

GAUSS' LAW

Class Activities: Gauss' Law

Discussion

Gauss vs Coulomb

Discussion re "which is more fundamental, Gauss or Coulomb" (and, why) Let them discuss. (Pointed out the Coulomb came first, historically. And that from one, you can show the other, in statics. But also pointed out Coulomb is *wrong*, but Gauss is always true, in non-static cases. Also pointed out Gauss is always true but not always *helpful* to solve for E in a given problem...)

Whiteboard

Charge distribution from E field

Whiteboard to compute the charge distribution from $E = c r(\text{vector})$ (which is also a clicker question) and then to compute $Q(\text{enclosed})$ of the resulting rho.

Tutorials

Divergence

Paul van Kampen – Dublin University (Tutorials 1-8, page 32)

In document "Tutorials 1-8"

Two students try to calculate the charge density, one uses Cartesian and one uses cylindrical. They calculate the divergence of the E field in both coordinate systems. They derive the divergence formula in cylindrical coordinates. Very nice tutorial.

Tutorials

Boundary condition activity

Oregon State University – Boundary E

Groups investigate how components of electric field change as you cross a boundary.

Tutorials

Gauss' Law

Oregon State University

Students working in small groups practice using Gauss' Law to determine the electric field due to several charge distributions.

Students practice using the symmetry arguments necessary to use Gauss' Law.

Which of the following are vectors?

(I) Electric field

(II) Electric flux

(III) Electric charge

A) (I) only

B) (I) and (II) only

C) (I) and (III) only

D) (II) and (III) only

E) (I), (II), and (III)

The space in and around a cubical box (edge length L) is filled with a constant uniform electric field, $\vec{E} = E\hat{y}$. What is the TOTAL electric flux $\oiint \vec{E} \cdot d\vec{a}$ through this closed surface?

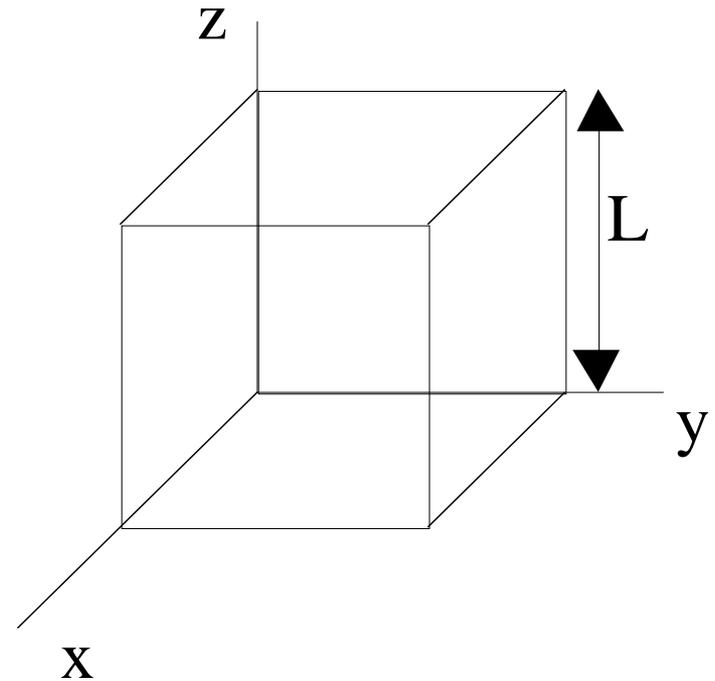
A. Zero

B. EL^2

C. $2EL^2$

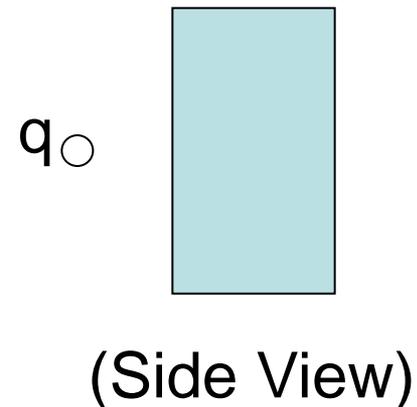
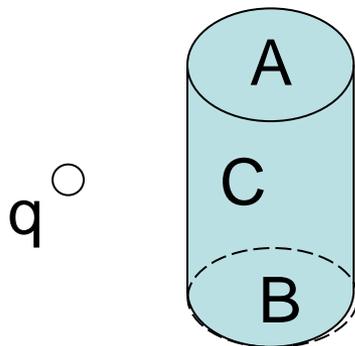
D. $6EL^2$

E. We don't know $\rho(r)$, so can't answer.



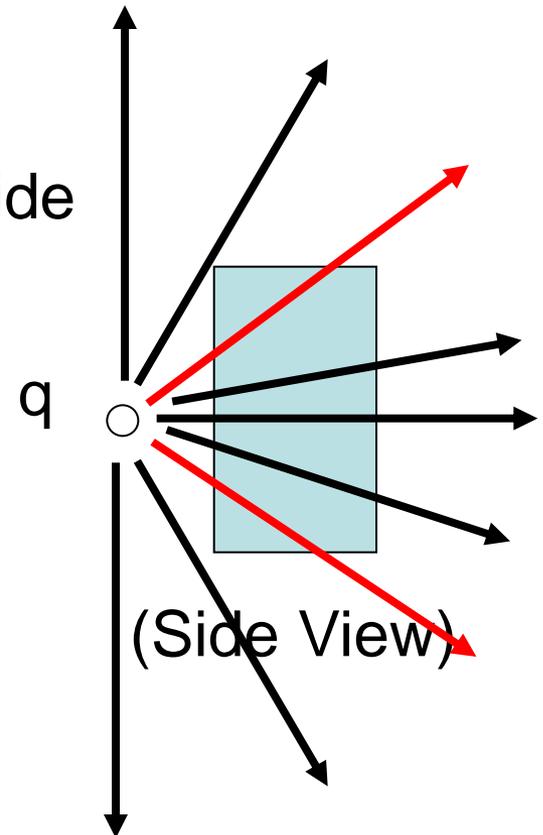
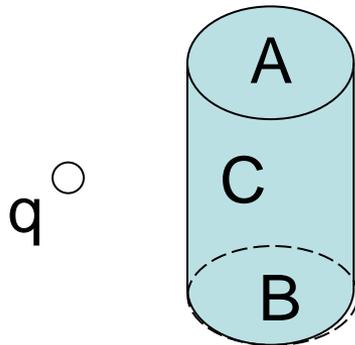
A positive point charge $+q$ is placed outside a closed cylindrical surface as shown. The closed surface consists of the flat end caps (labeled A and B) and the curved side surface (C). What is the sign of the electric flux through surface C?

- (A) positive (B) negative (C) zero
(D) not enough information given to decide

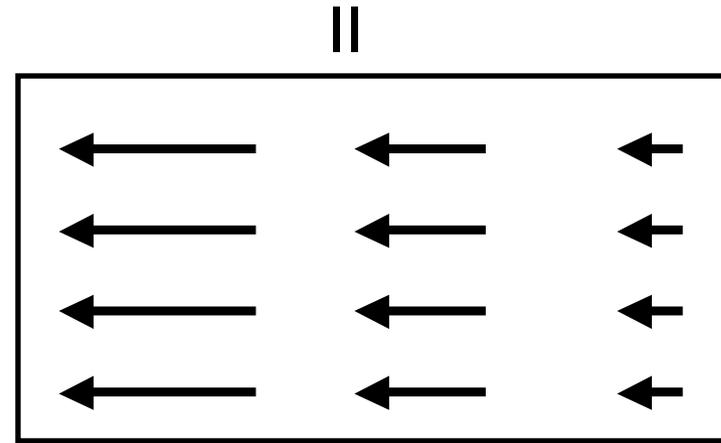
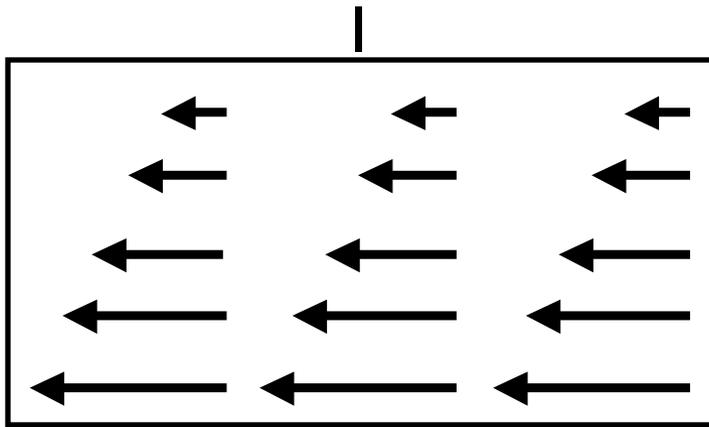


A positive point charge $+q$ is placed outside a closed cylindrical surface as shown. The closed surface consists of the flat end caps (labeled A and B) and the curved side surface (C). What is the sign of the electric flux through surface C?

