

Transformed E&M I homework

Polarization and Dielectrics, Field of Polarized Object (Griffiths Chapter 4)

Polarization, dipole moments

Question 1. Dipole moment and choice of origin

(Heald and Marion, 2-8 pg. 71)

Show that the electric dipole moment of a system of charges is independent of the choice of origin if the system has zero net charge (i.e., no monopole moment). If there is a net charge, show that there exists an origin such that the dipole moment is zero.

Question 2. Quadrupole moment of dipole

(Heald and Marion, 2-9 pg. 71)

Show that a simple finite dipole (charges $\pm q$ located at $z = \pm 1/2$) has zero quadrupole moment with respect to its center as the origin.

Question 3. Dipole above conducting plane

CALCULATION, EQUILIBRIUM VALUES (From Pollack and Stump, Electromagnetism, Problem 6.2)

Consider a dipole with moment p at a distance z_0 above a grounded conducting plane that is taken to define the $x - y$ plane. p is taken to lie in the $x - z$ plane, oriented at an angle θ to the normal (z) of the plane. Find the torque N on the dipole. What are the equilibrium

values of θ ? [Answer: $\vec{N} = -p^2 \sin \theta \cos \theta \hat{y} / (32\pi\epsilon_0 z_0^3)$]

Question 4. Force on dipole in E field

CALCULATION; CONNECTIONS BETWEEN TOPICS (From Pollack and Stump, Electromagnetism, Problem 6.4)

a. A dipole p with fixed magnitude is placed in an external electric field $E(r)$ that varies with position. Show that there is a force on the dipole, and find how this force depends on the orientation of the dipole and on the variation of the field. [Answer: $F = (p \cdot \nabla)E$]

b. Find the force and torque on a dipole in the field of a point charge. Let the charge q be at the origin, and the dipole $p = p (\sin \xi_0 \hat{x} + \cos \xi_0 \hat{z})$ be at the point $(0, 0, z_0)$. Also, find the force on q , and verify Newton's third law. [Answer: the force on q is

$$\mathbf{F} = \frac{pq}{4\pi\epsilon_0 z_0^3} (-\sin \xi_0 \hat{x} + 2 \cos \xi_0 \hat{z})$$

Question 5. Polarizability of air molecules – version 1

REAL WORLD, ESTIMATION (From Pollack and Stump, Electromagnetism, Problem 6.10)

The dielectric constant of air is 1.00059. From this, determine the mean polarizability of atmospheric molecules. Compare this result to the atomic polarizabilities listed, for example, in Griffiths Table 4.1.

Question 6. Polarizability of air molecules – version 2

REAL-WORLD (deGrand)

Use the Clausius-Mossotti equation to determine the polarizability of atoms in air molecules: O₂ and N₂. Note that the table in the book gives N₂ and “air” as a separate entries but the numbers are basically identical. Combine this result with the simple classical calculation we did in class (basically example 1 in chapter 4) to determine the average radius of an atom in an air molecule.

Question 7. Electric field of dielectric sphere

CALCULATION (From Pollack and Stump, Electromagnetism, Problem 6.23)

A hollow dielectric sphere, with dielectric constant κ , inner radius a and outer radius b , is placed in a uniform applied electric field $E_0 \hat{z}$. The presence of the sphere changes the field. Find the field in the three defined regions, i.e. $r < a$, $a < r < b$ and $r > b$. What is the field at the center of the spherical shell? What is the dipole moment of the dielectric medium? [for the last part, you'll find:

$$p_z = \frac{4\pi\epsilon_0 b^3 (a^3 - b^3) (2\kappa + 1) (\kappa + 1) E_0}{2a^3 (\kappa - 1)^2 - b^3 (2\kappa + 1) (\kappa + 2)}$$

Question 8. Dipole moment for charge distribution

CALCULATION

The electric dipole moment \mathbf{p} of a charge distribution $\rho(\vec{r})$ along the z-axis) is defined as

$$\mathbf{p} = \int d^3r \rho(\vec{r}) \mathbf{z}.$$

Suppose $\rho = \exp\left(-\frac{r}{a}\right) (A + B \cdot \cos(\theta) + B \cdot \sin(\theta) \cos(\phi))$. Find \mathbf{p} .

Question 9. Quadrupole moment of carbon dioxide

REAL-WORLD. DIPOLES.

