ELECTRIC DISPLACEMENT

If you put a dielectric in an external field $\mathbf{E}_{\text{ext}}$, it polarizes, adding a new field, $\mathbf{E}_{\text{induced}}$ (from the bound charges).

These superpose, making a total field $\mathbf{E}_{\text{tot}}$.

What is the vector equation relating these three fields?

$A)$ $\vec{\mathbf{E}}_{\text{tot}} + \vec{\mathbf{E}}_{\text{ext}} + \vec{\mathbf{E}}_{\text{induced}} = 0$

B) $\vec{\mathbf{E}}_{\text{tot}} = \vec{\mathbf{E}}_{\text{ext}} - \vec{\mathbf{E}}_{\text{induced}}$

C) $\vec{\mathbf{E}}_{\text{tot}} = \vec{\mathbf{E}}_{\text{ext}} + \vec{\mathbf{E}}_{\text{induced}}$

D) $\vec{\mathbf{E}}_{\text{tot}} = -\vec{\mathbf{E}}_{\text{ext}} + \vec{\mathbf{E}}_{\text{induced}}$

E) Something else!

4.5

We define "Electric Displacement" or "D" field: $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$.

If you put a dielectric in an external field $\mathbf{E}_{ext}$, it polarizes, adding a new field, $\mathbf{E}_{induced}$ (from the bound charges).

These superpose, making a total field $\mathbf{E}_{tot}$.

Which of these three E fields is the "E" in the formula for D above?

A) $\mathbf{E}_{ext}$ B) $\mathbf{E}_{induced}$ C) $\mathbf{E}_{tot}$

Linear Dielectric:

$$\mathbf{P} = e_0 c_e \mathbf{E}$$

c_e is the “Electric Susceptibility

(Usually small, always positive)

Linear Dielectric:

$$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$$

χ_e is the "Electric Susceptibility"

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon_0 \mathbf{E} + \epsilon_0 \chi_e \mathbf{E}$$

$$= \epsilon_0 (1 + \chi_e) \mathbf{E}$$

$$\equiv \epsilon_0 \epsilon_r \mathbf{E}$$

ϵ_r is the *dielectric constant*

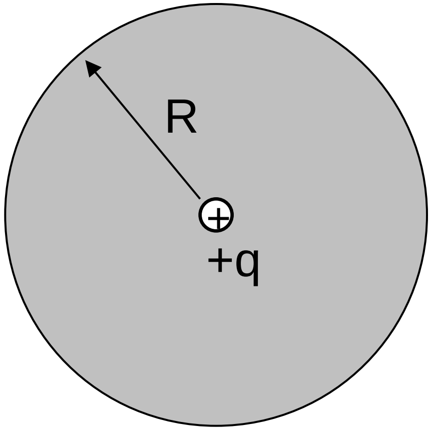
$\epsilon \equiv \epsilon_0 \epsilon_r$ is the *permittivity*

4.7c-alt

We define **D** $=\varepsilon_0$**E** + **P**, with

$$\oint \vec{D} \cdot d\vec{a} = Q_{\text{free, enclosed}}$$

A point charge +q is placed at the center of a dielectric sphere (radius R). There are no other free charges anywhere. What is |D(r)|?



A) $q/(4 \pi r^2)$ everywhere

B) $q/(4 \pi \varepsilon_0 r^2)$ everywhere

C) $q/(4 \pi r^2)$ for r<R, but $q/(4 \pi \varepsilon_0 r^2)$ for r>R

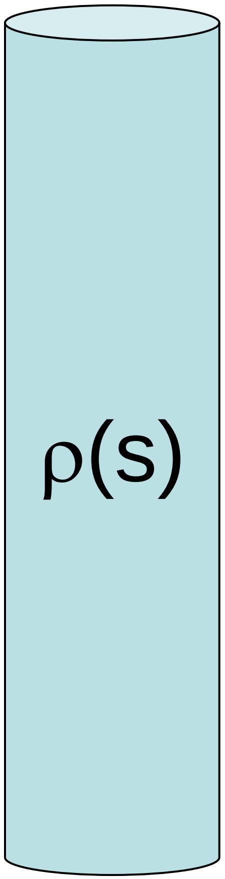
D) None of the above, it's more complicated

E) We need more info to answer!

