Transformed E&M I homework

Manetization (dimagnets,paramagnets, ferromagnets)

(Griffiths Chapter 6)

Magnetization, force on dipole

Question 1. Estimate field strength of magnet

Sanjoy Mahajan
Estimate the field strength of a permanent magnet

Question 2. Estimate magnetization of iron

Pollack and Stump, 9-3 pg. 350
Estimate the maximum magnetization (**M**) of iron. Assume the atomic dipole moment of an iron atom is due to two aligned (unpaired) electron spins. The mass density of iron is 7.87×10^3 kg/m^3, and the atomic mass is 55.85 u.

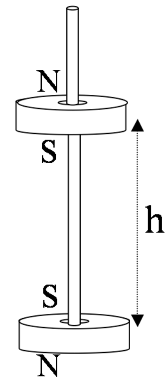
Question 3. Fields and strengths

A) Find the density ρ of mobile charges in a piece of speaker wire made of gold, assuming each atom contributes one conduction electron. (Look up any necessary physical constants.) Think about the definition of current and then *estimate* the average electron speed in a gold speaker wire carrying an ordinary current. *Your answer will come out quite slow.* If you flip on the stereo, and the speakers are, say, 2 meters away, would there be a noticeable "time lag" before you hear the speaker come on? Why/why not?

B) If you cut open this wire, you'll see that it is really two wires, each insulated, and wrapped close together in a single plastic cylinder (since you need a complete circuit, current has to flow TO and FROM the speaker, right?). Make reasonable guesses for the dimensions involved and estimate the size of the magnetic field in the space between the wires. How large a field to you expect at a distance of 1 meter from the pair of wires?

Assigned in SP08 (average score: a) 6.8, b) 6)
Assigned in FA08
Instructor notes: As always, students are uncomfortable with estimation, and part (A) generated many questions. At least some students did not know how physical constants about gold could give them an estimate for how many atoms were in a volume. The error on the estimation can be quite large (1-2 orders of magnitude) with an acceptable approximation scheme. The problem could be amended to state in part B that the length of the cable is assumed to be 2m.

Question 4. Force between magnets



A) Consider two small magnets (treat them as pointlike perfect dipoles with magnetic moments $\mathbf{m_1}$ and $\mathbf{m_2}$, to keep life as simple as possible). For the configuration shown ("opposite poles facing"), draw a diagram showing the relative orientation of the magnetic dipoles $\mathbf{m_1}$ and $\mathbf{m_2}$ of the two magnets. Then find the force between them as a function of distance h. Explain why the *sign* of the force does (or does not) make sense to you.

B) Now let's do a crude estimate of the strength of the magnetic moment of a simple cheap magnet. Assume the atomic magnetic dipole moment of each iron atom is due to a single (unpaired) electron spin. Find the magnetic dipole moment of an electron from an appropriate reference (and cite the reference). The mass density and atomic mass of iron are also easy to look up. Now consider a small, ordinary, kitchen fridge "button sized" magnet, and make a very rough estimate of its total magnetic moment **M**. Then use your formula from part A to estimate how high (shown as the distance h) one such magnet would "float" above another, if oriented as shown in the figure. Does your answer seem at all realistic, based on your experiences with small magnets? (Note that such a configuration is not *stable* - why not? I've seen toys like this, but they have a thin wooden peg to keep the magnets vertically aligned, so that's how I drew it in the figure.)

Assigned in SP08 (average score: a) 9.28, b) 8.3)
Assigned in FA08

Instructor notes: Some students had forgotten the formula $F = \nabla \cdot (m \cdot B)$, without which they were stuck. This is a good problem to probe students for what their physical intuitions say the answer should be. Some students say that the total force is the sum of the force acting on m2 by m1 and the force acting on m1 by m2 is F2.

Question 5. Paramagnetics and diamagnets

Make two columns, "paramagnetic" and "diamagnetic", and put each of the materials in the following list into one of those columns. Explain briefly what your reasoning is. Aluminum, Bismuth, Carbon, Air, a noble gas, an alkali metal, Salt, a superconductor, & water.

(You can look these up if you want to check your answers - but I just want a simple physical argument for how you classified them. If you do look them up, you'll find several in this list are not what you might expect. Write down briefly any thoughts about why a simple argument like you are using might not always work)

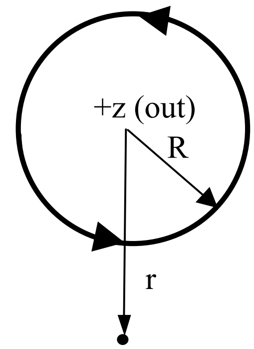
By the way - superconductors exhibit the "Meissner" effect, which means they prevent any external magnetic field from entering them - this is the source of "magnetic levitation". That might help you classify them!