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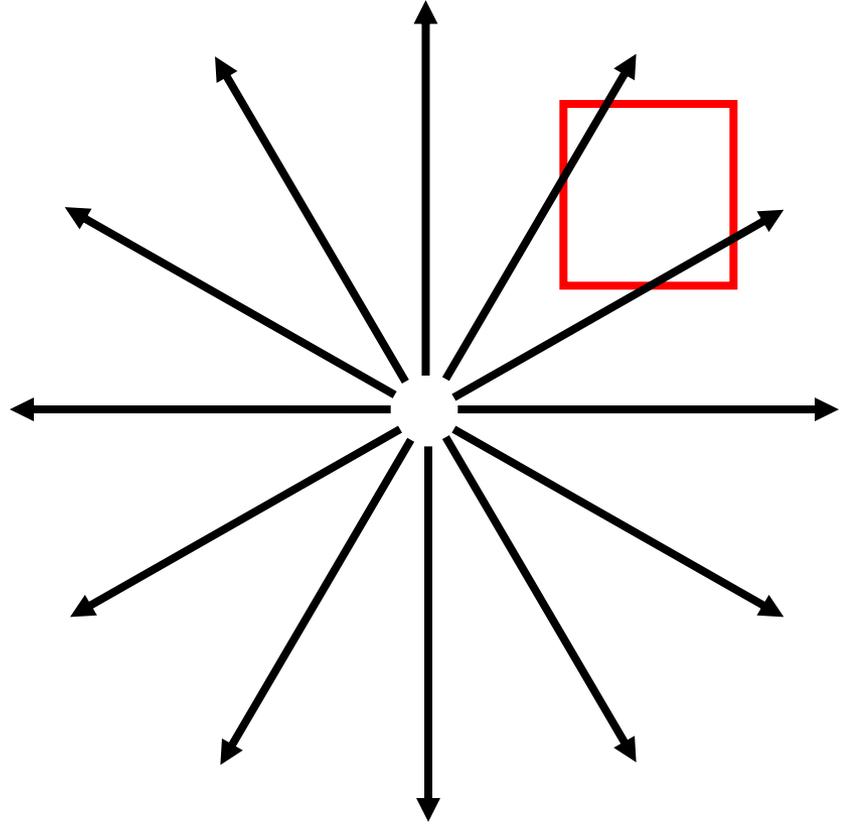
Which of the following two fields has zero divergence?



- A) Both do
 B) Only I is zero
 C) Only II is zero
 D) Neither is zero
 E) ??

What is the divergence of this vector field *in the boxed region*?

- A) Zero
- B) Not zero
- C) ???



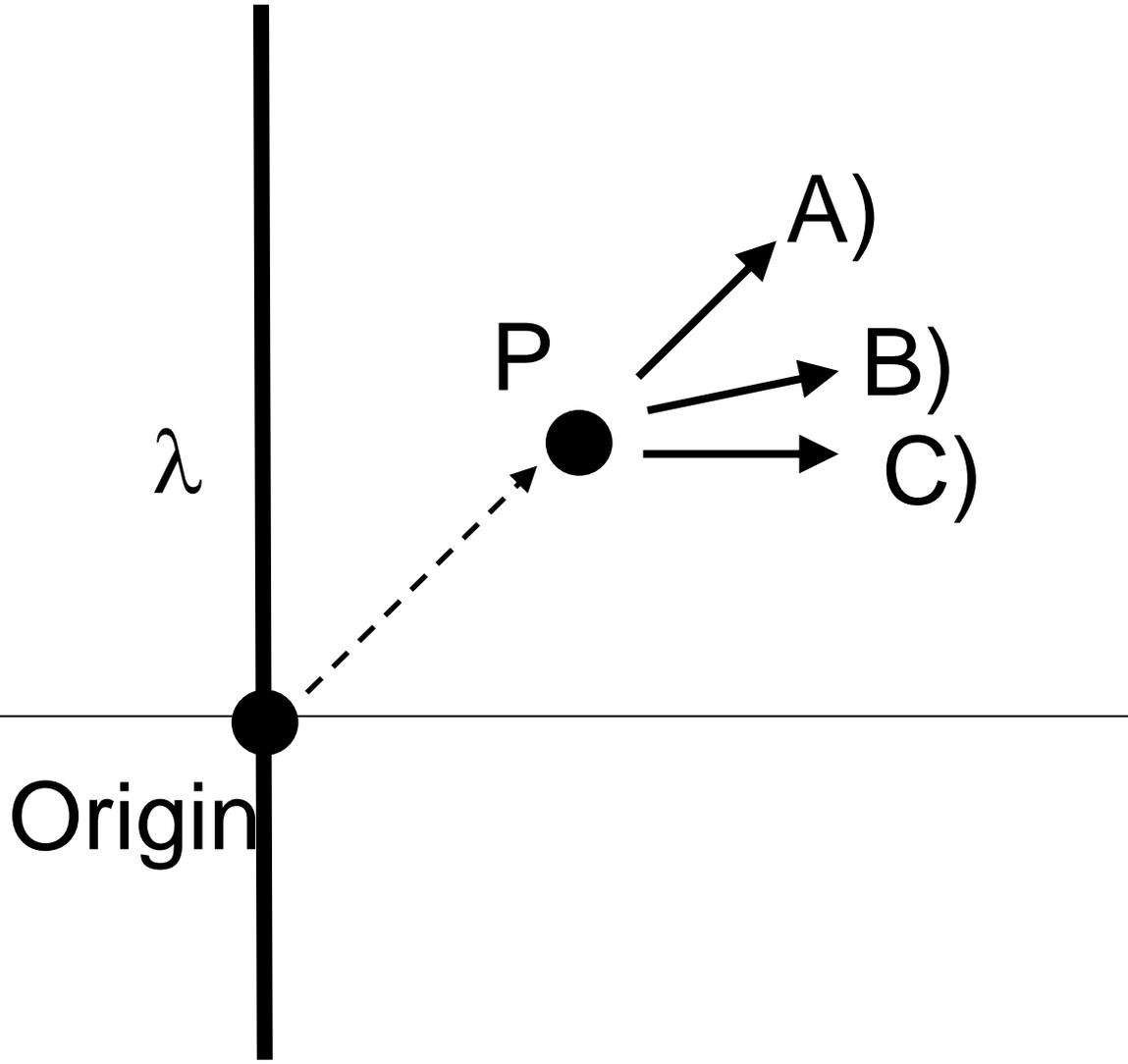
A Gaussian surface which is *not* a sphere has a single charge (q) inside it, *not* at the center. There are more charges outside.

What can we say about total electric flux through this surface $\oint \vec{E} \cdot d\vec{a}$?

- A) It is q/ϵ_0
- B) We know what it is, but it is NOT q/ϵ_0
- C) Need more info/details to figure it out.

2.17

An infinite rod has uniform charge density λ . What is the direction of the E field at the point P shown?

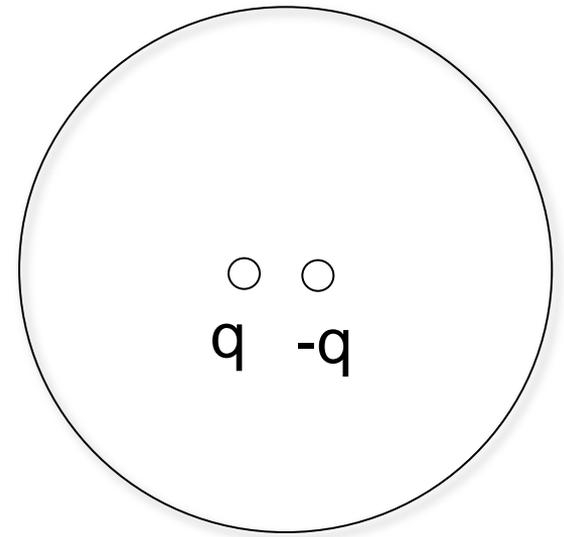


Deep questions to ponder

- Is Coulomb's force law valid for *all* separation distances? (How about $r=0$?)
- What is the physics origin of the r^2 dependence of Coulomb's force law?
- What is the physics origin of the $1/\epsilon_0$ dependence of Coulomb's force law?
- What is the physics origin of the $1/4\pi$ factor in Coulomb's force law?
- What really *is* electric charge?
- Why is electric charge quantized (in units of e)?
- What really is *negative* vs. *positive* electric charge (i.e. $-e$ vs. $+e$)?
- Why does the Coulomb force vary as the *product* of charges $q_1 q_2$?
- What really is the E -field associated with e.g. a point electric charge, e ?
- Are electric field lines real? Do they *really* exist in space and time?