4.8

A solid non-conducting dielectric rod has been injected ("doped") with a fixed, known charge distribution $\rho(s)$. (The material responds, polarizing internally)

When computing D in the rod, do you treat this $\rho(s)$ as the "free charges" or "bound charges" ?



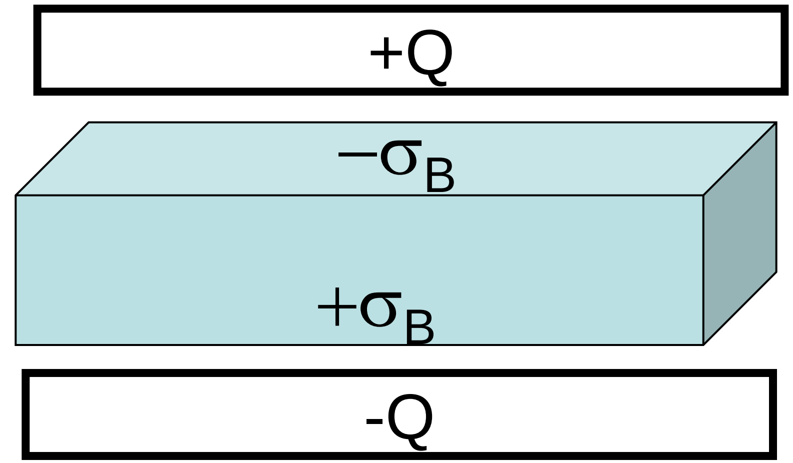
A) "free charge"
B) "bound charge"
C) Neither of these - $\rho(s)$ is some combination of free and bound
D) Something else.

4.6 A very large (effectively infinite) capacitor has charge Q.
A neutral (homogeneous) dielectric is inserted into the gap (and of course, it will polarize) .
We want to find **D** *everywhere.*
Which equation would *you* head to first?

i) $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

ii) $\oint \vec{D} \cdot d\vec{a} = Q_{free}$

iii) $\oint \vec{E} \cdot d\vec{a} = Q / \epsilon_0$



A) i B) ii C) iii
D) More than one of these would work OK.

4.6 b

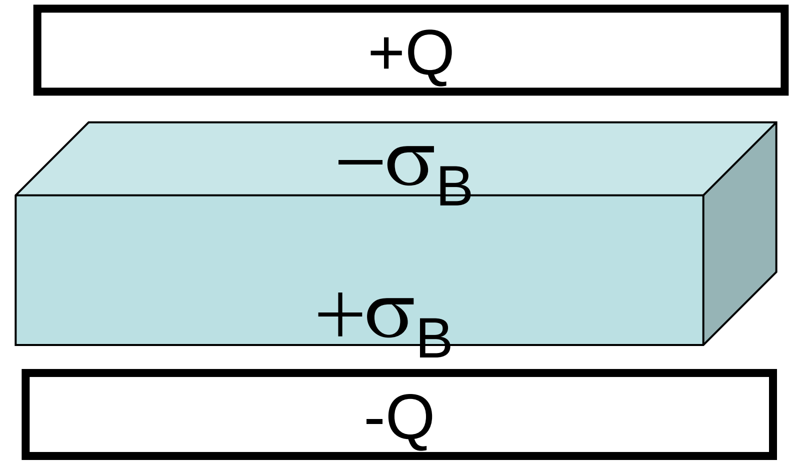
An ideal (large) capacitor has charge Q. A neutral dielectric is inserted into the gap (and of course, it will polarize)

We want to find **E** *everywhere*

(i) $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$

(ii) $\oint \vec{\mathbf{D}} \cdot d\vec{\mathbf{A}} = Q_{free}$

(iii) $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = Q/\varepsilon_0$



Which equation would *you* go to first?

A) i B) ii C) iii

D) Your call: *more* than 1 of these would work!

E) *Can't* solve, unless know the dielectric is linear!

