

Transformed E&M I homework

Potential (Griffiths Chapter 2)

Potential

Question 1. Pea sorting

ESTIMATION, REAL-WORLD (Lorrain 3-9)

PEA-SORTING

It is possible to separate normal seeds from discolored ones (or foreign objects) by means of a device that operates as follows. The seeds drop one by one between a pair of photocells. If the color is wrong, a needle deposits a small charge on the seed. The seeds then fall between a pair of electrically charged plates that deflect the undesired ones into a separate bin. (One such machine can sort about 2 metric tons per 24 hour day!)

Setup: If 100 seeds fall per sec, over what distance must they fall if they are to be spaced vertically by 20 mm when they pass between the photocells? (Neglect air resistance.)

Then: Assume the seeds are about 1 gram, and acquire a charge of 1nC (that's nano, 1E-9), the deflecting plates are parallel and 1cm apart, and the potential difference between them is 10,000 volts. How far should the plates extend below the charging needle if the charged seeds must deflect by 45 mm after leaving the plates? (Assume the charging needle and the top of the deflecting plates are close to the photocell.)

Assigned in SP08 (extra credit)

Instructor notes: Very few people did this. Most have the right approach but are off by a large amount. Some get a number which does not make physical sense.

Question 2. Electric potential of point charge – expansion

EXPANSION, EXPLANATION (C. Gwinn, UC Santa Barbara)

a) Find a series for the function $1/(1+\epsilon)$ as a sum over powers of ϵ , near the point $\epsilon = 0$. You may use Taylor series, or look up “binomial expansion” on en.wikipedia.org or elsewhere.

b) Write down the electric potential $V(z)$, along the z -axis for $-1 < z < 1$, for a point charge $+q$ on the z -axis at $z = +1$. Use your result from part a to express this as a

power series $\sum_{l=0}^{\infty} a_l z^l$

c) Express the potential for a point charge $+q$ on the z -axis at $z = +1$ as a sum of Legendre polynomials in $\cos \theta$ and powers of r , times coefficients a_l . Clearly, this potential depends only on r and θ , never on ϕ , so such an expression must exist.

Why is your expression different from that for a point charge at the origin, $V \propto 1/r$?

Could one say that a single charge has a dipole moment? Explain.

d) Express $1/(1+z)$ as a power series in $u = 1/z$, and use your results to find the potential in Legendre polynomials and powers of $1/r$. What is the region of convergence of this power series? What is that of your result from part c)?

Question 3. Screened Coulomb Potential

POTENTIAL; CONNECTIONS AND EXTENSIONS (Reitz 2-18)

Similar from (Berkeley, Stamper-Kurn)

Consider the “screened Coulomb potential” of a point charge of charge q that arises, e.g. in plasma physics:

$$V(\mathbf{r}) = \frac{q}{4\pi\epsilon_0} \frac{e^{-r/\lambda}}{r}$$

, where λ is a constant (called the screening length).

a) Determine the E-field $\mathbf{E}(\mathbf{r})$ associated with this potential.

b) Find the charge distribution $\rho(\mathbf{r})$ that produces this potential. (Think carefully about what happens at the origin!)

- Sketch this function $\rho(\mathbf{r})$ in a manner that clearly describes its characteristics (i.e. what’s the best way of representing this three-dimensional charge distribution? Use it, and explain what you’re plotting).

c) Show by explicit calculation over $\rho(\mathbf{r})$ that the net charge represented by this

distribution is zero (!) (If you don't get zero, think again about what happens at $r=0$)

- Verify this result using the integral form of Gauss’ law (i.e. integrate your electric flux over a *very large* spherical surface. By Gauss, that should tell you $Q(\text{enclosed})$)

Assigned in SP08 (average score: a) 4, b) 6.2, c) 3.5)

Assigned in FA08

Instructor notes: This one is clearly very hard (because there is a subtle trick at the origin, where a delta function appears!). Almost every student required instructor help in discovering the delta function. Students did not make a lot of effort to physically interpret their answer. The problem could be reworded to explicitly ask what the system “looks like” near the origin. It may also be worthwhile to tell or ask what is “screened” about this potential. Many students forget to use the correct direction when calculating E.

Question 4. Questions about potential and boundary value problems

THINKING DEEPER: Questions about Potential and boundary conditions (see

P435_Lect_03_QA.pdf in Univ of Illinois Questions for Lecture). There are some useful HW extension questions in here.