Look at carbon dioxide as an atom with positive carbon and two negative oxygens arranged in a line. Not charged so not monopole. Rest state has two positives in middle and negatives on each end so it's not a dipole. It's two dipoles pointing in opposite directions. -1 on oxygens and +2 on carbon. That's a linear quadrupole. It's a real thing. Now hit it with an electric field, displace central oxygen atom along line or perpendicular to line, now you have a dipole. Center of charge of positives/negatives aren't in the same place. So infrared radiation induces an oscillating dipole... get dipole radiation. More advanced is methane, CH₄, E field moves center and makes induced dipole.

Question 10. Dipole moment of oxygen

DIPOLE MOMENT, ENERGY, REAL-WORLD

Know electric dipole moment of oxygen that gives you an energy scale. Energy is dipole dot with E. Energy difference between dipole in one energy and other. Estimate energy scale. Compare with thermal energies. Estimate field strength in microwave oven. Charged metal plates.... Typical voltage scale.. maybe it's close to sparking because grape does spark. So then water molecule question can refer back to that. E is V/m from last week...

Question 11. Optical tweezers

DIPOLE, ENERGY, TORQUE, REAL-WORLD

Optical tweezers have strong E field in between. Gradient of E field... $F=(p \cdot \nabla)E$. Dipole in E field. Has a torque. Look at abstract Griffiths problem. Add a real world context to it. How heavy of a molecule can you levitate in middle of optical tweezers.

Question 12. Deflection of water and olive oil with a charged balloon

DIELECTRIC, REAL-WORLD

You want to levitate a single drop of water falling from a faucet. You bring a charged spherical balloon near water. Water is deflected, but not nearly as much as a olive oil would be. What is force on water drop assuming the water drop is neutral to begin with?

- Estimate if water were perfect conductor.
- Estimate if water were a dielectric.

Compare, and see which is most important. *(It seems that the conducting nature would be more important, however the olive oil is deflected more. We should try this in experiment, perhaps even as class demo. Deionized water is a dielectric but real water is a conductor).*

Question 13. Faraday's Ice pail experiment

CAPACITORS, POLARIZATION

Look up Faraday's ice pail experiment in physics demos. Glass jar, filled with conducting water, wrap in Al foil and charge. Capacitor. Remove glass jar from Al foil and pour out water. Another one... the glass jar has an induced polarization and that's stored and it discharges into new one. Nice intro to polarization.

Question 14. Energy of dipole in E field; instability

DIPOLE, REAL WORLD EXTENSION (U WASHINGTON)

This is similar to a problem from Griffiths but with a real-world explanation.

1* Let there be a uniform electric field in the z direction, E . The energy of an electric dipole in such a field is $-p_z E$. Suppose that p_z could only take on two values, $p_z = \pm \hat{p}$. This is analogous to the behavior of the magnetic dipole of an electron which can only take two values. One wants to measure the average value of p_z when the system is at temperature T . Show that this average value is

$$\bar{p}_z = \hat{p} \tanh(\hat{p}E/k_B T).$$

Note that for $\hat{p}E/k_B T \gg 1$, $\bar{p}_z \rightarrow \hat{p}$, as you would expect. In the opposite limit in which $\hat{p}E/k_B T \ll 1$, $\bar{p}_z \rightarrow \hat{p}(\hat{p}E/k_B T)$ which is much smaller in magnitude than \hat{p} . This is due to the fact that the dipole will tumble due to thermal fluctuations.

Field of polarized object, bound charge

Question 15. Cylindrical capacitor, bound charges

CALCULATION (from deGrand)

- (a) Find the capacitance per unit length of a long cylinder with an inner conductor of radius a , an outer conductor of inner radius b and outer radius c , a material with dielectric constant K in between, and a free charge per unit length λ on each conductor (of opposite signs on either). (b) What and where are the polarization charges?

Question 16. Cylindrical capacitor in dielectric oil

CONNECTIONS BETWEEN PHYSICS FIELDS (deGrand)

Two long coaxial metal tubes of inner radius a , and outer radius b , and height h stand vertically in a pan of dielectric oil of mass density ρ and susceptibility X . The inner cylinder is maintained at a potential V and the outer cylinder is grounded. To what height h does oil rise up in the space between the cylinders?