Assigned in SP08 (average score: 97.5%)

Instructor notes: We had some very nice discussions about this question, with several students doing some good reasoning. Some were excited to be able to use some chemistry (bonding orbitals and unpaired electrons) to predict magnetic behavior. Some students weren't sure if this question was asking them to derive the classification of materials as dia- or para-magnetic, or look it up. Superconductors caused a quandary of classification.

Question 6. E from B

A very long solenoid of radius R and n turns per unit length, has a current I(t) in the CCW direction and I is ***increasing with time*** (see diagram).
A) Solve for the electric field (direction and magnitude) everywhere.
B) An electron (mass m, charge -e) is released from rest at a point outside the solenoid a distance r > R from the center. What is the initial acceleration (direction and magnitude) of the electron? What would be the initial acceleration of the electron if it had an initial velocity inward (along radius) of magnitude v_o?



Assigned in FA08

Question 7. Explain susceptibilities of noble gasses

Pollack and Stump, 9-7 pg. 351
The magnetic susceptibilities of the first three noble gasses are X_m (He) = -1.1 x 10^{-9}, X_m (Ne) = -3.9 x 10^{-9}, and X_m (Ar) = -1.1 x 10^{-8}. The magnetic susceptibilities of the first three rare earth elements are X_m (La) = 5.33 x 10^{-5}, X_m (Ce) = 1.50 x 10^{-3}, and X_m (Pr) = 3.34 x 10^{-3}. Explain these results.

Question 8. Orbital frequency of Bohr model

Pollack and Stump, 9-1 pg. 350
In the Bohr model the radius of the first Bohr orbit is 0.529 x 10^{-10} m. Calculate the orbital frequency if the orbital magnetic moment is 1 Bohr magneton. Compare the corresponding electron speed to the speed of light. [Answer: 1/137]

Question 9. Magnetic field of the earth

Lorrain, Dorson, Lorrain, 16-2 pg. 288

A sphere of radius R is uniformly magnetized in the direction parallel to a diameter. The external field of such a sphere is similar to that of the earth.

Show that the equivalent surface current density is the same as if the sphere carried a uniform surface charge density σ and rotated at an angular velocity ω such that $\sigma\omega R = M$.

Question 10. Field inside a tubular magnet

Lorrain, Dorson, Lorrain, 16-9 pg. 290

You are asked to design a permanent magnet that would supply a magnetic field in a cylindrical volume about 20 millimeters in length and 20 millimeters in diameter. Someone suggests a tubular magnet, magnetized along its length that would surround this volume. What do you think of this suggestion?

Question 11. Fridge magnet and dipole force

REAL-WORLD, MAGNETIC DIPOLES
The Physics Teacher, 45, 348-351, Sept 2007 [available from Stephanie]

You can measure the force between magnetic dipoles by puttinga fridge magnet on the top of a scale and bringing another magnet in from above so that they repel. In this way, you can measure force versus distance. You will find that the force between the two dipoles is given by the inverse 4^{th} power of the distance between them. Why is this? What would be the force/distance relationship between two magnetic monopoles, if they existed? [Answer: inverse 2^{nd} power]. What would you expect the force/distance relationship between the two magnetic dipoles to be if you turned the magnet on its side? [Answer: still inverse 4^{th}]. When the two magnets are brought extremely close together (on the order of the diameter of the magnet), the force/distance relationship is no longer inverse 4^{th} power. What would you expect it to be, and why? [Answer: two infinite planes]. [Note that this is an easy experiment to do, and if you plot the force and distance on log/log paper you get a clear line with a slope of -4]. Now, what if we have a magnet and a spherical iron ball. As the iron ball approaches the magnet, the strength of the magnetization induced in the iron increases. Thus, the induced magnetic dipole becomes stronger as the magnet is brought closer. What do you expect the force/distance relationship to be in this case?

Question 12. Magnetic trapping of ultracold atoms

CALCULATION; MAGNETIC DIPOLES, REAL WORLD (Berkeley; Stamper-Kurn)
Magnetic trapping of ultracold atoms
The next set of problems concern the magnetic trapping of ultracold atoms, specifically atoms of 87 Rb (one of the rubidium isotopes). In one of their internal states (the F = 1 hyperfine state), such atoms have a magnetic dipole moment of m = mB /2 where mB = e h/2me is the Bohr magneton. Magnetic trapping is achieved under the condition that the atomic magnetic moment remains anti-aligned with the magnetic field as the atom moves around in space, thus producing a trapping potential of the form U (r) = m |B|.