Linear Dielectric:

$$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$$

χ_e is the "Electric Susceptibility"

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

$$= \epsilon_0 (1 + \chi_e) \mathbf{E}$$

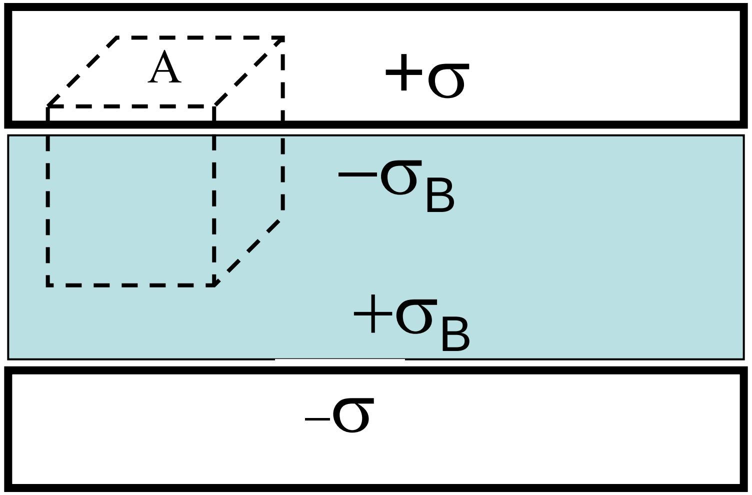
$$\equiv \epsilon_0 \epsilon_r \mathbf{E}$$

ϵ_r is the *dielectric constant*

MD8-4

An ideal (large) capacitor has charge Q.
A neutral linear dielectric is inserted into the gap.
We want to find **D** in the dielectric.

$$\oiint \vec{D} \cdot d\vec{a} = Q_{free}$$

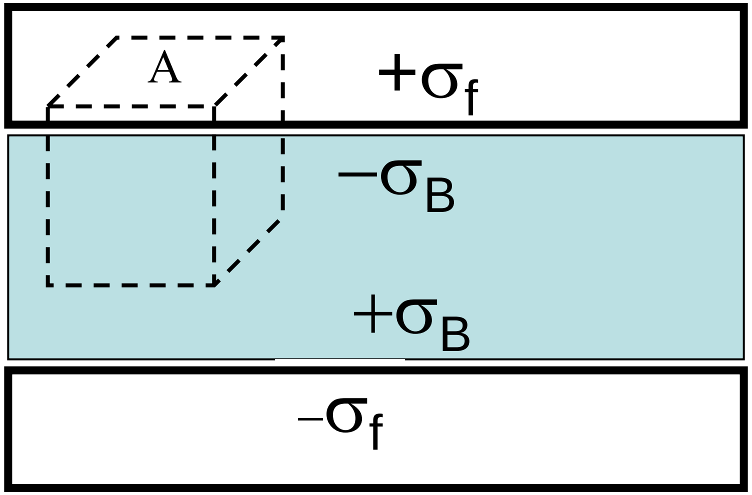


For the Gaussian pillbox shown, what is $Q_{free,enclosed}$?

A) σA
B) $-\sigma_B A$
C) $(\sigma - \sigma_B)A$
D) $(\sigma + \sigma_B)A$
E) Something else

An ideal (large) capacitor has charge Q.
A neutral linear dielectric is inserted into the gap.
We want to find **D** in the dielectric.

$$\oiint \vec{D} \cdot d\vec{a} = Q_{free}$$

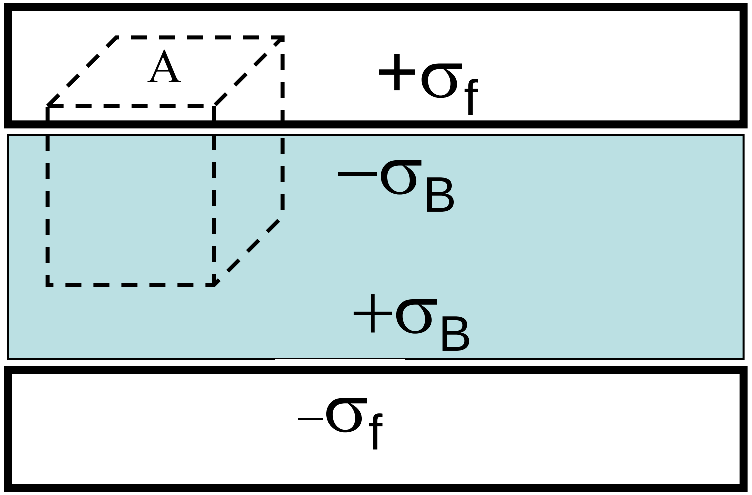


For the Gaussian pillbox shown, what is $Q_{free,enclosed}$?