Question 17. Bound charges I

Consider a long insulating rod, (a dielectric cylinder), radius a . Suppose that the rod has no free charge but has a *permanent* polarization $\mathbf{P}(s, \theta, z) = C \mathbf{s}$ ($= C s \hat{\mathbf{s}}$), where \mathbf{s} is the usual cylindrical radial vector from the z -axis, and C is a positive constant). Neglect end effects: the cylinder is long. *Note that this rod is NOT a linear dielectric. Only very special materials can have a permanent polarization. Barium titanate, BaTiO_3 , is one such material. A rod with a permanent polarization is called an "electret".*

- A) Calculate the bound charges σ_b and ρ_b (on the surface, and interior of the rod respectively). What are the units of C ? Sketch the charge distribution of the rod.
 B) Next, use these bound charges (along with Gauss' law) to find the electric field inside and outside the cylinder. (Direction and magnitude)
 C) Find the electric displacement field \mathbf{D} inside and outside the cylinder, and verify that "Gauss's Law for \mathbf{D} " (Eqn 4.23, p. 176) works.

Assigned in SP08 (average score: a) 4.67, b) 4.43, c) 4.09)

Assigned in FA08

Instructor notes: This problem appears to be useful practice, though most students in the transformed class did not have conceptual problems with bound charge. Some confusion about whether D should vanish in those two problems (and why); we spent some time in several groups talking about "when can you get D easily" and when can't you?

Question 18. Bound charges II

Consider now a hollow insulating rod, with inner radius a and outer radius b .

Suppose now that the rod has a different permanent polarization, namely $\mathbf{P}(s,\theta,z) =$

$\frac{C}{s^2} \hat{\mathbf{s}}$ for $a < s < b$, $C =$ positive constant (not the same constant as in Q1). *Again, this*

is not an ordinary linear dielectric. It has a permanent polarization.

A) We have vacuum for $s < a$ and $s > b$. What does that tell you about \mathbf{P} in those regions?

Find the bound charges σ_b and ρ_b (σ_b on the inner AND outer surfaces of the hollow rod, and ρ_b everywhere else. Use these bound charges, along with Gauss' law, to find the electric field everywhere in space. (Direction and magnitude)

B) Use Griffiths' Eq 4.23 (p. 176) to find \mathbf{D} everywhere in space. (This should be quick - are there any *free* charges in this problem?) Use this (simple) result for \mathbf{D} (along with Griffiths basic definition/relation of \mathbf{E} to \mathbf{D} , Eq 4.21) to find \mathbf{E} everywhere in space. *(This should serve as a check for part a, and shows why sometimes thinking about \mathbf{D} fields is easier and faster!)*

Assigned in SP08 (average score: a) 8.9, b) 4.57)

Assigned in FA08

Instructor notes: Students need this practice with reasoning the direction of \mathbf{E} , \mathbf{D} , and \mathbf{P} .

Question 19. Math of bound charges

Using Griffiths' formal definition of bound charge distributions, eqns 4.11 and 4.12 (p. 167-8) prove (mathematically) that if you have a dielectric which starts off neutral, and then it gets polarized, that the total bound charge is still exactly zero. Explain how this formal mathematical result makes (simple) physical sense. Look back at all previous questions, does the total charge in those problems satisfy what you found here? *(This problem need not be too long!)*

Assigned in SP08 (average score: 91.43%)

Assigned in FA08

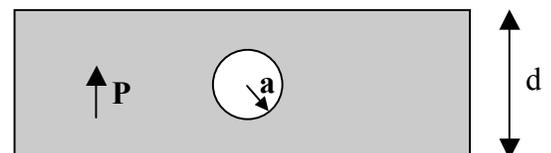
Instructor notes: This is a fairly easy problem for students.

Question 20. Superposition

Consider a large slab (infinite in the xy directions) of dielectric material which has thickness d , and has uniform polarization $\mathbf{P} = k \hat{\mathbf{z}}$. Assume there are no free charges anywhere (so this polarization \mathbf{P} is permanent). The polarization is perpendicular to the surfaces of the slab.

A) Compute the electric field everywhere due to the charges arising from the polarization of the slab.

B) Now, assume that at the midplane of the sheet is a



spherical hole of radius a . The inside of the bubble contains vacuum: it has zero polarization. Find the net electric field everywhere *inside* the hole.

Hint: Carving out a cavity is the same as superimposing an object of the same shape but opposite polarization.

C) Assume there are still no free charges anywhere. Find the bound surface charge on the surface of the hole. Then, find (or at least describe) the electric field *outside* the slab.

Hints: Griffiths' Examples 3.9 and 4.2 may be useful.

Assigned in SP08 (average score: a) 3.62, b) 6.86, c) 3.38)

Assigned in FA08

Instructor notes: This is a hard problem. Lots of confusion about the approach, whether they had to rederive the results from Griffiths for a polarized sphere, how/what superposition means... Lots of students try to solve the Laplace equation by using separation of variables, but few of them got the right answer.