1) First, let's consider a simple kind of magnetic trap known as the spherical quadrupole magnetic trap.

a) Consider two electromagnet coils sharing the same axis and separted by d, each of radius a and N turns, each coil carrying current I running in the opposite direction as that of the other coil. Placing the origin at the midpoint between the two coils, calculate the magnetic field B(x, y, z) in the region near the origin, keeping just the lowest order terms. Note that to determine the field for radial displacements (i.e. in the x or y directions), you can consider the symmetry of the problem and apply the condition of zero-divergence for the magnetic field.

b) You should find a spherical quadrupole field with field gradients in the radial and axial directions determined by properties of the coils which create the field. A spherical quadrupole field can be written as

$$B(x, y, z) = B (x\ \hat{x} + y\ \hat{y} - 2z\ \hat{z}) \quad (1)$$

where B is the radial field gradient. Now put in some real numbers. Consider coils of radius a = 2.5 cm separated by d = 2a/√3 (the optimal separation), each of 20 turns running I = 92 amperes. What are the axial and radial gradients (G/cm)?

2) Majorana losses: In the spherical quadrupole magnetic field described above, the potential U (r) = m |B| is clearly minimized at a point, and, thus, particles with their

magnetic moment anti-aligned with the field will be trapped by such a field. However, a problem with the spherical quadrupole trap is the fact that it is always lossy. Magnetic trapping requires that the atomic magnetic moment always follow the local orientation of the magnetic field. If the field is changing its direction faster than an atom's orientation can keep up, the atomic orientation may change, causing the atom to be ejected from a magnetic trap. This type of loss is given the name Majorana loss.

a) Consider a 87 Rb atom moving with velocity v and impact parameter b in the x − y plane of a spherical quadrupole trap. Give an estimate for the critical impact parameter b0 below which the atom has a high probability of being lost, and above which the atom has a low probability of being lost. To do this, consider the maximal rate at which the orientation of the field is changing – this quantity has units of s−1 . Compare this with the Larmor precession frequency of the atom in the local magnetic field (this frequency is zero at the origin and larger elsewhere).

b) Now we will estimate the Ma jorana loss rate for atoms at a certain temperature trapped in a spherical quadrupole trap. To do this, consider that the treatment above determines an ellipsoidal volume within which atoms at a given velocity may be lost from the trap. A loss rate can be calculated as the product nσv where n is the density of the atomic gas, σ is the area of the ellipsoid (which differs slightly if approached from different directions; just take some characteristic area), and v is the typical velocity of atoms. To estimate the volume occupied by the gas (you'll need this to determine the density n), consider how large is the volume in which the trap potential U (r) is smaller than kB T where kB is the Boltzmann constant. Similarly, to obtain a typical velocity, consider the velocity at which the kinetic energy is kB T . Having made all these estimates, find the loss rate per atom given the temperature T and the radial field gradient B .

c) Again, let's put in some numbers. Consider a gas of 87 Rb trapped in a spherical quadrupole trap with a radial gradient of 120 G/cm. What is the Ma jorana loss rate per atom at temperatures of 1, 10, and 100 μK?

Magnetic Dipoles (also included in Chapter 5)

Question 13. Magnetic field of spinning charged sphere

Lorrain, Dorson, Lorrain, 14-14 pg. 261

A conducting sphere of radius R is charged to a potential V and spun about a diameter at an angular velocity ω.

(a) Show that the surface current density is $\alpha = \varepsilon_0 \omega V \sin\theta = M \sin\theta$, where M is $\varepsilon_0 \omega V$.

(b) Find the magnetic flux density B_0 at the center.

(c) What is the numerical value of B_0 for a sphere 100 millimeters in radius, charged to 10.0 kilovolts, and spinning at 10,000 turns per minute?

(d) Show that the dipole moment is $\frac{4}{3}\pi R^3 M\hat{z}$ where $\hat{z}$ is a unit vector along the axis, related to the direction of rotation by the right-hand screw rule.

(e) What current flowing through a loop 100 millimeters in diameter would have the same dipole moment?

Question 14. Square loop- far away

A) In a previous homework, we considered a square current loop (current I running around a wire bent in the shape of a square of side a) sitting flat in the x-y plane, centered at the origin. (You found B at the center). Now redo that problem but find **B**(0,0,z) (mag and direction), i.e. a distance z above the center. (Check yourself by setting z=0 and making sure you get the correct result.)