An electric dipole

($+q$ and $-q$, small distance d apart) sits centered in a Gaussian sphere.

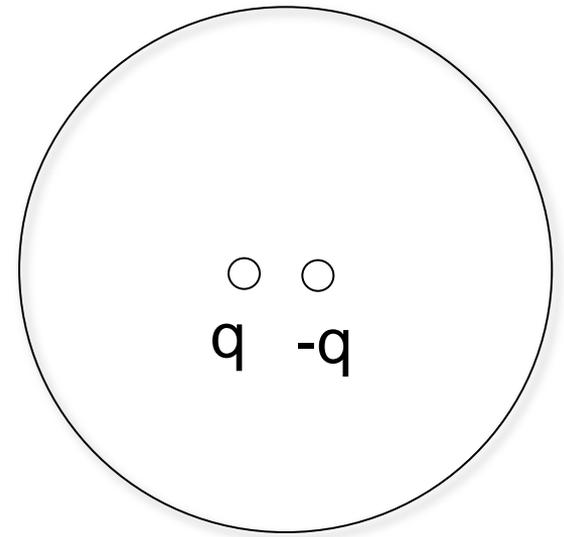


What can you say about the flux of E through the sphere, and $|E|$ on the sphere?

- A) Flux=0, $E=0$ everywhere on sphere surface
- B) Flux =0, E need not be zero *everywhere* on sphere
- C) Flux is not zero, $E=0$ everywhere on sphere
- D) Flux is not zero, E need not be zero...

An electric dipole

(+q and -q, small distance d apart)
sits centered in a Gaussian sphere.



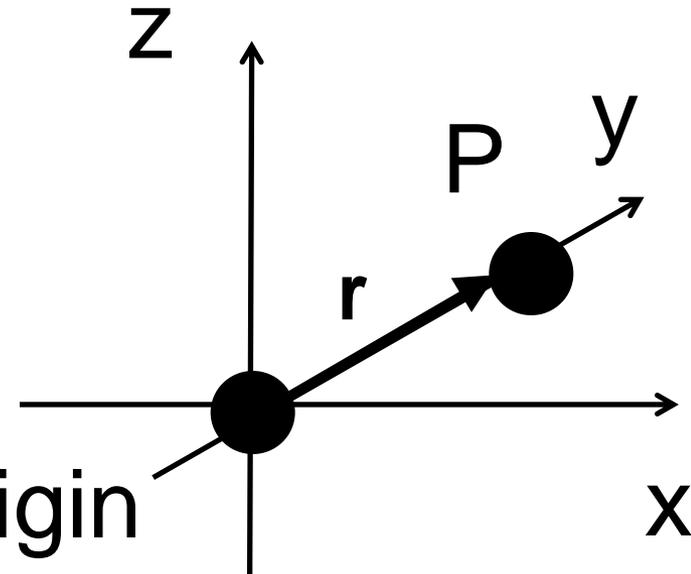
$$\oint_{surf} \vec{E} \times d\vec{a} = \frac{Q_{inside}}{\epsilon_0}$$

What can you say about the flux of E through the sphere, and $|E|$ on the sphere?

- A) Flux=0, $E=0$ everywhere on sphere surface
- B) Flux =0, E need not be zero on sphere
- C) Flux is not zero, $E=0$ everywhere on sphere
- D) Flux is not zero, E need not be zero...

1.1

In spherical coordinates, what would be the correct description of the position vector “ \mathbf{r} ” of the point P shown at $(x,y,z) = (0, 2 \text{ m}, 0)$



A) $\vec{\mathbf{r}} = (2 \text{ m}) \hat{r}$

B) $\vec{\mathbf{r}} = (2 \text{ m}) \hat{r} + \rho \hat{q}$

C) $\vec{\mathbf{r}} = (2 \text{ m}) \hat{r} + \rho \hat{q} + \rho \hat{j}$

D) $\vec{\mathbf{r}} = (2 \text{ m}) \hat{r} + \rho / 2 \hat{q} + \rho / 2 \hat{j}$

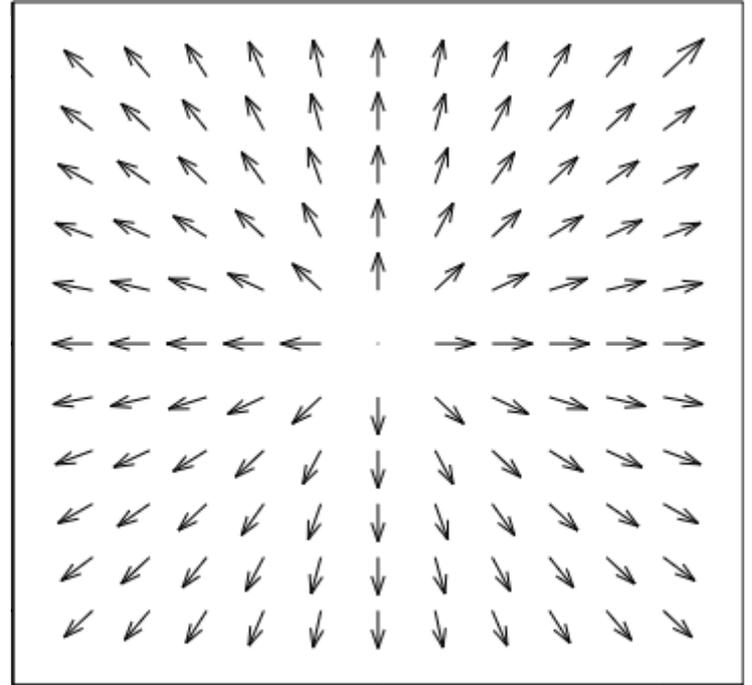
E) None of these

-- WEEK 3 --

Divergence of the E-field, work and energy

Consider the vector field

$$\vec{V}(\vec{r}) = c \hat{r}$$



where $c = \text{constant}$.

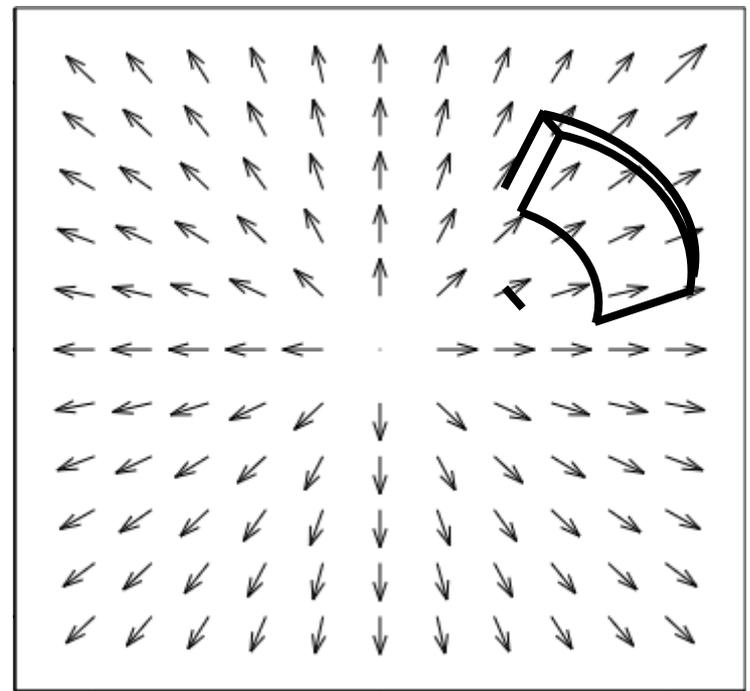
The divergence of this vector field is:

- A) Zero everywhere except at the origin
- B) Zero everywhere including the origin
- C) Non-zero everywhere, including the origin.
- D) Non-zero everywhere, except at origin (zero at origin)

(No fair computing the answer. Get answer from your brain.)