A) $\sigma_f A$
B) $-\sigma_B A$
C) $(\sigma_f - \sigma_B)A$
D) $(\sigma_f + \sigma_B)A$
E) Something else

An ideal (large) capacitor has charge Q.
A neutral linear dielectric is inserted into the gap.
We want to find **D** in the dielectric**.**

$$\oint\!\!\oint \vec{D}\cdot d\vec{a} = Q_{free}$$

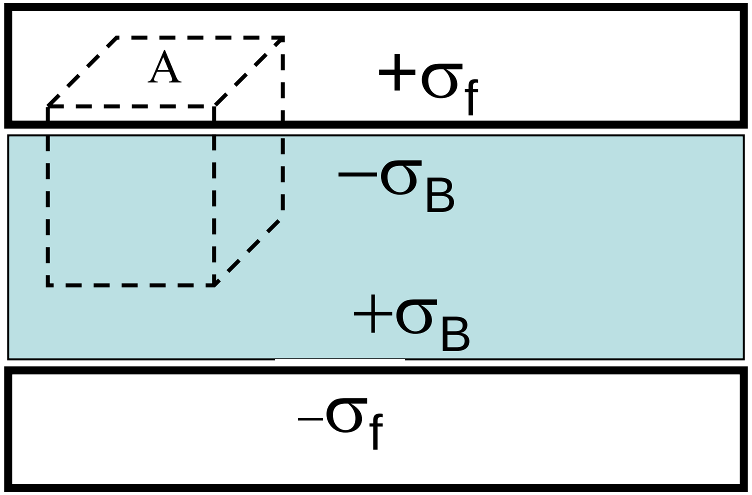


Is D zero INSIDE the metal?
(i.e. on the top face of our cubical Gaussian surface)

A) It must be zero in there
B) It depends
C) It is definitely NOT zero in there…

An ideal (large) capacitor has charge Q.
A neutral linear dielectric is inserted into the gap.
We want to find **D** in the dielectric.

$$\oiint \vec{D} \cdot d\vec{a} = Q_{free}$$



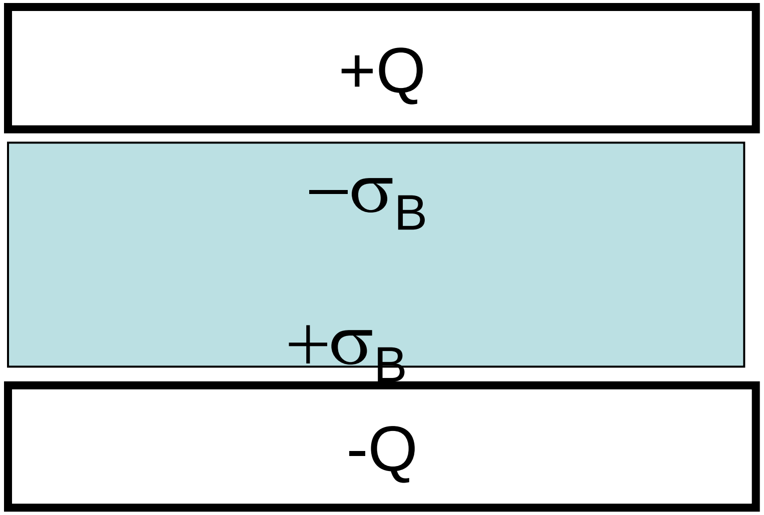
What is |**D**| in the dielectric?

