Transformed E&M I homework

Magnetic Vector Potential
(Griffiths Chapter 5)

Magnetic Vector Potential A

Question 1. Vector potential inside current-carrying conductor

Lorrain, Dorson, Lorrain, 15-1 pg. 272
Show that, inside a straight current-carrying conductor of radius R,

$$A = \frac{\mu_0 I}{2\pi}\left(1 - \frac{\rho^2}{R^2}\right)$$

If A is set equal to zero at $\rho = R$.

Question 2. Multipoles, dipole moment

The vector potential of a small current loop (a magnetic dipole) with magnetic moment $\vec{m}$ is $\vec{\mathbf{A}} = \frac{\mu_0}{4\pi}\frac{\vec{\mathbf{m}}\times\hat{\mathbf{r}}}{r^2}$

A) Assume that the magnetic dipole is at the origin and the magnetic moment is aligned with the +z axis. Use the vector potential to compute the B-field in spherical coordinates.
B) Show that your expression for the B-field in part (a) can be written in the coordinate-free form $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[\,3\left(\vec{m}\cdot\hat{r}\right)\hat{r} - \vec{m}\,\right]$

Note: The easiest way to do this one is by assuming the coordinate-free form and then showing *that you get your expression in part (a) if you define your z axis to lie along **m**, rather than trying to go the other way around. Coordinate free formulas are nice, because now you can find B for more general situations.*

Question 3. Vector Potential of infinite sheet

Lorrain, Dorson; Example, Pair of Long Parallel Currents pg. 252
Figure 14-10 shows two long parallel wires separated by a distance D and carrying equal currents ***I*** in opposite directions. To calculate ***A*,** we use the above result for the ***A*** of a single wire and add the two vector potentials:

$$A = \frac{\mu_0 I}{2\pi}\left(\ln\frac{L}{\rho_a} - \ln\frac{L}{\rho_b}\right) \qquad (14\text{-}37)$$

Do Bx, By, and Bz at midpoint stretch?

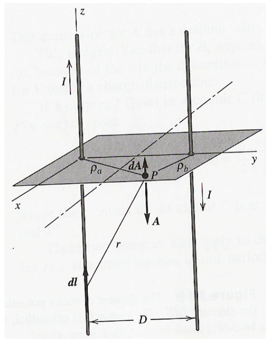


Figure 14-10 Pair of long parallel wires carrying currents of the same magnitude in opposite directions. The vector *A* is zero in the vertical plane that passes through the dashed line, and it points upward on the left and downward on the right.

Question 4. Vector Potential of infinite sheet

CHECK ANSWER, CALCULATION (deGrand)

Find the vector potential above and below an infinite current sheet carrying a uniform current density *K* in the x direction. Check your answer by verifying that computing $\vec{B} = \nabla \times \vec{A}$ gives the known result.

Question 5. Vector potential / magnetic field from dipole

MATH PROOF (deGrand)

Starting with the formula for the vector potential from a magnetic dipole

$$A(r) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

Show that you can write the magnetic field as

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\left(-\frac{\vec{m}}{r^3} + 3\frac{(\vec{m}\cdot\vec{r})\vec{r}}{r^5}\right)$$

Question 6. Vector potential of infinite solenoid

INTEGRAL, ANSWER CHECK (deGrand)

Find the vector potential due to an infinite solenoid of radius R, current I, carrying N' turns per unit length. Try to avoid looking up the answer. It is useful to begin by showing

$$\oint \vec{A}\cdot d\vec{l} = \int \vec{B}\cdot\hat{n}\,dA$$

where the LHS integral is around the edge of the open surface whose area is A.

4) [10 points] As a check of your answer in (3), compute $\vec{\nabla}\times\vec{A}$ in cylindrical coordinates and show that you get the correct $\vec{B}$.

Question 7. Vector potential I

A) A very long wire is a cylinder of radius R centered on the z-axis. It carries a uniformly distributed current I_0 in the +z direction. Assuming $\nabla\cdot\vec{\mathbf{A}} = 0$ (the Coulomb gauge), and choosing A=0 at the edge of the wire, show that the vector potential *inside the wire* could be given by $A(s) = c\,I_0\left(1-\frac{s^2}{R^2}\right)$. Find the constant c (including units.)

What is the vector direction of **A**? (Does it make sense in any way to you?) Is your answer unique, or is there any remaining ambiguity in **A**? *(Note that I'm not asking you to derive **A**. Just check that this choice of **A** works.)*
B) What is the vector potential *outside* that wire? (Make sure that it still satisfies $\nabla \cdot \vec{\mathbf{A}} = 0$, and make sure that **A** is continuous at the edge of the wire, consistent with part (a).)
Is your answer unique, or is there any remaining ambiguity in **A(**outside)?
C) Plot or sketch A(s) vs. s , showing the behavior of A both inside and outside the wire.

Assigned in SP08 (average score: a) 6.85, b) 7.5)
Assigned in FA08
Instructor notes: **Many** students did not try to convince themselves that this form of A does work. A few students did not use the boundary condition that A(s=R)=0, thus missing the "R" in the Ln.

Question 8. Vector potential II

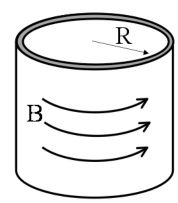
A) Griffiths Fig 5.48 (p. 240) is a nice, and handy, "triangle" summarizing the mathematical connections between **J**, **A**, and **B** (like Fig. 2.35 on p. 87) But there's a missing link, he has nothing for the left arrow from **B** to **A**. Notice that the *equations* defining **A** are really very analogous to the basic Maxwell's equations for **B**:

$$\nabla \cdot \vec{\mathbf{B}} = 0 \quad \Leftrightarrow \quad \nabla \cdot \vec{\mathbf{A}} = 0$$

$$\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}} \quad \Leftrightarrow \quad \nabla \times \vec{\mathbf{A}} = \vec{\mathbf{B}}$$

So **A** depends on **B** in the same way (mathematically) the **B** depends on **J**. (Think Biot-Savart.) Use this idea to just write down a formula for **A** in terms of **B** to finish off that triangle.

B) We know the B-field everywhere inside and outside an infinite solenoid (which can be thought of as either a solenoid with current per length n I or a cylinder with surface current density K = n I). Use the basic idea from part (a) to quickly and easily write down the vector potential **A** in a situation where **B** looks analogous to that, i.e. $\vec{\mathbf{B}} = C\delta(s-R)\,\hat{\varphi}$, with C constant. (Sketch this **A** for us, please) *(You should be able to just see the answer; no nasty integral needed.) It's kind of cool - think about what's going on here. You have a previously solved problem, where a given **J** led us to some **B**. Now we immediately know what **A** is in a* very different *physical situation, one where **B** happens to look like **J** did in that previous problem.*



To discuss: What physical situation creates such a B field? (This is a little tricky, think carefully.)

Assigned in SP08 (average score: a) 8.6, b) 6.1)
Assigned in FA08
Instructor notes: This is a tough question. Many people had trouble understanding the analogy. For a few, I had to explicitly point out the analogy, drawing the diagram above and then drawing a solenoid so that it popped out at you. Note that this analogy is part of the tutorial, but students were still struggling with it. Many said that since

$\nabla^2\vec{A}=\vec{B}$ then they plugged in formula to find that $\vec{A}=\int\nabla\times\vec{B}$. This works, and you can use a vector identity to arrive at $\vec{A}=\int J\times r$.

Lots of questions on the physical situation in part B, and many needed instructor coaching. Many students forgot that A=constant is only INSIDE the solenoid. Also the majority give a solenoid-like picture of A and B without sketching a graph.