Consider the vector field

$$\vec{V}(\vec{r}) = c \hat{r}$$



where $c = \text{constant}$.

$$\nabla \cdot \vec{V}(\vec{r}) = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 V_r) + \dots (\text{involving } V_\theta \text{ and } V_\phi) \text{ in spherical coords.}$$

The divergence of this vector field is:

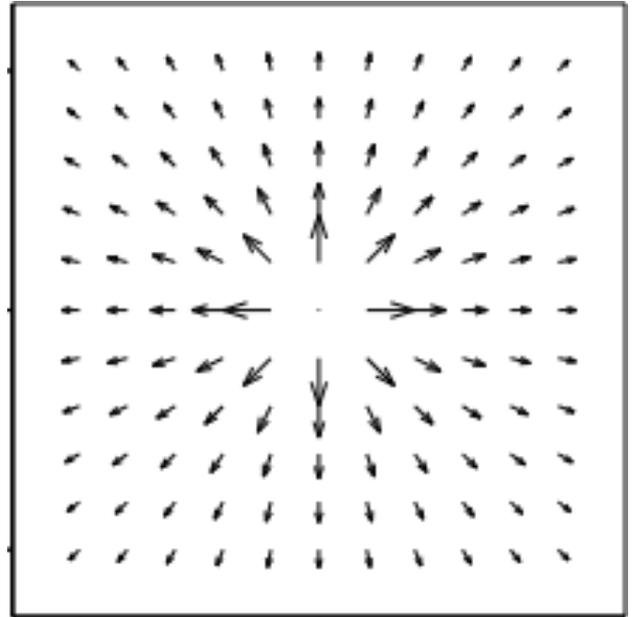
- A) Zero everywhere except at the origin
- B) Zero everywhere including the origin
- C) Non-zero everywhere, including the origin.
- D) Non-zero everywhere, except at origin (zero at origin)

(No fair computing the answer. Get answer from your brain.)

Consider the 3D vector field

$$\vec{V}(\vec{r}) = c \frac{\hat{r}}{r^2}$$

in spherical coordinates,
where $c = \text{constant}$.



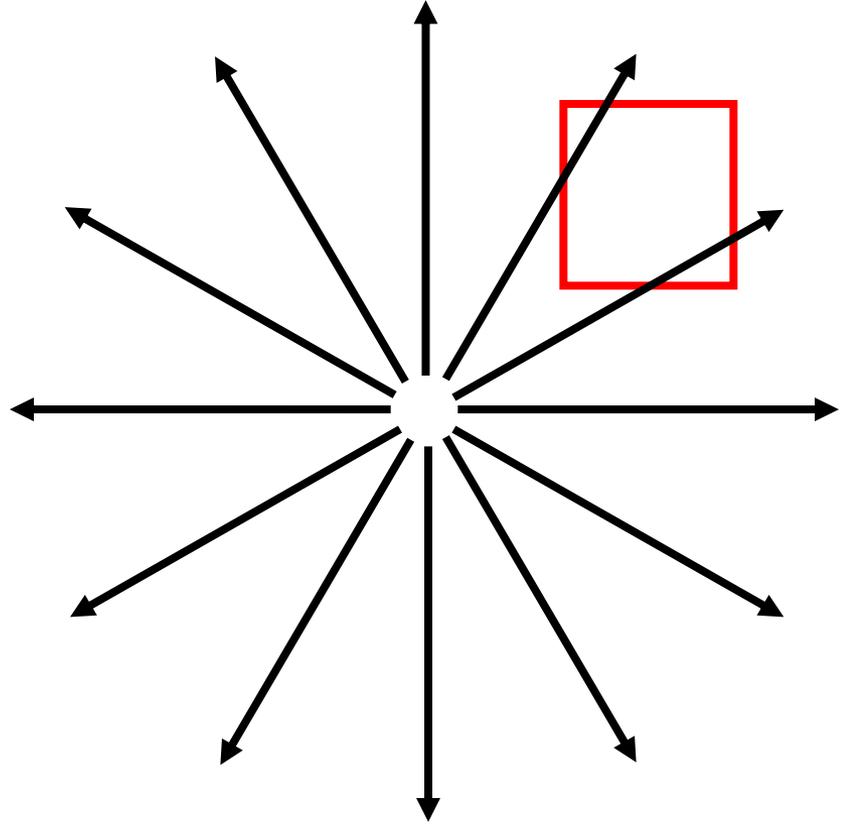
The divergence of this vector field is:

- A) Zero everywhere except at the origin
- B) Zero everywhere including the origin
- C) Non-zero everywhere, including the origin.
- D) Non-zero everywhere, except at origin (zero at origin)

(No fair computing the answer. Get answer from your brain.)

What is the divergence of this vector field *in the boxed region*?

- A) Zero
- B) Not zero
- C) ???



What is the value of $\int_{-\infty}^{\infty} x^2 d(x - 2) dx$

A) 0

B) 2

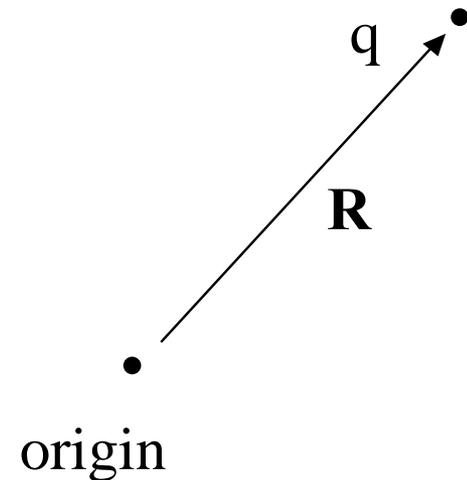
C) 4

D) ∞

E) Something different!

A point charge (q) is located at position \mathbf{R} , as shown. What is $\rho(\mathbf{r})$, the charge density in all space?

- A) $\rho(\mathbf{r}) = q \delta^3(\mathbf{R})$
- B) $\rho(\mathbf{r}) = q \delta^3(\mathbf{r})$
- C) $\rho(\mathbf{r}) = q \delta^3(\mathbf{r} - \mathbf{R})$
- D) $\rho(\mathbf{r}) = q \delta^3(\mathbf{R} - \mathbf{r})$



E) None of these/more than one/???

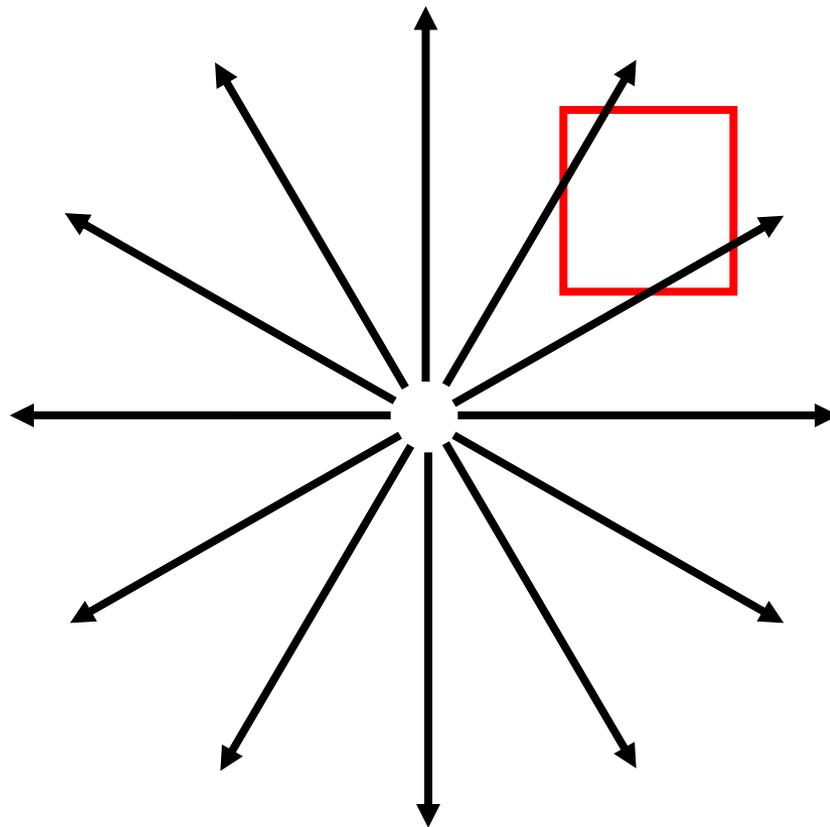
What are the units of $\delta(x)$ if x is measured in meters?

- A) δ is dimensionless (‘no units’)
- B) [m]: Unit of length
- C) [m²]: Unit of length squared
- D) [m⁻¹]: 1 / (unit of length)
- E) [m⁻²]: 1 / (unit of length squared)

What are the units of $\delta^3(\vec{r})$ if the components of \vec{r} are measured in meters?