A) σ_f B) $2\sigma_f$ C) $\sigma_f / 2$

D) $\sigma_f + \sigma_b$ E) Something else

4.6d

An ideal (large) capacitor has charge Q.
A neutral linear dielectric is inserted into the gap.
Now that we have **D** in the dielectric**,**
what is **E** inside the dielectric ?



A) $\mathbf{E} = \mathbf{D}\, \varepsilon_0\, \varepsilon_r$

B) $\mathbf{E} = \mathbf{D}/\varepsilon_0\varepsilon_r$

C) $\mathbf{E} = \mathbf{D}\, \varepsilon_0$

D) $\mathbf{E} = \mathbf{D}/\varepsilon_0$

E) Not so simple! Need another method

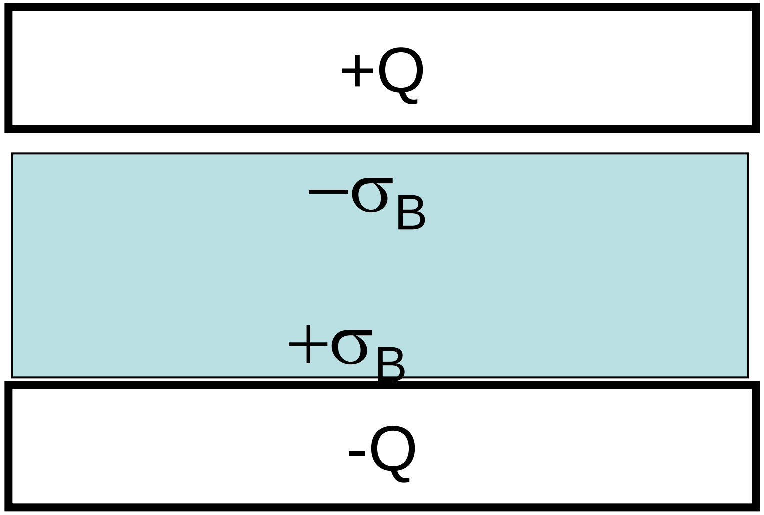
Linear Dielectric:

$$\mathbf{D} = e_0 e_r \mathbf{E}$$

e_r is the *dielectric constant*

4.6e An ideal (large) capacitor has charge Q.
A neutral *linear* dielectric is inserted into the gap (with given dielectric constant)
Now that we have **D** in the dielectric**,**

what is **E** in that *small gap* *above* the dielectric ?



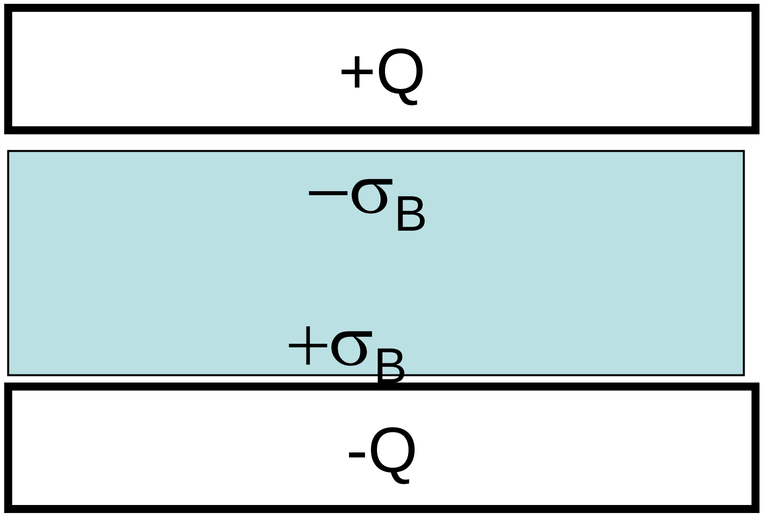
A) $\mathbf{E} = \mathbf{D}\,\varepsilon_0\,\varepsilon_r$
B) $\mathbf{E} = \mathbf{D}/\varepsilon_0\varepsilon_r$
C) $\mathbf{E} = \mathbf{D}\,\varepsilon_0$
D) $\mathbf{E} = \mathbf{D}/\varepsilon_0$
E) Not so simple! Need another method