B) Take your result from part (a) and now take the limit z >> a , finding an approximate simple formula for B_z (0,0, large z). Do the same for a circular current loop of radius a, and compare. (The exact expression is derived in Griffiths, Eq. 5.38, you don't have to rederive that, just consider the large-z limit) Check both answers by comparing with Griffiths dipole approximation (Eq. 5.86), looking only along the +z-axis of course. *Notice how simple everything gets far away, and how your two expressions for very different wire shapes differ only by the constant out front, which should go like* $m = I \times$ (area of loop) *This is the magnetic dipole moment.*

Assigned in SP08 (average score: a) 9.15, b) 6.05)
Assigned in FA08
Instructor notes: **Most student** questions here were about taking the limit of B as z>>a, including some confusion about what parameter is small. Students are not universally comfortable with the binomial expansion (even though one can simply approximate in this problem without doing the whole expansion). Half the students did this problem by integrating from Biot-Savart.

Question 15. Magnetic moment of spinning disk

Pollack and Stump, 8-31 pg. 303

A thin disk of radius R carries a surface charge σ. It rotates with angular frequency ω About the z axis, which is perpendicular to the disk and through its center. What is B along the z axis? What is the magnetic moment of the spinning disk? [$m_z = \pi\sigma\omega R^4 / 4$]

Question 16. Magnetic moment and B of spinning charged disk

Pollack and Stump, 8-32 pg. 304

A charged annular disk with inner radius *a* and outer radius *b*, and surface charge density σ, lies in the xy plane with its center at the origin. It rotates about the *z* axis with angular velocity ω.

(a) What is the magnetic moment **m**?

(b) What is **B** in the xy plane at distance *p* from the origin, where *p* >> *a* ,b?

(c) Find B_z (0, 0, z) for all values of z.

Question 17. The TOP trap

CALCULATION; MAGNETIC DIPOLES, REAL WORLD (Berkeley; Stamper-Kurn)

3) The TOP trap: Losses from spherical quadrupole traps made evaporative cooling of atoms in magnetic traps ineffective until some manner was employed to avoid these losses. As you found above, the problem becomes ever worse as the temperature of the gas gets lower, thus precluding the ultra-low temperatures needed to reach Bose-Einstein condensation. One solution, employed by the team of Cornell and Wieman in Colorado for the first achievement of Bose-Einstein condensation in a gas, was the TOP (time-averaged orbiting potential) trap. Consider that a spherical quadrupole trap is formed, and then an additional bias field is applied of the form

$$B_{TOP}(x, y, z) = B_0 \cos \omega t\, \hat{x} + B_0 \sin \omega t\, \hat{y} \quad (2)$$

Atoms held in a time-varying magnetic trap of this type will experience a time-averaged potential at every point so long as (1) the field rotates at a rate faster than the motional time-scale of atoms so that an atom experiences a local average of the magnitude of the field, and (2) the field rotates at a rate slower than the Larmor precession frequency so that atomic magnetic moment remains anti-aligned with the magnetic field.

a) Assume that these conditions are met. What is the time-averaged magnitude of the magnetic field near the origin? You should find that this magnitude has a minimum at the origin, and varies quadratically away from the origin with a fixed ratio of trap curvatures in the different directions.

b) Now put in some numbers. Suppose on top of the 120 G/cm radial gradient spherical quadrupole field, the time-varying TOP field has a magnitude of 10 G and rotates at a rate of $\omega = 2\pi \times 7.5$ kHz. What are the oscillation frequencies of a ^{87}Rb atom in this trap? (Note: if a one dimensional potential has the form $U(x) = (1/2)m\Omega^2 x^2$, then a particle of mass m will oscillate harmonically in this potential with oscillation angular frequency Ω). Do the field settings chosen satisfy the two conditions for adiabaticity described above?

c) Finally, the critical temperature T_c for Bose-Einstein condensation for atoms in a harmonic trap is determined by the geometric mean of trapping frequencies $\bar{\omega}$ and the number of atoms N by the relation

$$N = \zeta(3)\left(\frac{k_B T_c}{\hbar\bar{\omega}}\right)^3 \quad (3)$$

where k_B is the Boltzmann constant and ζ is the Riemann Zeta function ($\zeta(3) \approx 1.202$). This equation can be interpreted as stating that once the number of quantum states available to a gas at a temperature T_c roughly equals the number of particles, multiple occupancy of quantum states occurs and quantum statistical effects such as Bose-Einstein condensation ensue. Consider the above TOP trap with 2×10^4 atoms. What is the Bose-Einstein condensation transition temperature?

References for magnetic trapping and Bose-Einstein condensation include the following:

- *Pethick and Smith, Bose-Einstein condensation in Dilute Gases (Cambridge University Press, New York, 2002).*
- *Metcalf and van der Straten, Laser Cooling and Trapping (Springer, New York, 1999).*
- *M.H. Anderson, et al., "Observation of Bose-Einstein Condensation in a Dilute Atomic Vapor," Science 269, 198 (1995).*