

Transformed E&M I homework

Gauss' Law, Divergence and Curl of E (Griffiths Chapter 2)

Gauss' Law

Question 1. E field of infinite plane

SUPERPOSITION, SKETCH, REASONING. (OSU Paradigms, Vector Fields HW 10)

1. We know that the electric field everywhere in space due to an infinite plane of charge with charge density located in the xy -plane at $z = 0$ is

$$\vec{E}(z) = \begin{cases} +\frac{\sigma}{2\epsilon_0} \hat{z} & z > 0 \\ -\frac{\sigma}{2\epsilon_0} \hat{z} & z < 0 \end{cases}$$

(Mentally check that this is true for both positive and negative values of σ .)

- Sketch the z -component of the electric field as a function of z .
- Draw a similar picture, and write a function that expresses the electric field everywhere in space, for an infinite **conducting** slab in the xy -plane, of thickness d in the z -direction, that has a charge density $+\sigma$ on each surface.
- Repeat for a charge density $-\sigma$ on each surface.
- Now imagine two **conductors**, one each of the two types described above, separated by a distance L . Use the principle of superposition to find the electric field everywhere. Discuss whether your answer is reasonable. Does it agree with the known fact that the electric field inside a conductor is zero? Has superposition been correctly applied? Is Gauss' Law correct? Try to resolve any inconsistencies.

Question 2. Questions about Gauss

GAUSS (Griffiths by Inquiry Lab1)

What are the advantages and disadvantages of using Gauss' Law to find the electric field instead of using Coulomb's Law (Griffiths Eq 2.8)? What role does symmetry play? Consider these questions in finding the electric field everywhere for a conducting sphere, a uniformly charged sphere, and a sphere with charge distribution varying as r^n , all with radius r_0 and total charge Q . Do the same for three cubes with the same properties.

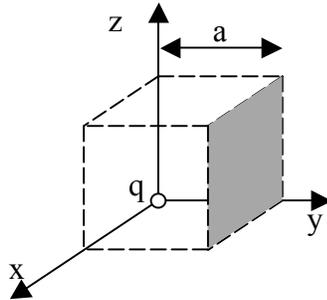
Question 3. Electric flux

A charge q at the origin is at the back corner of an imaginary cube of edge length a .

What is the electric flux through the shaded region (the plane at $y = a$)? Work out the answer two ways:

A) By Gauss's Law.

B) By direct integration. (Hint: set up the problem with Cartesian coordinates, not spherical coordinates. Rewrite $\hat{r} = \frac{\vec{r}}{r}$ in terms of Cartesian coordinates and proceed.)



Assigned in FA08

Question 4. E from $1/\sqrt{r}$ charge density

GAUSS (Shadowitz p 94#3)

Let $\rho = \frac{a}{\sqrt{r}}$ from $r=0$ to $r=R$ in spherical coordinates. Find E .

The charge density is infinite at $r=0$ but the electric field is finite there. Is this case physically realizable?

Question 5. charge enclosed

GAUSS (Reitz 2-13)

Have to look this one up

Here's a change in electric field, what is the charge enclosed.

Question 6. Integrate charge density to find Q

CHARGE DENSITY; CALCULATION

The surface charge density on a sphere of radius R is $\sigma = \sigma_0 \sin^2(\theta) \cos^2(\phi)$.

Find $Q = \int dA \sigma(\vec{r})$.

Question 7. Questions about Gauss' Law

THINKING DEEPER: Questions about Gauss' Law (see P435_Lect_02_QA.pdf in Univ of Illinois Questions for Lecture) – maybe some good HW extension questions in here.

Question 8. Charge density of charge distributions

CHARGE DENSITY; DELTA FUNCTION (Zimmerman; have solns)

What is the *volume* charge density $\rho(r)$ for the following charge distributions? Hint: delta functions may be useful.

(a) a charge Q uniformly distributed on a spherical shell of radius r_0

(b) An infinitely long line of charge density in the xy plane lying on the line $x=2y+3$

Question 9. Spherical charge distributions

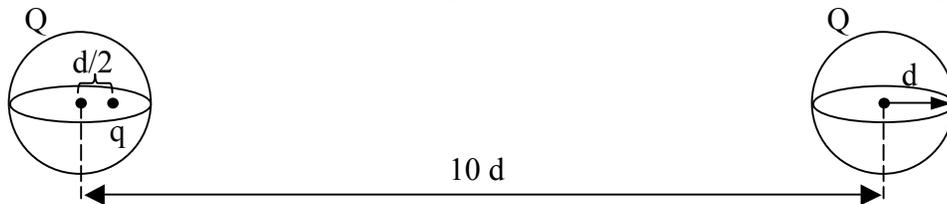
For parts A) and B), consider a sphere of radius R , centered on the origin, with a radially symmetric charge distribution $\rho(r)$.

A) What $\rho(r)$ is required for the E-field in the sphere to have the power-law form $E(r) = c r^n$, where c and n are constants? The case $n = -2$ is special. How so? Some values of n are unphysical since these would lead to an infinite amount of charge in the sphere. Which values of n are physically allowed?