4.6f

An ideal (large) capacitor has charge Q.
A neutral linear dielectric is inserted into the gap
(with given dielectric constant)

Where is E **dis**continuous?

i) near the free charges on the plates
ii) near the bound charges on the dielectric surface

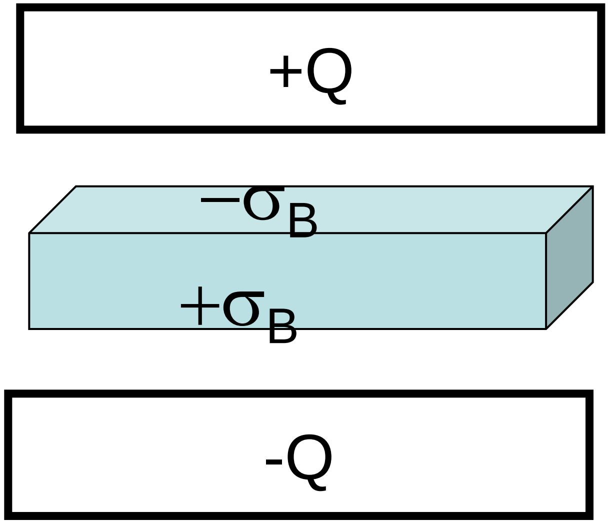


A) i only
B) ii only
C) both i and ii (but nowhere else)
D) both i and ii but also other places
E) none of these/other/???

4.6g

An ideal (large) capacitor has charge Q. A neutral *linear* dielectric is inserted into the gap (with given dielectric constant)

Where is E discontinuous?



i) near the free charges on the plates

ii) near the bound charges on the dielectric surface

A) i only

B) ii only

C) both i and ii (but nowhere else)

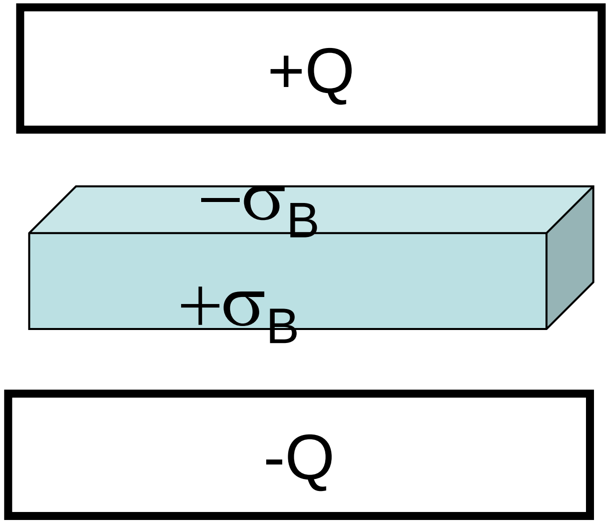
D) both i and ii but also other places

E) none of these/other/???

4.6g

An ideal (large) capacitor has charge Q. A neutral *linear* dielectric is inserted into the gap (with given dielectric constant)

Where is D discontinuous?



i) near the free charges on the plates

ii) near the bound charges on the dielectric surface

A) i only

B) ii only

C) both i and ii (but nowhere else)

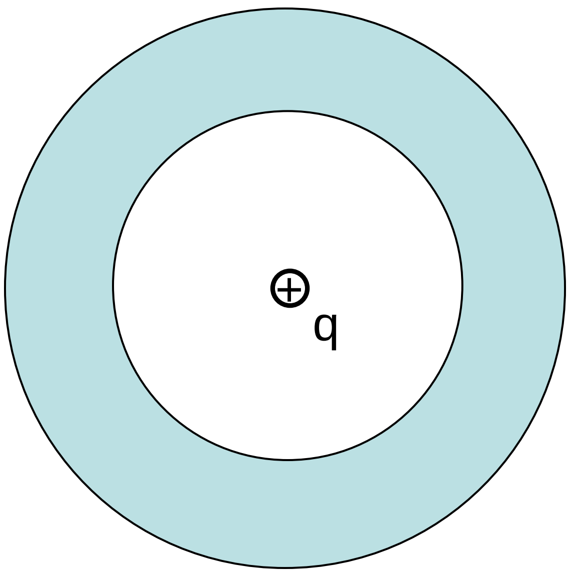
D) both i and ii but also other places

E) none of these/other/???