- A) [m]: Unit of length
- B) [m²]: Unit of length squared
- C) [m⁻¹]: 1 / (unit of length)
- D) [m⁻²]: 1 / (unit of length squared)
- E) None of these.

What is the divergence of this vector field *in the boxed region*?



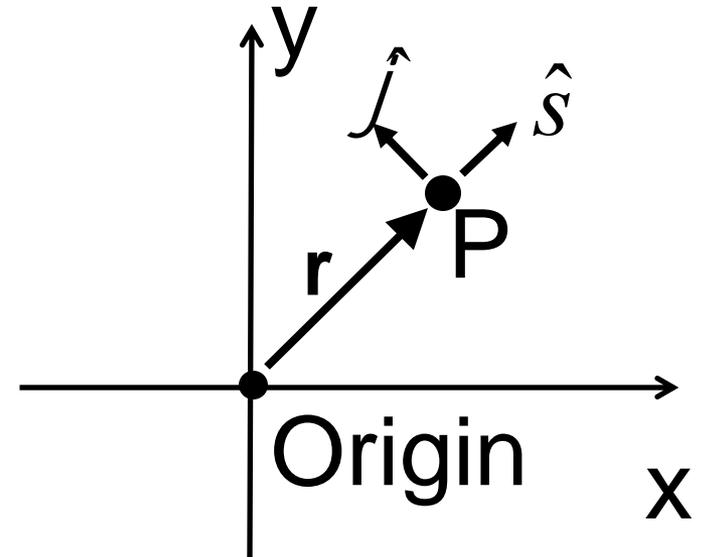
(It's zero there.)

Divergence is nonzero where “ ρ ” is!

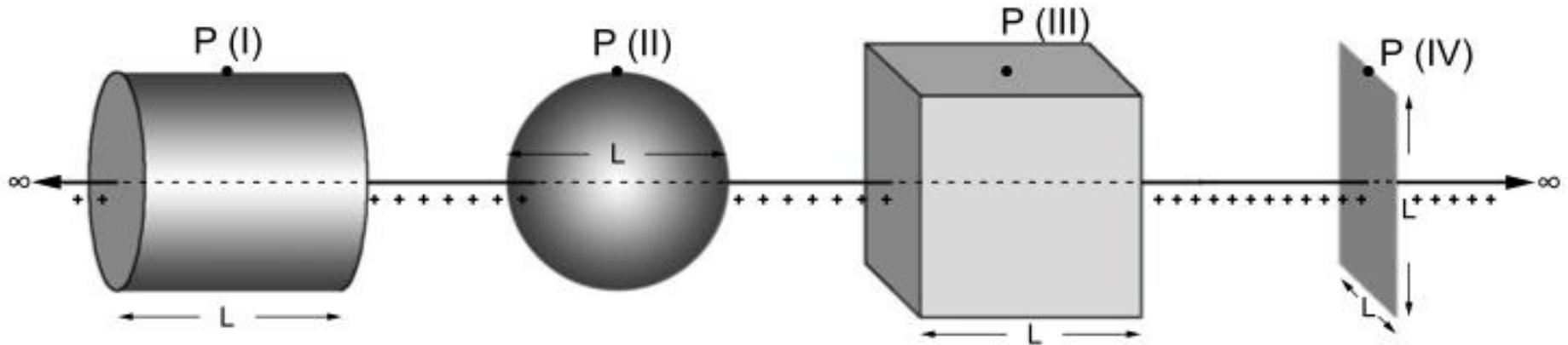
In cylindrical (2D) coordinates, what would be the correct description of the position vector “ \mathbf{r} ” of the point P shown at $(x,y) = (1, 1)$

- A) $\vec{\mathbf{r}} = \sqrt{2} \hat{s}$
- B) $\vec{\mathbf{r}} = \sqrt{2} \hat{s} + \rho / 4 \hat{j}$
- C) $\vec{\mathbf{r}} = \sqrt{2} \hat{s} - \rho / 4 \hat{j}$
- D) $\vec{\mathbf{r}} = \rho / 4 \hat{j}$

E) Something else entirely



4 surfaces are coaxial with an infinitely long line of charge with uniform λ . Choose all surfaces through which $F_E = \lambda L / \epsilon_0$



- A) I only B) I and II only C) I and III only
 D) I, II, and III only E) All four.

You have an E field given by
 $\mathbf{E} = c \mathbf{r} / \epsilon_0$, (Here $c = \text{constant}$,
 $\mathbf{r} = \text{spherical radius vector}$)

What is the charge density $\rho(r)$?

- A) c B) $c r$ C) $3 c$
D) $3 c r^2$
E) None of these is correct

Given $\mathbf{E} = c \mathbf{r}/\epsilon_0$,

($c = \text{constant}$, $\mathbf{r} = \text{spherical radius vector}$)

We just found $\rho(r) = 3c$.

What is the total charge Q enclosed by an imaginary sphere centered on the origin, of radius R ?

Hint: Can you find it two DIFFERENT ways?

A) $(4/3) \pi c$

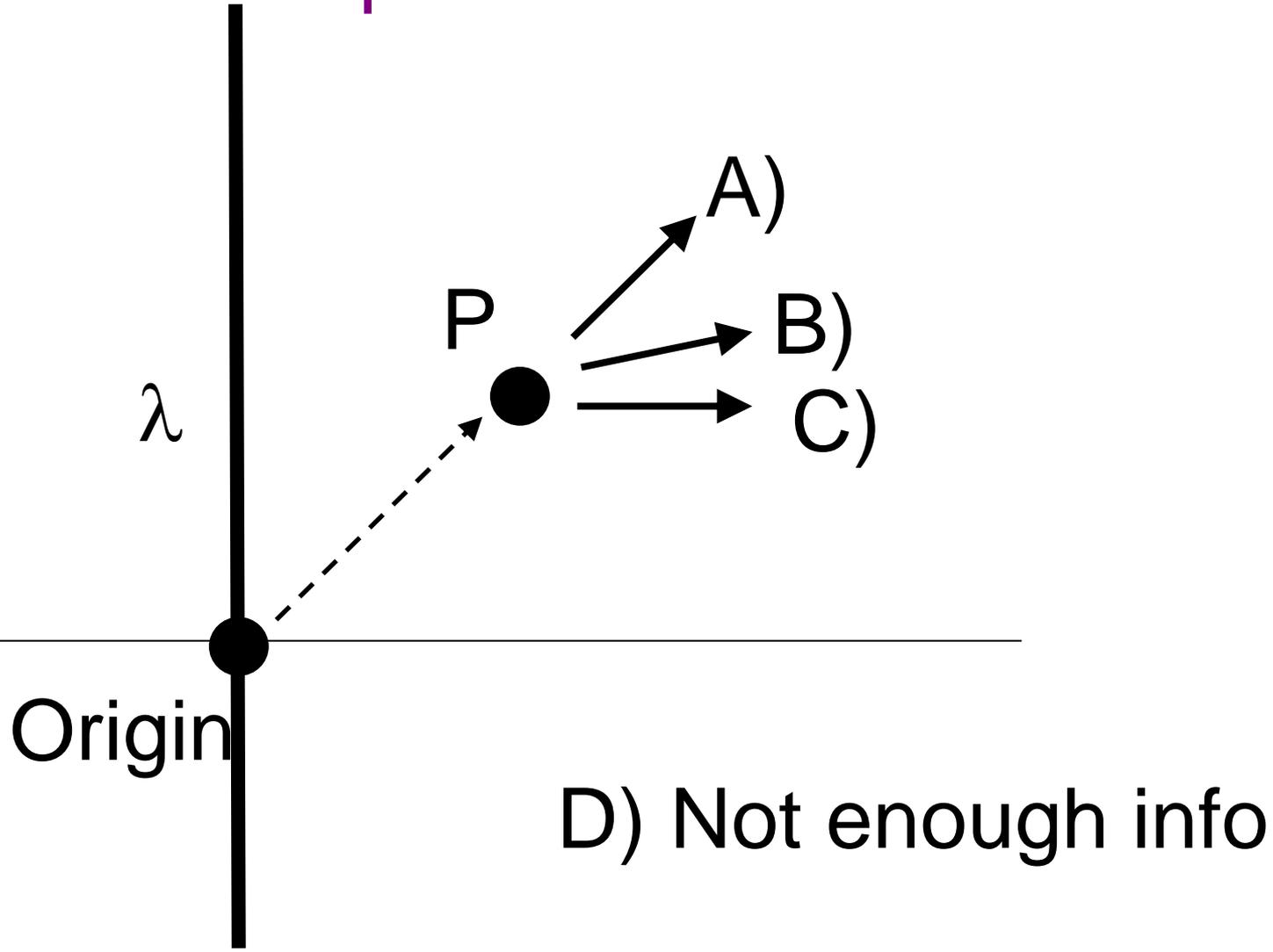
B) $4 \pi c$

C) $(4/3) \pi c R^3$

D) $4 \pi c R^3$

E) None of these is correct

An infinite rod has uniform charge density λ . What is the direction of the E field at the point P shown?

