

# Transformed E&M I homework

## Work and Energy (Griffiths Chapter 2)

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### Work & Energy

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#### Question 1. Energy of point charge distribution

Imagine a small square (side "a") with four point charges  $+q$ , one on each corner.

Calculate the total stored energy of this system (i.e. the amount of work required to assemble it).

Then, calculate how much work it takes to "neutralize" these charges by bringing in *one* more point charge ( $-4q$ ) from far away and placing it right at the *center* of this square.

*(When studying crystal structures, it is sometimes convenient to model them as rectangular grids of charged ions, this problem forms the starting point for such a model)*

**Assigned in SP08 (average score: 8.6)**

**Assigned in FA08**

Instructor notes: This seemed to be an easy question for students.

#### Question 2. Energy of a continuous charge distribution

A) Find the total electrostatic energy stored in a uniformly charged sphere of radius  $R$  and total charge  $Q$ . (Note: this is uniform throughout the whole volume; it's not a shell.) Express your answer in terms of  $Q$  and  $R$ . There are many different ways to do this; I want you to use two different methods so you can check yourself.

i) figure out  $E(r)$  and then use Griffiths Eq. 2.45 (being careful to integrate over *all space*, not just where the charge is!)

ii) figure out  $V(r)$  and then use Griff. 2.43 (what region do you need to integrate this over?)

B) Sketch  $V(r)$  vs  $r$  and directly below your  $V(r)$  plot, sketch  $E(r)$  vs.  $r$ . Indicate clearly the functional forms both inside and outside the sphere. Your plots of  $V(r)$  and  $E(r)$  should line up vertically so that their relationship is clear. How are  $V(r)$  and  $E(r)$  related?

C) According to Einstein, a static electron has a rest mass energy  $E = mc^2$ . It is tempting (though wrong!) to imagine that the electron is NOT really a point charge, but instead is a tiny sphere, with uniform charge density, whose total electrostatic energy (found above) EQUALS this mass energy of the electron. In other words, the idea would be that the rest energy of the electron is purely electromagnetic.

What radius would an electron have to have, for this to work out? Give formula and numerical answer. *Don't take your result too literally; a correct model must take quantum mechanics into account.*

D) Compute the electrostatic energy of an atomic nucleus with  $Z$  protons and  $N$  neutrons, or in other words,  $A = N + Z$  nucleons. Use an approximation for the nuclear radius of  $R = (1.2 \times 10^{-15} \text{ m}) A^{1/3}$ . You can assume that the nucleus is a uniformly charged sphere. Convert your answer, which is likely in Joules, to the energy units of

MeV. (Give your result in units of MeV times  $Z^2 / A^{1/3}$ ) Now use this result to estimate the *change* of electrostatic energy when a uranium nucleus undergoes fission into two roughly equal-sized nuclei. Go online and check what the experimental answer is. What does this tell you about the source of the energy released in a nuclear explosion ? Is it mostly electrostatic?

**Assigned in SP08 (average score: ai) 8.4, aii) 7.2)**

**Assigned in FA08**

Instructor notes: (a) There was a lot of confusion on this question. Such as, why can't we use  $V(r=0)=0$ ? Why do you stop integrating  $\rho * V$  at the edge, when there is clearly more energy "outside". Why do we integrate over all space of (a) but just the charge density for (b)? We had to move charge through all space in order to create the charge distribution, so why didn't we have to integrate over the space outside the sphere? Which  $V$  do we use -  $V(r)$  or  $V(R)$ ? Some wanted to use  $V(R)$  even outside the sphere. Students are having a lot of difficulty conceptualizing the formulas for energy physically.

Also, most students calculated  $V(r)$  by setting  $V(0) = 0$ , and just integrating from  $r=0$  to  $r=r$ . Firstly, they didn't recognize that this was different from what we usually do ( $V=0$  at infinity) or that this was what they were implicitly assuming by doing that integral. The reason that didn't work was because in the formula for energy, the voltage is implicitly set to zero at infinity, but that formula is not transparently understood.