Question 5. Sketch gradient of scalar potential

POTENTIAL, SKETCH

Find the gradient of the scalar potential $\phi(x, y, z) = \alpha xy$. Provide a clear sketch of the contour lines of ϕ in the \hat{x} - \hat{y} plane and a representation of its gradient field. Such a field

is known as a radial quadrupole field, and is used in the focusing of charged particle beams and of dipolar (electric or magnetic) particles.

b. Provide a clear sketch (in the \hat{x} - \hat{y} plane) of the vector field expressed in cylindrical coordinates as $E = \alpha \rho \hat{\rho}$ where α is a constant. Calculate its divergence. If E is an electric field, what charge distribution generates it?

Question 6. Potential above infinite flat sheet

CALCULATION (deGrand)

Find the electrostatic potential a distance z above infinite flat sheet carrying a uniform charge per unit area σ by (a) integrating E (b) directly integrating the “standard formula” for V . In part (b) you will have to be very careful to compute a difference of potentials between two points, or something similar, to avoid integrals which are infinite!

Question 7. Potential of infinitely long straight wire

CALCULATION (deGrand)

Find the electrostatic potential a distance r from an infinitely long straight wire containing a uniform charge per unit length λ by (a) integrating E (b) directly integrating the “standard formula” for V . In part (b) you will have to be very careful to compute a difference of potentials between two points, or something similar, to avoid integrals which are infinite!

Question 8. Electric field of infinite wire

COULOMB, COMPARISON, REASONING. (OSU Paradigms, Symmetries HW 6)

4. (a) Find the electric field around an infinite, uniformly charged, straight wire, starting from the expression for the electrostatic potential that we found in class:

$$V(\vec{r}) = \frac{2\lambda}{4\pi\epsilon_0} \ln \frac{r_0}{r}$$

- (b) Find the electric field around a finite, uniformly charged, straight wire, at a point a distance r straight out from the midpoint, starting from the expression for the electrostatic potential that we found in class:

$$V(\vec{r}) = \frac{\lambda}{4\pi\epsilon_0} \left[\ln \left(L + \sqrt{L^2 + r^2} \right) - \ln \left(-L + \sqrt{L^2 + r^2} \right) \right]$$

- (c) Find the electric field around an infinite, uniformly charged, straight wire, starting from Coulomb's Law.
- (d) Find the electric field around a finite, uniformly charged, straight wire, at a point a distance r straight out from the midpoint, starting from Coulomb's Law.

Question 9. Electric field due to ring of charge

WRITING, REASONING, LIMITING CASES. (OSU Paradigms, Vector Fields HW8)

We have rubrics and discussion from OSU on their writing assignments.

6. Writing Assignment: Write a complete solution to the following problem. Keep the Guiding Questions and the Evaluation Rubric in mind when writing this assignment.

- (a) Find an integral expression (that Maple could evaluate) for the electric field everywhere in space due to a ring of charge. Assume the ring has a uniform charge density, a radius R , and that the total charge on the ring is Q .
- (b) Find the electric field and the electric potential due to this ring of charge for all locations on the z -axis. Comment on your answers.
- (c) In your discussion section, explore the similarities and differences between your answer for the potential and your answer for the electric field. There are many things one might discuss such as the relationship of the physics to the mathematics and/or various limiting cases. Make your own professional choice about what to discuss. Since you don't have the results of other students, it would not be relevant to comment about them.

Question 10. Potential of ring of charge

WRITING, REASONING, EXPANSION. (OSU Paradigms, Symmetries HW 5)

We have rubrics and discussion from OSU on their writing assignments.

2. (a) Starting with the integral expression for the electrostatic potential due to a ring of charge

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \frac{\lambda R d\phi'}{\sqrt{r^2 + R^2 - 2rR \cos(\phi - \phi') + z^2}}$$

find the value of this potential everywhere along the axis of symmetry.

- (b) Find the electrostatic potential everywhere along the axis of symmetry due to a finite disk of charge with uniform (surface) charge density σ . Start with your answer to part (a)
 - (c) Find two nonzero terms in a series expansion of your answer to part (b) for the value of the potential very far away from the disk.
 - (d) Find the electrostatic potential due to an infinite disk, using your results from part (b).
3. (a) Give an expression for the electric potential $V(\vec{r})$ at a point \vec{r} due to a point charge located at \vec{r}' .
- (b) Give an expression for the electric field $\vec{E}(\vec{r})$ at a point \vec{r} due to a point charge located at \vec{r}' .
 - (c) Working in your favorite coordinate system, compute the gradient of V .
 - (d) Write several sentences comparing your answers to the last two questions.