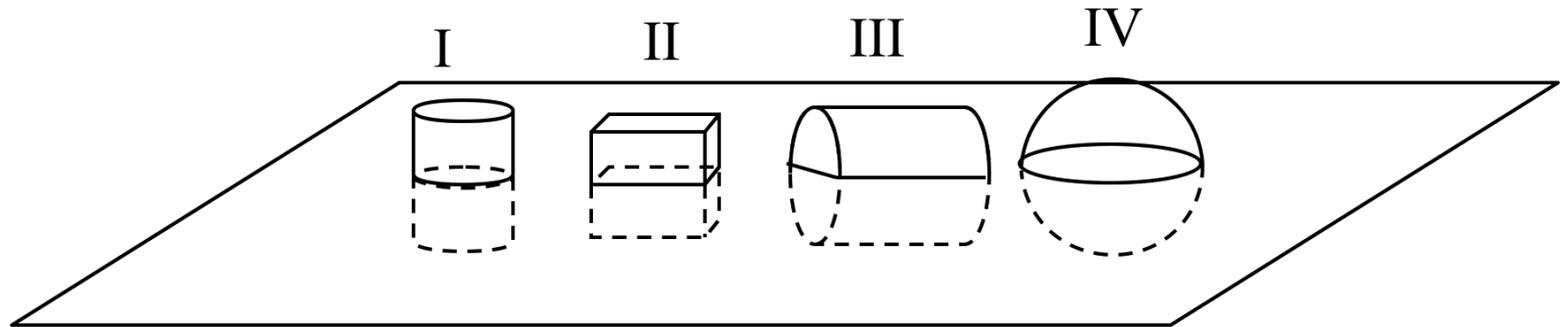


Consider these four closed gaussian surfaces, each of which straddles an infinite sheet of constant areal charge density σ .

The four shapes are

I: cylinder II: cube III: cylinder IV: sphere

For which of these surfaces does gauss's law, $\oiint \vec{E} \cdot d\vec{A} = Q_{\text{enclosed}} / \epsilon_0$ help us find E near the surface??



- A) All B) I and II only C) I and IV only D) I, II and IV only
E) Some other combo!

2.26

A spherical *shell* has a uniform positive charge density on its surface. (There are no other charges around)

What is the electric field *inside* the sphere?

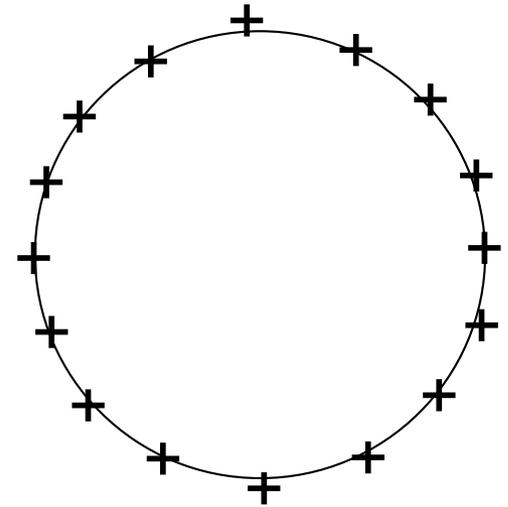
A: $\mathbf{E}=0$ everywhere inside

B: \mathbf{E} is non-zero

everywhere in the sphere

C: $\mathbf{E}=0$ only at the very center, but non-zero elsewhere inside the sphere.

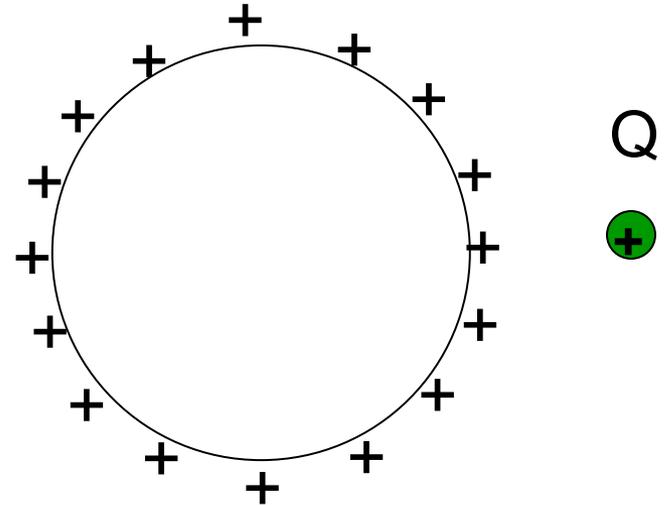
D: Not enough info given



We place a charge Q just outside an insulating, spherical shell (*First fixing all surface charges uniformly around the sphere, and keeping them there*)

What is the electric field *inside* the sphere?

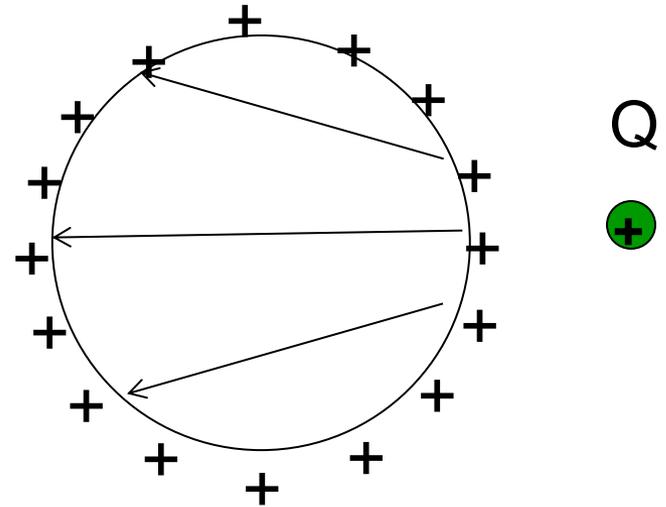
- A: 0 everywhere inside
- B: non-zero everywhere in the sphere
- C: Something else
- D: Not enough info given



We place a charge Q just outside an insulating, spherical shell (*fixing* all surface charges uniformly around the sphere)

What is the electric field *inside* the sphere?

- A: 0 everywhere inside
- B: non-zero everywhere in the sphere
- C: Something else
- D: Not enough info given



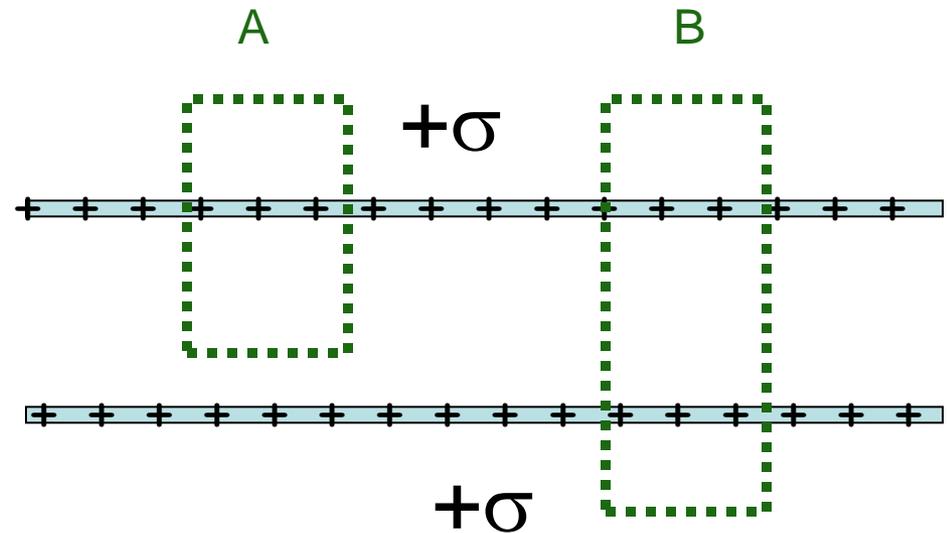
When you are done with “white sheet”,
page 1, side 1, Click A

When you are done with both sides, Click B

If you are done with the YELLOW sheet, click C

Given a pair of very large, flat, charged plates with surface charge densities $+\sigma$. Using the two Gaussian surfaces shown (A and B), what is the E field in the region OUTSIDE the plates?

