nearby compass needle, H. C. Oersted wrote: "If the distance of the connecting wire does not exceed three-quarters of an inch from the needle, the declination of the needle makes an angle of about 45^{o}. If the distance is increased the angle diminishes proportionally. The declination likewise varies with the power of the battery." About how large a current, in amperes, must have been flowing in Oersted's "connecting wire"? Assume the horizontal component of the earth's field in Copenhagen in 1820 was the same as it is today, 0.2 gauss.

Question 6. Quadrupole focusing

Pollack and Stump, 8-4 pg. 297

A magnetic Quadrupole field can be used as a focusing field for a charged particle beam. The cross section of the pole faces is shown in Fig. 8.29. The pole faces are hyperbolas of the form xy = constant. There are two north poles and two south poles, marked on the figure. The dimension of the magnet perpendicular to the cross section is ℓ. The magnetic field in the region $0 \le z \le \ell$ is

$$\mathrm{B}(x,y,z) = b(y\hat{i} + x\hat{j})$$

in which b > 0; the field is 0 for z < 0 and z > ℓ. Particles enter from negative z with velocity $v_0 = v_0\hat{k}$ and are deflected by the force $F = qv_0 \times B$. (Neglect the small components v_x and v_y in calculating the force.)

(a) Sketch the **B** field lines in the xy plane.

(b) Explain qualitatively why **B** produces focusing in the x direction and defocusing in the y direction, assuming the beam particles are positively charged.

(c) Write the equations of motion for a beam particle with charge q and mass m, using the approximate force given above. Solve for x as a function of z for z > 0, assuming $x = x_0$ and $v_x = 0$ at z = 0. Sketch a graph of x(z).

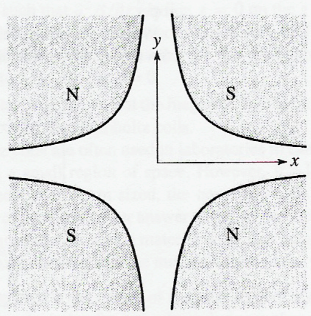


FIGURE 8.29 **Exercise 4.** A quadrupole focusing magnet. The hyperbolic curves are the boundaries of the pole faces. A charged particle beam moves out of the page.

Question 7. Unification

Along with the necessary addition made by Maxwell, Ampere's law in full glory reads

$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$ (We haven't talked much about the second term yet, since we've been focused on statics this term, but it's there!)
Take the divergence of this equation, and show that electric charge is conserved globally. *I think* both *these results are pretty cool, and carry very deep messages about the nature and unification of electricity and magnetism,* and *their connection to special relativity!*

Assigned in SP08 (average score: 9.78)
Assigned in FA08
Instructor notes: This question is problematic and should be rewritten to be used. It is too abstract and turned into a math problem without much sense making. Most students did not remember the current conservation formula. Even as a math problem, I saw them e.g. moving d/dt past the divergence, and when I asked them "why is that legit", they couldn't tell me! And very few students seemed to notice that Del dot del cross B vanishes!

Question 8. Force on long solenoid

DISCONTINUITIES, REAL-WORLD, EXTENSION (From Reitz, Milford and Christy, Foundations of Electromagnetic Theory, 4th Ed., Problem 8.5)
Suppose a very long solenoid carries a current of 10 A and has 1000 turns per cm. Find the radial force per unit length f on one turn of the winding. Careful: the magnetic field for an infinite solenoid is discontinuous – how did we handle such discontinuities to determine the pressure on the surface of a charged conductor? Such forces pose severe design constraints on the construction of large, typically superconducting, electromagnets.

Question 9. Bubble chamber

REAL WORLD (COPYRIGHT C. Singh)
This could be a clicker and lead into a HW question. Could ask them to estimate momentum of electron using a ruler.
The figure shows the track of an electron in a bubble chamber which is immersed in a uniform magnetic field that points out of the plane of the paper. Which one of the following statements is the most convincing reason why the radius of the trajectory of the elctron is decreasing (it is spiraling inward)?

(a) the work done on the electron by the magnetic field decreases its velocity
(b) Initially the electron's velocity makes an angle *less* than 90^0 to the magnetic field so the electron loses energy until the velocity becomes perpendicular to the field
(c) Initially the electron's velocity makes an angle *greater* than 90^0 to the magnetic field so the electron loses energy until the velocity becomes perpendicular to the field
(d) Collisions with the gas present in the bubble chamber decreases the electron's velocity.
(e) The electron's inertia (mass) prevents its radius from decreasing all at once to the final radius required to equlbrate with the magnetic field present

Question 10. Magnetic monopoles

Suppose magnetic monopoles DO exist. (They *might*! We just haven't found direct experimental evidence for them yet)

First, start off by just writing down Maxwell's equations in electro- and magneto-statics and also write down the usual Lorentz force law for a charge in E and B fields.

Now *modify these (5) equations* to allow for the possibility of magnetic "charges", or magnetic monopoles, existing in nature.

Think carefully about the *units* of any/all new constants you introduce. (Try to express things in terms of SI base units, I think that helps a lot) In particular - what would be a good guess for the SI units for your new "magnetic charge?" (The "force law" you write down may be quite helpful in this respect).

Based just on units, can you make some guesses for the new constants? (Can you make the full set of equations look especially symmetrical and lovely?) Can you perhaps see why physicists have looked so long and hard for magnetic monopoles?

As a result of these equations, write down a "Coulomb's law" for the force between static magnetic monopoles.

You may decide that there is some ambiguity in your answers to many of these questions - what experiments can you propose to settle any questions?

Assigned in SP08 (average score: 45.71% based on 8 students attempting)
Instructor notes: **Students can correctly find the** magnetic charge and magnetic current but very few people can get force correctly.