(b) Quite a few questions about the \*language\* in ii) What is a "nucleon"? What is "A"? Why  $A^{1/3}$ ? The problem as stated was just a little confusing, they weren't quite sure how to get started or what exactly I was asking. This problem could be reworded. Also, some students started to use Coulomb's Law (instead of Gauss) to calculate  $E$  inside, and were still unsure how to set up Gauss' Law.

### Question 3. Voltage of surface of the sun

ENERGY, REAL-WORLD (Lorrain et al, p 45/46)

The surface of the sun has a voltage due to the escape of electrons from its surface. Both the electrons and protons on the surface of the sun are hot, so they have the same kinetic energy ( $T$ ) as defined by the temperature on the surface of the sun. However, since the electrons have a lower mass than the protons, they are more likely to escape. They escape until electrical equilibrium is reached. All remaining electrons and protons must have a total energy ( $E=U+T$ ) below a certain threshold,  $E_{\text{escape}}$ , in order to remain on the surface of the sun. If both electrons and protons that remain on the surface have an equal likelihood of escape, then they will all have the same potential energy ( $U$ ). If you calculate that potential, you will find that the potential on the surface of the sun is about 1000 V. Use this information to estimate the net charge of the sun.

*We have to decide how much information to give them to solve this problem.*

### Question 4. Speed of electron in hydrogen

REAL-WORLD, ESTIMATION, FORCE

Hydrogen is 1 angstrom in diameter, estimate E force on an electron using  $F=ma$  and  $mv^2/r$ . How fast is the electron going by electrical forces? Is it faster than c? (this is a trivial problem but they have to look up these numbers).

### Question 5. Compute radius of electron

REAL-WORLD (Downloaded from Reed)

One might imagine that the electron's mass is electrostatic in origin. Assume that the electron is a uniform charged sphere; use your results from problem 11.3 (finding energy stored in uniformly charged sphere of radius  $R$  and charge  $q$ ) and the rest energy of the electron ( $mc^2$ ) to compute the classical radius  $r_0$ . You might find it convenient to express your answer in terms of the *fine structure constant* and *Bohr radius* before plugging in numbers:

$$\alpha = \frac{e^2}{4\pi\epsilon_0}$$

### Question 6. Calculate radius of electron 2

REAL WORLD; CALCULATION (Zimmerman, have solns)

ELECTRON MASS 3

The mass of an electron is  $m_e = 9.11 \times 10^{-31}$  kg. The relativistic mass energy of an electron is  $m_e c^2$ . Given the apparent infinite electrostatic energy associated with a point charge, it is tempting (though wrong) to view the electron not as a point charge, but as a sphere of uniform charge density whose electrostatic energy is equal to the mass energy of an electron. What would the radius of such a ball of charge be? (The electron's charge, which has the special symbol  $e$ , is  $-1.60 \times 10^{-19}$  coulombs).

### Question 7. Energy to hold together the electron

REAL-WORLD; CALCULATION (deGrand)

ELECTRON MASS 2

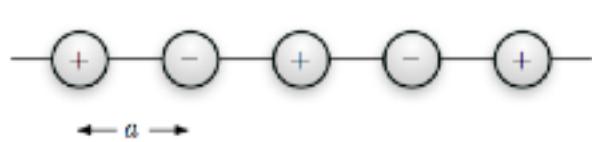
In the pre-quantum days people thought of the electron as a little charges sphere of radius  $R$  (called the "classical electron radius") and tried to set the rest mass of the electron  $m_e c^2$  equal to the electrostatic binding energy. Assume for a change of pace that the electron is a charged shell of radius  $R$  with a uniform surface charge density. Find (a) the energy required to hold the shell together (many ways to do this) (b) the formula relating  $R$  and  $m_e c^2$  and (c) a numerical value for  $R$  (it will be about  $10^{-13}$  cm)

### Question 8. Work to assemble chain of charges

ESTIMATION, SERIES EXPANSION (Downloaded from Reed)

Consider an infinite chain of alternating point charges  $q$  and  $-q$  as shown below.

Working from Equation 2.41 in Griffiths, find the work per particle required to assemble such a configuration. You can look up the sum or work out the Taylor series for  $\log(1 + x)$  to recognize it.



This calculation would provide a good estimate of the electrostatic energy stored in an ionic crystal (such as NaCl). In dimensions greater than one, the calculation of these sums (referred to as Coulomb-Madelung sums) must be carried out numerically.