Question 21. Hole in dielectric material

CALCULATION, SUPERPOSITION? (deGrand)

A dielectric material with constant K has a spherical hole of radius a drilled in it. Far from the hole, in the dielectric, there is an electric field \vec{E}_0 . Find the electric field inside the hole.

Question 22. Dielectric series capacitor 1

CALCULATION (deGrand)

The dielectric series capacitor is a parallel plate capacitor of surface area A and thickness $d = d_1 + d_2$, between which a dielectric slab of thickness d_1 and constant K_1 and a dielectric slab of thickness d_2 and constant K_2 are inserted. Assume free charges on the top and bottom of $\pm\sigma$. Find (a) \vec{D} in each slab (b) \vec{E} in each slab (c) \vec{P} in each slab (d) the potential between the plates (e) the location and the value of all bound charges (f) the capacitance.

Question 23. Dielectric series capacitor 2

CALCULATION (deGrand)

The dielectric series capacitor is a parallel plate capacitor of width w , depth l , and thickness d , between which a dielectric slab of constant K is inserted for a width s (see figure). We put a voltage V between the plates. Find [4 points each] (a) \vec{E} in each slab (b) \vec{D} in each slab (c) the free charge density in each region (d) the bound surface charge density in each region (e) the capacitance. Neglect all fringing fields in your calculations.

Question 24. E field of polarized cylinder

CALCULATION (deGrand)

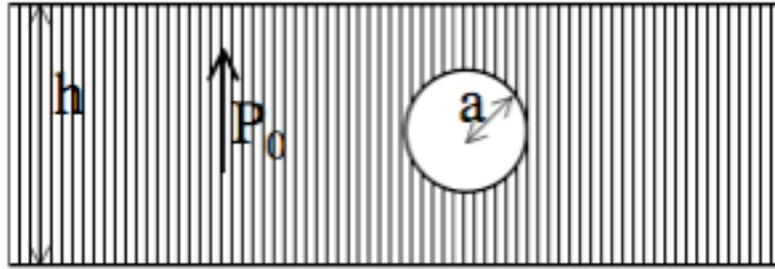
Suppose you have a right circular cylinder of length L and radius R , and in it the polarization P is a constant oriented along the length of the cylinder. Find the electric field outside the cylinder in its axis (a distance z from the center).

Question 25. Bound charge of polarized sheet with bubble

CALCULATION; CONNECTIONS BETWEEN IDEAS (C. Gwinn, UC Santa Barbara)

A large sheet of material of thickness h has uniform polarization $P_0 \hat{z}$. The polarization is perpendicular to the surfaces of the sheet. This problem contains no free charge whatsoever.

- Find the bound charge on the sheet. What is the electric field inside the sheet?
- At the midplane of the sheet is a spherical bubble, of radius a . The inside of the bubble contains vacuum: it has zero polarization. Find the bound surface charge at the surface of the bubble.



- Find the electric field outside the sheet. Also, find the net electric field inside the bubble. (Note: the sheet is not a conductor. You may use results for spherical dipole distributions, such as Example 3.9 or Example 4.2, so long as you are careful!)

Question 26. Charge densities on dielectric cube

POLARIZATION, CALCULATION (deGrand)

Suppose a dielectric cube of sides of length a is centered on the origin, and suppose that it has a polarization $\vec{P} = a\vec{r}$ where $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$. Compute the surface and volume charge densities and show that the total charge of the cube is really zero.

Linear dielectrics

Question 27. Electric field of dielectric media; images

CONNECTIONS BETWEEN CONCEPTS; EXPLANATIONS (From Pollack and Stump, Electromagnetism, Problem 6.28)

Consider two insulating media with dielectric constants $\epsilon_1 / \epsilon_0 = \kappa_1$ and $\epsilon_2 / \epsilon_0 = \kappa_2$, placed together with a planar interface between them. In the region of κ_1 there is an infinite line charge with linear charge density λ , parallel to the interface at perpendicular distance a . Use the method of images to find the electric field in both media. Show that the force per unit length on the line charge is

$$\frac{dF}{dL} = \frac{\lambda^2(\kappa_1 - \kappa_2)}{4\pi\epsilon_1 a(\kappa_1 + \kappa_2)}$$

Note that the sign of the force depends on the difference $\kappa_1 - \kappa_2$. Explain physically what's going on here.

Question 28. Using E field to align system of slabs

REAL WORLD (U WASHINGTON)