- A) $\sigma/2\epsilon_0$
- B) σ/ϵ_0
- C) $2\sigma/\epsilon_0$
- D) $4\sigma/\epsilon_0$
- E) It depends on the choice of surface



Click only when you are done with the yellow sheet:

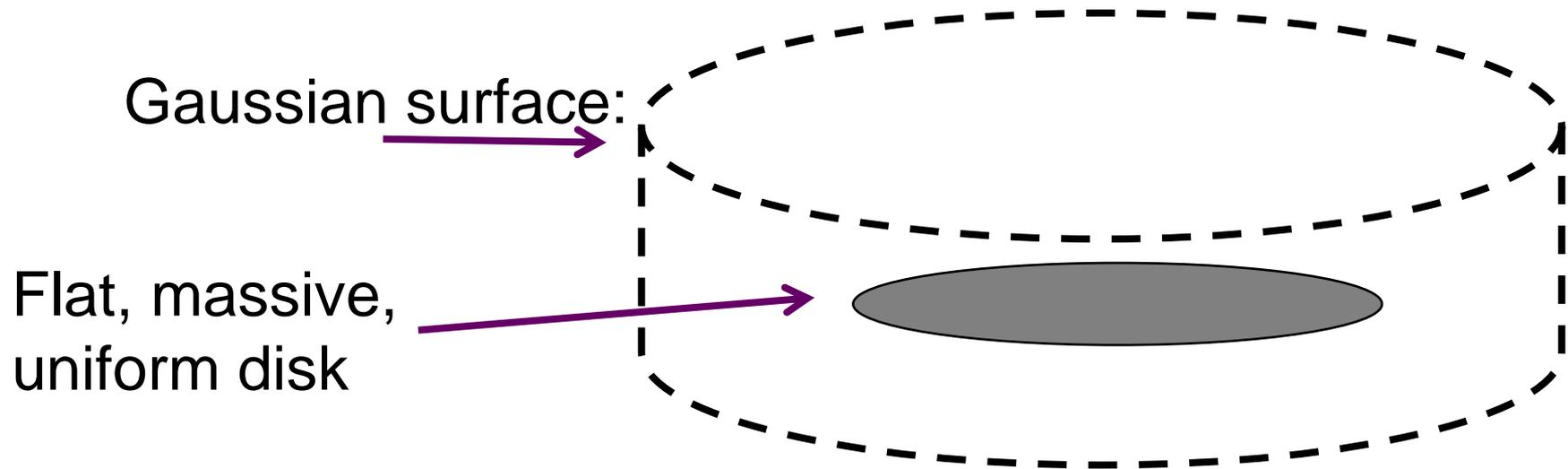
What is the answer to the last question:

iv) Where is this E-field's divergence non-zero?

- A) $\text{div}(\mathbf{E})$ is nonzero everywhere
- B) $\text{div}(\mathbf{E})$ is nonzero Nowhere (i.e. 0 everywhere)
- C) $\text{div}(\mathbf{E})$ is nonzero everywhere OUTSIDE the pipe
- D) $\text{div}(\mathbf{E})$ is nonzero on the surface of the pipe only
- E) Something else!

I would like to find the E field directly above the center of a uniform (finite) charged disk.

Can we use Gauss' law with the Gaussian surface depicted below?

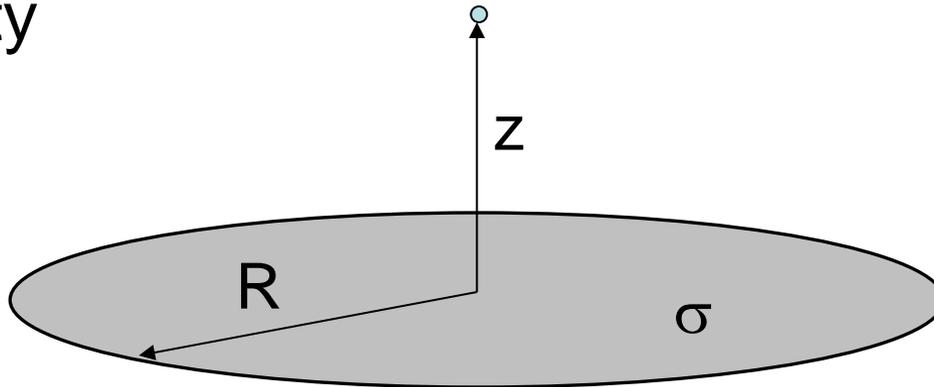


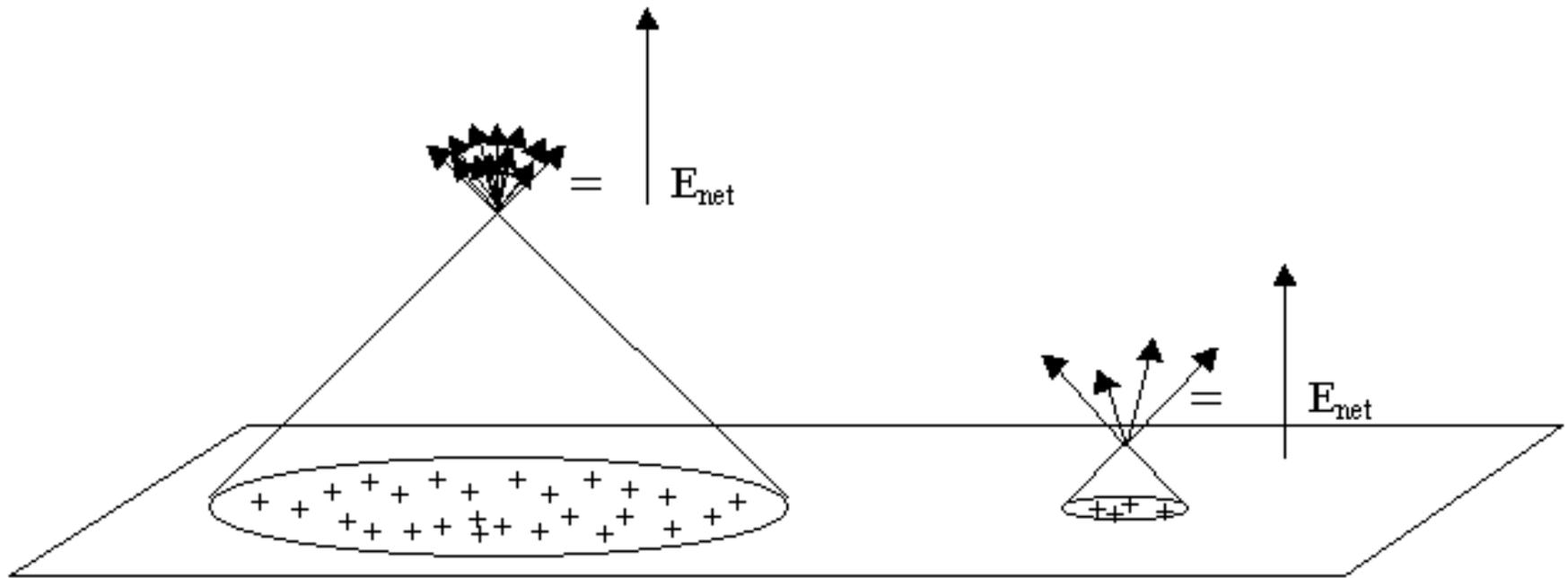
- A) Yes, and it would be pretty easy/elegant...
- B) Yes, but it's not easy.
- C) No. Gauss' law applies, but it can not be used here to compute E directly.
- D) No, Gauss' law would not even apply in this case

Consider the z -component of the electric field E_z at distance z above the center of a uniformly charged disk (charge per area = $+\sigma$, radius = R).

In the limit, $z \ll R$, the value of E_z approaches

- A) zero
- B) a positive constant
- C) a negative constant
- D) +infinity
- E) -infinity

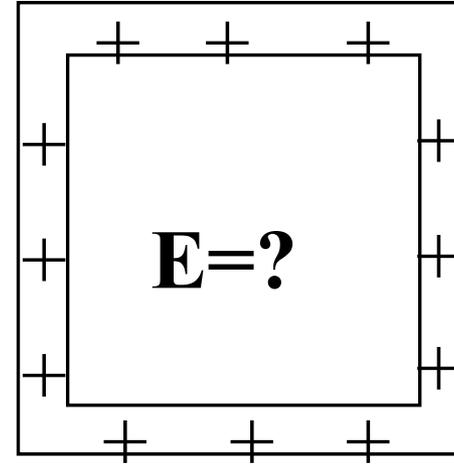




The E-field near an infinite plane of charge is constant.