Question 11. Finding V via relaxation method

RELAXATION METHOD (U WASHINGTON)

1† Here is a simple relaxation problem that can be done with a hand calculator. Consider a right-triangular space with potential $V = 0$ on the two perpendicular sides, each of length unity, and $V = 120$ on the hypotenuse. At the corners take $V = 60$. Divide the space into a grid, as shown, with six rows and six columns. There are only six points at which the potential is unknown. Use the relaxation method to calculate the potential at these six points starting with the initial guess show. Show your results after seven iterations.

60					
0	120				
0	0	120			
0	0	0	120		
0	0	0	0	120	
0	0	0	0	0	60

2* Use the relaxation method to calculate the potential within a square with sides of length $L = 1$ ($0 \leq x, y \leq 1$). The potentials on the boundaries are fixed at $V(0, y) = V(x, 0) = 0$, and $V(1, y) = V(x, 1) = 240$. Set the potential $V(0, 1) = V(1, 0) = 120$. Use a 12 by 12 grid of points with the first and last row and column fixed by the boundary conditions. Stop the iterations after the root mean square change in potential from one iteration to the next is less than 0.001.

Question 12. Cathode ray tube

ESTIMATION, VOLTAGE, FORCE (Lorrain 3-5, adapted)

Estimate the downward deflection of an electron as it travels from back of CRT to the front. 10000 volts. They should estimate it – maybe ½ m from front to bck, but don't give them that. If your TV screen has pixel density of (300 x 300?) do the engineers have to worry about this? (Downward deflection is less than one pixel).

Question 13. Electrical levitation

VOLTAGE

Electrical levitation of small conducting sphere. Micrometer diameter copper. Can you levitate that between two capacitor plates. What sort of voltage do you need to levitate a copper sphere the size of a red blood cell?

And can ask stability questions. Displace a mm above middle, what happens.

Question 14. Jacob's Ladder

VOLTAGE, ESTIMATION

(Describe the Jacob's Ladder setup). The spark heats the air, and convection pulls the spark up the rails. At the bottom of the rails are two spherical copper balls, one positively charged and one negatively charged, a distance d apart. (Can model these as two point charges, or spherical capacitors). The two balls are brought together until a spark forms. Estimate the voltage difference you need between the balls to create a spark (or, estimate the distance d at which a spark is first seen; or estimate the E field as a function of distance).

Question 15. Van de Graaf

VOLTAGE AND CHARGE

Estimate number of charges on surface of copper conductor. Put voltage on a conductor, like van de graaf, and figure out how much charge on it. Is that an appreciable fraction of conduction electrons?

Question 16. Gradient and curl of field

POTENTIAL – CHAPTER 2 AND 5?

(Give a field which has no div or curl, ask them to show that it can be written as the gradient of a scalar AND the curl of a vector. Find the scalar and vector potentials?)

Question 17. Find charge distribution given potential

LAPLACIAN; CALCULATION (Zimmerman; have solns)

What charge distribution gives a potential in cylindrical coordinates $V(r) = r^2 \exp(-\frac{r}{a})$?

$$(\nabla^2 V = -\frac{\rho}{\epsilon_0})$$

Question 18. Finding voltage from charge distribution

a) Find a formula for the electrostatic potential $V(z)$ everywhere along the symmetry-axis of a charged ring (radius a , centered on the z -axis, with uniform linear charge density λ around the ring) Please use the method of direct integration (Griffiths 2.30, on p. 85) to do this, and set your reference point to be $V(\infty)=0$.

- Sketch $V(z)$.

- How does $V(z)$ behave as $z \rightarrow \infty$? (Don't just say it goes to 0. HOW does it go to zero?

Does your answer make physical sense to you? Briefly, explain.)

b) Use your result from part a for $V(0,0,z)$ to find the z -component of the E field anywhere along the z -axis.

- What is the Voltage at the origin?

- What is the E field right at the origin?

- Do both of these results (for V , and E , at the origin) make physical sense to you, and are they consistent with each other? Explain briefly!

Assigned in SP08 (average score: a) 8, b) 8.4)

Assigned in FA08

Instructor notes: Many students are not thinking about the integral form for finding voltage as being a "sum over dq's." Many people set up the integral form for voltage, but then their "lambda dL" became "lambda dz" so the integral was difficult. A lot of people miss the point about the consistency between E and V at $z=0$. Only about 20% of students get the correct graph in (a). A common mistake is to draw a sharp peak at $z=0$ or blowing up at $z=0$. Still some issues about expanding for large z .