MD8-5

A point charge +q is placed at the center of a neutral, linear, homogeneous, dielectric teflon shell. Can D be computed from its divergence?

$$\oint \vec{D} \cdot d\vec{a} = Q_{free}$$

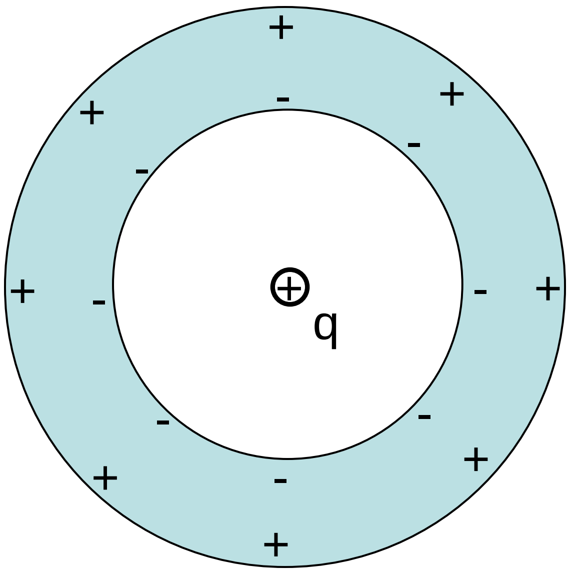


A) Yes
B) No
C) Depends on other things not given

MD8-6 A point charge +q is placed at the center of a neutral, linear, homogeneous dielectric teflon shell.
The shell polarizes due to the point charge.

Is the curl of the polarization **P** zero everywhere?

$$\oint \vec{P} \cdot d\vec{l} = 0 \quad \text{for every possible loop?}$$

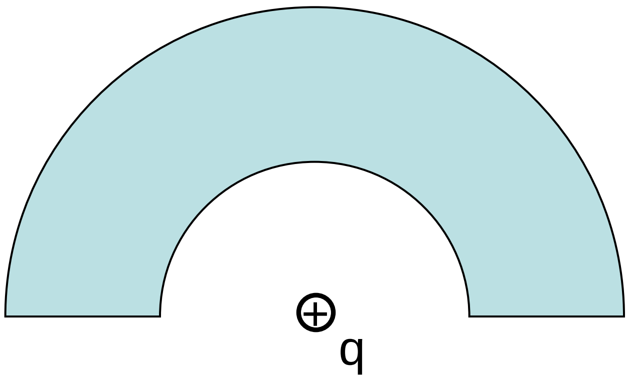


A) Yes
B) No
C) Depends on other things not given

MD8-7

A point charge +q is placed at the center of a neutral, linear, dielectric **hemispherical** shell. Can D be computed from its divergence?

$$\oint \vec{D} \cdot d\vec{a} = Q_{free}$$

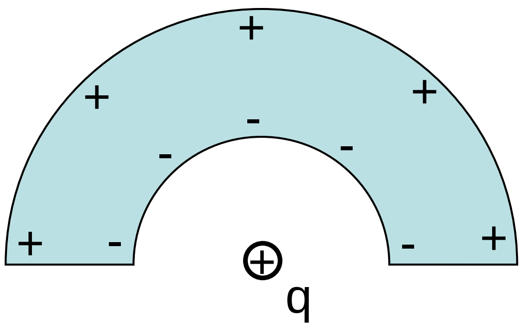


A) Yes
B) No
C) Depends on the inner radius of the dielectric.

A point charge +q is placed at the center of a neutral, linear, dielectric shell. The shell polarizes due to the point charge.