A cubical non-conducting *shell* has a **uniform** positive charge density on its surface. (There are no other charges around)

What is the field inside the box?



A: $\mathbf{E}=0$ everywhere inside

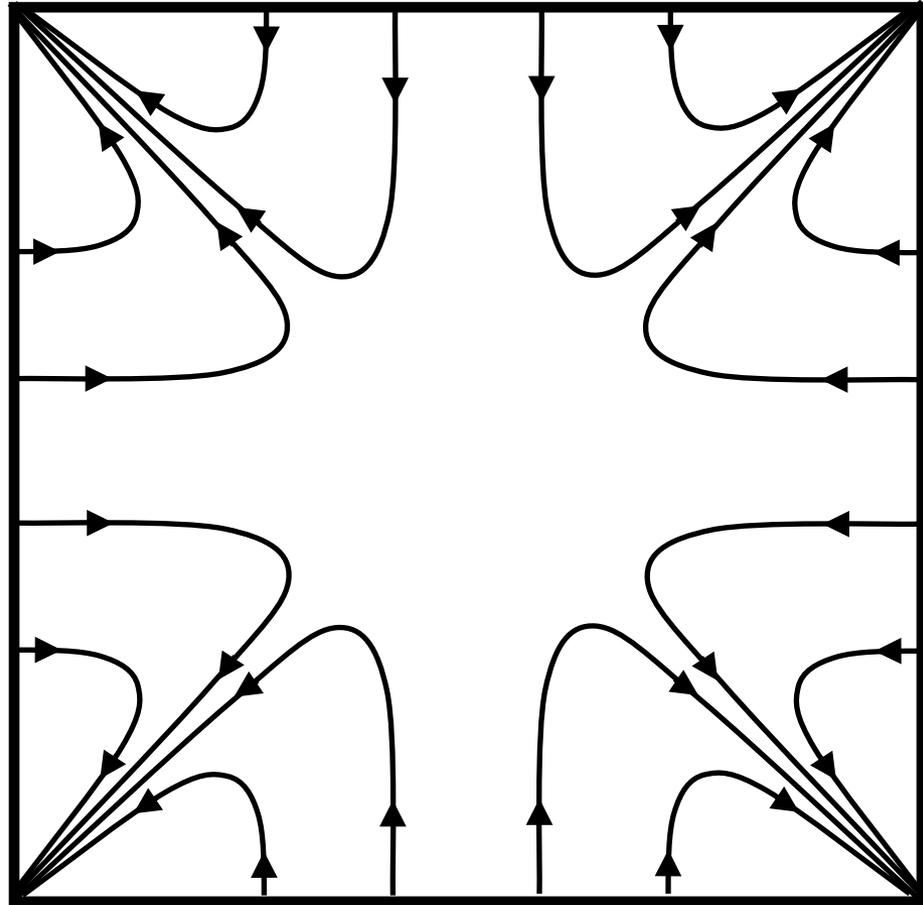
B: \mathbf{E} is non-zero everywhere inside

C: $\mathbf{E}=0$ only at the very center, but non-zero elsewhere inside.

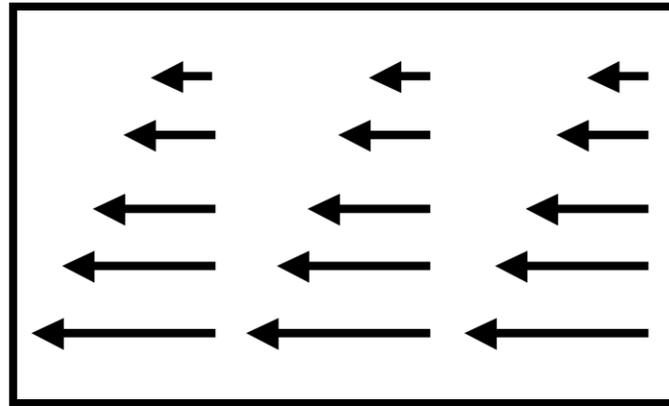
D: Not enough info given

E-field inside a cubical box with a **uniform** surface charge.

The E-field lines sneak out the corners!

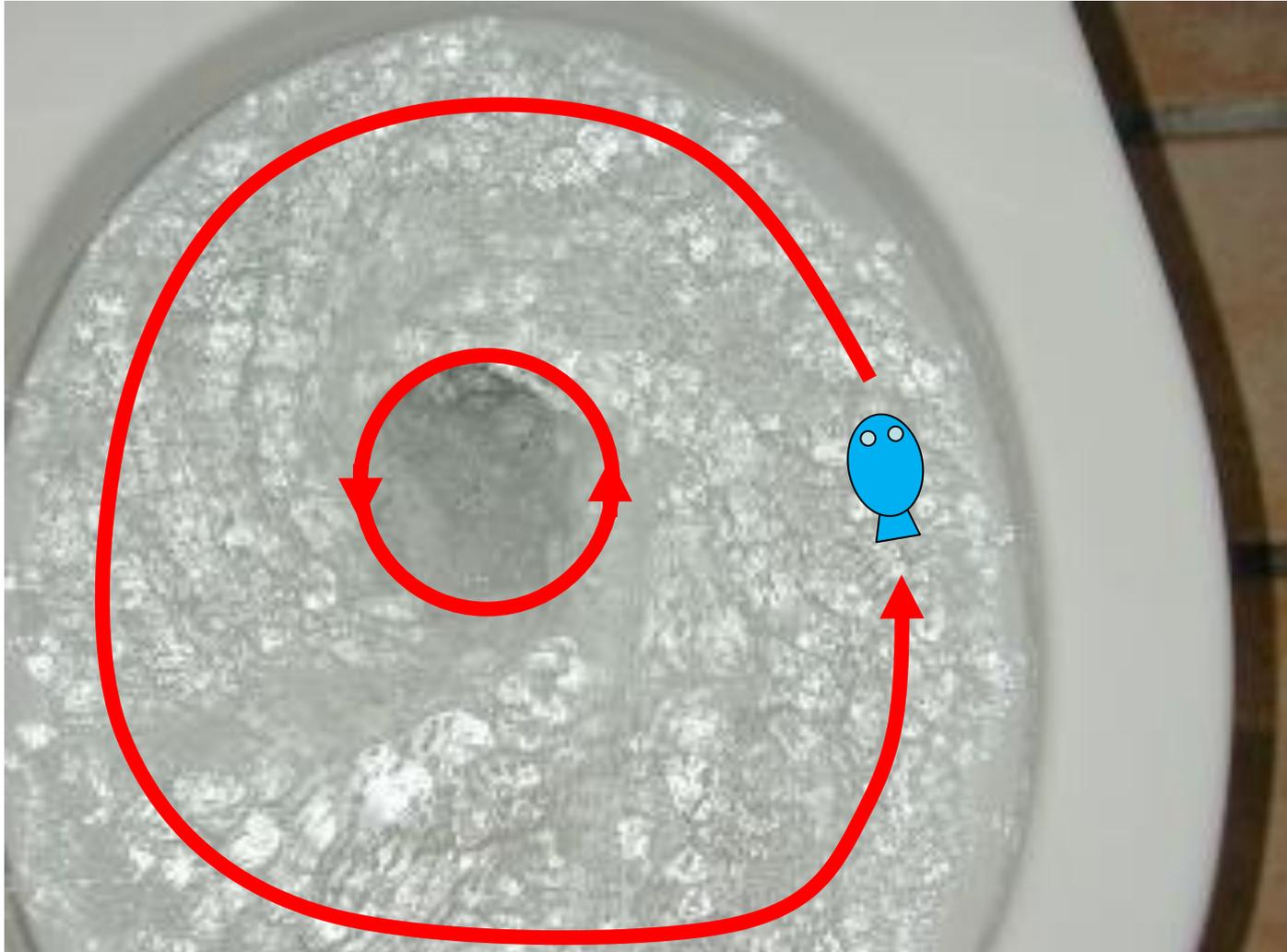


What is the curl of this vector field, in the region shown below?



- A. non-zero everywhere
- B. Non-zero at a limited set of points
- C. zero curl everywhere
- D. We need a formula to decide for sure

Common vortex



Feline Vortex