Question 19. Calculating voltage from E field

Last week, we investigated the electric field outside an infinite line that runs along the z-axis, $\vec{E} = \frac{2\lambda}{4\pi\epsilon_0} \frac{\vec{s}}{s^2}$.

a) This field may look similar to Q2 above, but it is *not* the same - how is it different?

- Find the potential V(s) for points a distance "s" away from the z-axis.

(Note: you will have to be very careful to compute a difference of potentials between two points, or something similar, to avoid integrals which are infinite! You'll discuss this in part b)

- Check your answer by explicitly taking the gradient of V to make sure it gives you \vec{E} .

b) Briefly discuss the question of "reference point": where did you set V=0? Can you use $s=\infty$, or $s=0$, as the reference point, V(s)=0, here?

- How would your answer change if I told you that I wanted you to set V=0 at a distance $s=3$ meters away from the z-axis?

- Why is there trouble with setting $V(\infty)=0$? (our usual choice), or $V(0)=0$ (often our second choice).

c) A typical Colorado lightning bolt might transfer a few Coulombs of charge in a stroke. Although lightning is clearly not remotely "electrostatic", let's pretend it is - consider a brief period during the stroke, and assume all the charges are fairly uniformly distributed in a long thin line. If you see the lightning stroke, and then a few seconds later hear the thunder, make a *very rough* estimate of the resulting voltage difference across a distance the size of your heart. (For you to think about - why is this not worrisome?)

What's the model? I am thinking of a lightning strike as looking rather like a long uniform line of charge... You've done the "physics" of this in the previous parts! (But e.g., you need a numerical estimate for λ . How long might that lightning bolt be? For estimation problems, don't worry about the small details, you can be off by 3, or even 10, I just don't want you off by factors of 1000's!)

Assigned in SP08 (average score: a) 7.4, b) 7.4, c) 8)

Instructor notes: Students often plugged away at the problem without asking why you couldn't set $V(\infty)=0$; they could do the problem but not interpret it. So Many people argue that $s=\infty$ is not a good reference point because the line charge extends to infinity, thus creating a singularity; however, the line charge extends in z, not s.

Question 20. Calculating voltage from E field

A) Use Gauss's law to show that the electric field a distance s from an infinite line of

uniform charge density λ is $\vec{E} = \frac{\lambda}{2\pi\epsilon_0} \frac{\hat{s}}{s}$.

B) Find the voltage difference between any two points near the line. Check your answer by explicitly taking the gradient of V to compute \vec{E} .

C) Briefly discuss the question of "reference point". Where can you set V=0 in this problem? Can you use $s = \infty$, or $s = 0$, as the reference point, V(s)=0? Explain, briefly.

D) Suppose a lightning stroke briefly created a long line of charge with a charge per length of say, 1 C/100 m. You see the stroke and a couple of second later you hear it. Estimate the voltage difference caused by the lightning across an object the size of your heart, where you are standing. Does the answer surprise you?

Assigned in FA08

Question 21. Allowed E fields

Which of the following two static E-fields is physically *impossible*. Why?

i) $\mathbf{E} = c(2x\hat{\mathbf{i}} - x\hat{\mathbf{j}} + y\hat{\mathbf{k}})$

ii) $\mathbf{E} = c(2x\hat{\mathbf{i}} + z\hat{\mathbf{j}} + y\hat{\mathbf{k}})$

where c is a constant (with appropriate units)

For the one which IS possible, find the potential $V(r)$, using the *origin* as your reference point (i.e. setting $V(0)=0$)

- Check your answer by explicitly computing the gradient of V .

Note: you must select a specific path to integrate along. It doesn't matter which path you choose, since the answer is path-independent, but you can't compute a line integral without having a particular path in mind, so be explicit about that in your solution.

Assigned in SP08 (average score: 8)

Assigned in FA08

Instructor notes: Students still struggle with the mechanics of doing a line integral. For example, they'd say $\int z dy = zy$, without noticing that $z=0$ along the segment they were integrating over!

Question 22. Shells of charge

Two charged droplets of toner ink behave as two *shells* of uniform surface charge density. (Toner is an insulating material, not metal.) Shell 1 has total charge Q_1 , radius R_1 , with its center at position \mathbf{r}_1 . Shell 2 has Q_2 , R_2 , and \mathbf{r}_2 . Write expressions for the voltage $V(\mathbf{r})$ everywhere: outside both spheres, inside sphere 1, and inside sphere 2. (Hint: use curly r notation, but define your symbols carefully.) Is the voltage inside shell 1 constant?