B) What kind of charge distribution is required for the radial E-field inside the sphere to be of constant magnitude; that is, what $\rho(r)$ produces $E(r) = \text{constant}$ (inside only)?

C) The following problem is from the 2001 Physics GRE exam. Students were expected to solve the problem in just a few minutes!

Two spherical, nonconducting, and very thin shells of uniformly distributed positive charge Q and radius d are located a distance $10d$ apart. A positive point charge q is placed inside one of the shells at a distance $d/2$ from the center, on the line connecting the centers of the two shells, as shown in the figure. What is the net force on the charge q ?



Assigned in FA08

Question 10. Delta functions and charge distributions

a) On the previous homework we had two point charges: $+3q$ located at $x=-D$, and $-q$ located at $x=+D$.

Write an expression for the *volume* charge density $\rho(\mathbf{r})$ of this system of charges.

b) On the previous homework we had another problem with a spherical surface of radius R (Fig 2.11 in Griffiths) which carried a uniform surface charge density σ . Write an expression for the *volume* charge density $\rho(\mathbf{r})$ of this charge distribution. (Hint: use spherical coordinates, and be sure that your total integrated charge comes out right.) (Verify explicitly that the units of your final expression are correct)

c) Suppose a linear charge density is given to you as $\lambda(x) = q_0\delta(x) + 4q_0\delta(x-1)$

Describe in words what this charge distribution looks like.

How much total charge do we have?

Assuming that q_0 is given in Coulombs, what are the *units* of all other symbols in this equation (including, specifically the symbols λ , x , the delta function itself, the number

written as "4" in front of the second term, and the number written as "1" inside the last δ function.

It can also be tricky to translate between the math and the physics in problems like this, especially since the idea of an infinite charge density is not intuitive. Practice doing this translation is important (and not just for delta functions!)

Assigned in SP08 (average score: a) 7.6, b) 7.6, c) 8.8)

Assigned in FA08

Instructor notes: This problem brings up several fundamental difficulties with charge and charge density. Students struggle to write point charges as volume charge density. They are uncertain whether this should be a 1D delta or a 3D delta function. One explanation that worked is that a 3D delta function constrains in 3D to a point, 2D constrains in 2D to a line, and 1D is a plane. In part (b), asking for rho with a surface charge density, still evoked a lot of confusion about whether the argument of the delta function should be a vector, a scalar, or what). Several students wrote that rho is the integral of charge density.

Question 11. Delta functions and Gauss

The electric field outside an infinite line that runs along the z-axis is equal to $\vec{E} = \frac{2\lambda}{4\pi\epsilon_0 s} \hat{s}$

in cylindrical coordinates. (This is derived in Griffiths Example 2.1)

- Find the divergence of the E field for $s > 0$.
- Calculate the electric flux out of an imaginary "Gaussian" cylinder of length "L", and radius "a", centered around the z axis. Do this 2 different ways to check yourself: by direct integration, and using Gauss' law)
- Given parts a and b, what is the divergence of this E field?

[Hint 1: use cylindrical coordinates. Hint 2: your answer can't be zero everywhere! Why not?]

Notation note: s is the symbol Griffiths uses for "distance from the z axis" in cylindrical coordinates. I may sometimes use the symbol r for that quantity (if it's clear I won't confuse it with spherical coordinate), or sometimes even ρ (if I don't think I'll confuse it with a charge density!)

Assigned in SP08 (average score: a) 8.88, b) 8.88, c) 7.44)

Instructor notes: The dimensionality of the delta function is a good point of discussion in this problem, as students struggle with it. The divergence of the E field in part (c) was troublesome. Writing the divergence as the delta function was something new (instead of rho). Some students have trouble articulating the relationship between divergence (part c) and flux (part b). The fact they were writing the flux in part (b) but the divergence in part (c) threw them. Most solve the problem by writing rho(r) as a delta function, then using the fact that the divergence of E is rho/epsilon-nought, without explicitly making a connection between flux and divergence. It is tricky to solve for the constant in front of the delta function (eg., divergence of $E = C \cdot \delta(s)$).

Question 12. Gravitational field and divergence

DIVERGENCE, GAUSS

GRAVITATIONAL FIELD

A gravitational field (\vec{g}) has a divergence proportional to the mass density $\rho(r)$ for $0 < r \leq R$, $\nabla \cdot \vec{g} = -G\rho$. The divergence of the field \vec{g} is equal to zero for $r > R$.

- Sketch the gravitational field for $r < R$.
- Calculate \vec{g} .
- Give an example of a physical situation that would give rise to such a field.