### Question 9. Energy of cylindrical capacitor

CALCULATION (deGrand)

Find the electrostatic energy per unit length stored in a long cylindrical capacitor (inner radius  $a$ , outer radius  $b$ ) with a potential difference between the plates equal to  $V$ , in two ways: (a) Find  $C$  and use  $W = \frac{1}{2}CV^2$ . (b) Use  $W = (\frac{\epsilon_0}{2}) \int E^2 d^3x$ : you must find and integrate  $E$ . In this part, finish by expressing the charge per unit length  $\lambda$  in terms of  $V$  to show that you get the same answer in (b) that you got in (a).

### Question 10. Energy of cylindrical capacitor 2

CALCULATION (Zimmerman, have solns)

A very long coaxial capacitor has an inner conducting cylinder with radius  $a$  and an outer conducting cylinder with radius  $b$  and a potential difference  $V$  is maintained between the conductors. You may treat these as infinite cylindrical cells. What is the energy stored per unit length in the capacitor? Work this two ways:

- (a) Find the capacitance  $C$  and use  $W=1/2CV^2$
- (b) Integrate the energy density stored in the electric field

### Question 11. Energy of charged sphere ; atomic radius

REAL-WORLD (From Pollack and Stump, Electromagnetism, Problem 3.28)

- a. Show that the electrostatic energy of a uniformly charged solid sphere, with total charge  $Q$  and radius  $R$  is  $3Q^2 / (20\pi\epsilon_0 R)$ .
- b. Use the result above to compute the electrostatic energy of an atomic nucleus with  $Z$  protons and a total of  $A$  nucleons, using an approximation for the nuclear radius as  $R = (1.2 \times 10^{-15} \text{ m}) A^{1/3}$ . Give your result in units of MeV times  $Z^2 / A^{1/3}$ .
- c. Calculate the change of electrostatic energy when a uranium nucleus undergoes fission.  
*Did you know that the energy released in a nuclear explosion is primarily electrostatic ?*

### Question 12. Work to assemble charges on vertices of icosahedron

WORK, CALCULATION, NEW SITUATION (C. Gwinn, UC Santa Barbara)

A regular icosahedron is a solid geometrical figure with 20 triangular faces. 5 faces meet at each of the 12 vertices. The triangles are equilateral triangles, and all the vertices are identical. (Useful fact: The vertices of an icosahedron lie at the corners of 3 mutually perpendicular golden rectangles. The shorter edges of the rectangles are 6 of the edges of the icosahedron, and the longer edges are longer by a factor of  $\phi = \sqrt{5}/2 + 1/2 \approx 1.618$ .)



- a) Suppose that a charge  $+q$  is placed at each of the 12 corners of a regular icosahedron,

with length of side  $a$ . Find the work required to assemble this charge distribution.

b) A negative charge  $-Q$  is placed at the center of the icosahedron. Find the value of  $Q$  for which the total work to assemble this charge configuration is 0.

For fun: Find the net potential  $V$  at  $r \rightarrow \infty$  of icosahedron plus  $Q$ .

### Question 13. Energy of sodium chloride crystal

REAL-WORLD; COMPUTATION (U WASHINGTON)

2\* The energy of a sodium chloride crystal of  $N$  atoms can be calculated from

$$U = \frac{Ne^2}{4\pi\epsilon_0 a} S,$$

where  $a$  is the lattice constant, and  $S = \lim_{p \rightarrow \infty} S_p$  as  $p \rightarrow \infty$  with

$$S_p = \frac{1}{2} \sum_{l=-p}^p \sum_{m=-p}^p \sum_{n=-p}^p \frac{f_l^p f_m^p f_n^p}{(l^2 + m^2 + n^2)^{1/2}}.$$

The term with  $l = m = n = 0$  is excluded from the sum and  $f_n^p = (-1)^n$  for all  $p$ . Write some computer program in whatever language you are familiar with and calculate  $S_p$  for  $1 \leq p \leq 10$  and present your results as a graph of  $S_p$  vs.  $p$ . You will note that each successive cube has charge of magnitude  $e$ . You should find that your result oscillates and converges slowly.