Question 13. E field near hydrogen atom

REAL-WORLD (deGrand)

a) Find the E-field in a hydrogen atom. Quantum mechanics tells us that the electron is effectively "smeared out", so the electron's resulting contribution to the charge density is $\rho_0(r) = \rho_0 \exp(-\frac{2r}{a_0})$ where a_0 is the Bohr radius. (Hint: Don't forget to normalize ρ so the electron's charge is $-e$. This charge density is not uniform, so you must integrate to get enclosed charge. Also, don't forget there is also a pointlike proton in the middle of this atom!) Sketch and briefly discuss your result (compare the E field you get to what you'd have from the proton alone)

b) Having done the above, briefly discuss the advantages and disadvantages of using Gauss' Law to find the electric field instead of using Coulomb's Law (Griffiths Eq 2.8)? What role does symmetry play?

Note: sketch means sketch - just a rough plot which shows key features (e.g. what's it do near the origin? Near infinity?) We don't want to just calculate E fields; we want to be able to imagine what they look like, so sketching fields is important. (You can always use a program (like Mathematica) to check your sketch, but try on your own first...)

Assigned in SP08 (average score: A0 6.36, b) 6.36)

Assigned in FA08

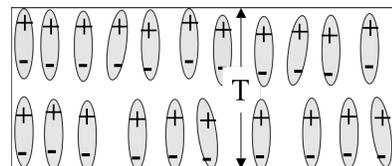
Instructor notes: Students struggle with this problem, and almost everyone does the normalization integral by stopping at $r=a_0$ (the Bohr radius). Many people did not attempt part (b). This problem brings out a common student misconception, that the charge distribution has to be symmetric in order to use Gauss' Law, whereas the symmetry in the E field (and thus the E field dotted with da) is the important symmetry.

Question 14. Smectic-C liquid crystals

IS THIS ONE GAUSS? REAL WORLD (deGrand)

Smectic-C liquid crystals are made of long rod like molecules with a positive head and a negative tail which pack together to form a long, thin sheet as shown. The volume charge density is obviously complicated, but can be quite successfully modeled with the rather simple form $\rho(x,y,z) = \rho_0 \sinh(z/z_0)$,

i.e. uniform in x and y , but varying in z , with $z = 0$ defined to be the middle of the sheet. (That's the "hyperbolic sinh" function) Find the electric field everywhere in space, in terms of the constants ρ_0 , z_0 and the sheet thickness T , and sketch it. Where is it biggest?

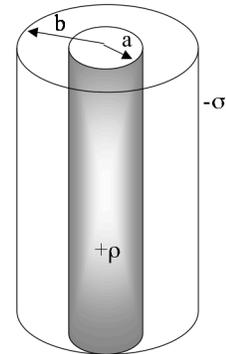


Assigned in SP08 (extra credit)

Question 15. Electric field of coaxial cable

REAL-WORLD

A long coaxial cable carries a uniform volume charge density ρ throughout an inner cylinder (radius a) and a uniform surface charge density σ on the outer cylinder (radius b). The cable is overall electrically neutral. Find \mathbf{E} everywhere in space, and sketch it.



b) Last week you considered how much charge a child's balloon might hold before sparking. Now let's consider the same question for a real life coax cable. Making any reasonable guess as to physical dimensions for a cable like one that might attach your stereo to your TVs - what would be the maximum static charge you could put onto that cable?

(This is *estimation* - I don't care if you're off by a factor of 2, or even 10, but would like to know the rough order of magnitude of the answer!) From where to where do you expect a spark to go, if it *does* break down?

Assigned in SP08 (average score: a) 6.96, b) 6.94)

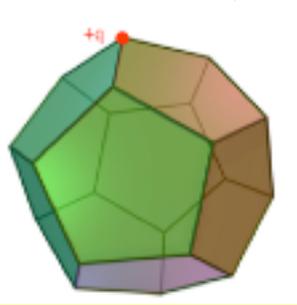
Assigned in FA08

Instructor notes: Again, many students sketch the field lines instead of producing a graph. For those who do graph it, only a few people correctly sketch the E field going to zero suddenly. As on previous estimation problems, many students did not attempt part (b). *Typo error in Pollock solution: for E between a and b , the factor of 2 should be on the numerator.*

Question 16. Point charge at corner of dodecahedron

FLUX, SYMMETRY, ARGUMENT (C. Gwinn, UC Santa Barbara)

A plane can be tiled with equilateral triangles, squares, or hexagons. Analogously, 3-space can be completely filled with cubes or regular dodecahedra. Suppose that a point charge $+q$ is placed at one corner of a dodecahedron, with length of each edge a .



1. What is the total electric flux to emerge from the 12 faces? What fraction of that flux emerges from the 3 pentagons adjacent to the charge?
2. Argue that, if a charge $+q$ is placed at each corner of a dodecahedron, then the electric field at the center of the dodecahedron is zero. What is the E -field if one of the 20 charges is removed? [Useful fact: 8 of the 20 corners of a dodecahedron lie on a cube. The edges of the cube are diagonals of the regular pentagons, that are the faces of the dodecahedron. The ratio of the length of this diagonal to the length of

one of the edges of the dodecahedron is the Golden Ratio, $\phi = (\sqrt{5}+1)/2 \approx 1.618\dots$]
For fun: Can you remove two charges and have zero E-field? Three? Four? Five?
And so on!