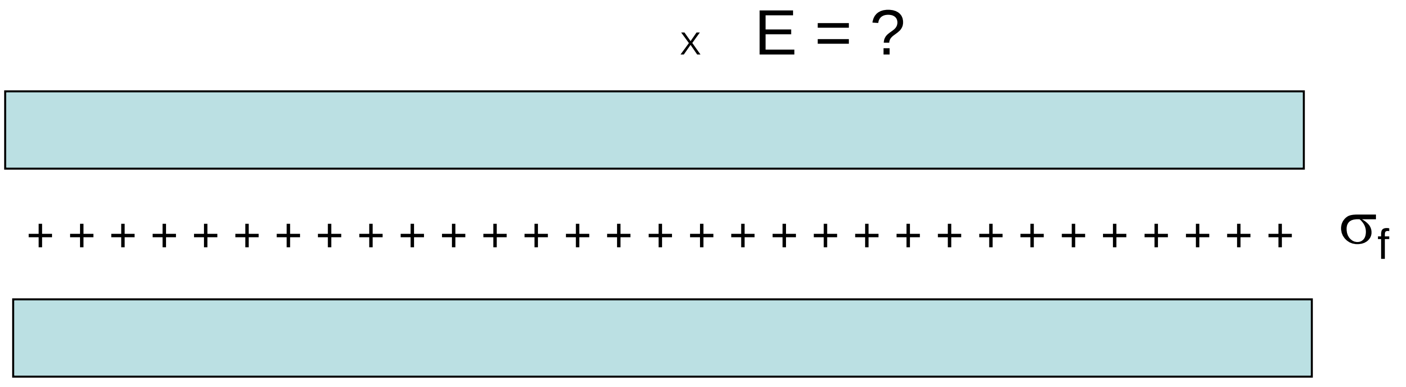
Is the curl of the polarization **P** zero everywhere?

$$\oint \vec{P} \cdot d\vec{l} = 0 \quad \text{for every possible loop?}$$



A) Yes
B) No
C) Depends on the inner radius of the dielectric.

An infinite plane of charge with surface charge density σ_f is between two infinite slabs of neutral linear dielectric (of dielectric constant ε_r), as shown. The "bare" E-field, due only to the plane of free charge, has magnitude $E_o = \sigma_f / 2\varepsilon_0$



What is the magnitude of the E-field in the space above the top dielectric at the point x ?

A) $E = E_0$ B) $E > E_0$ C) $E < E_0$

4.11 We argued that C goes UP by a factor of ε_r if you fill a capacitor with dielectric. What happens to the stored energy of a capacitor if it's filled with a dielectric?

A) It goes up

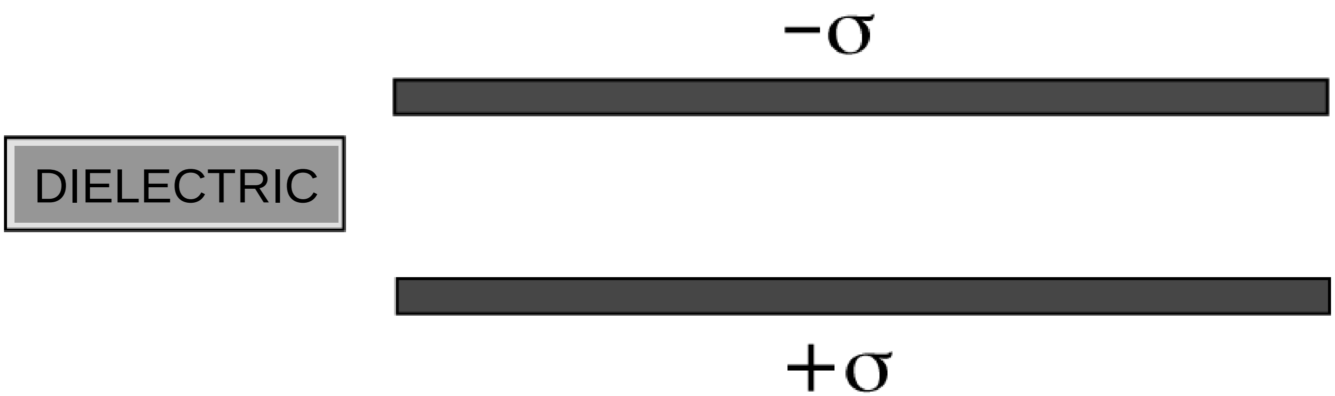
B) It goes down

C) It is unchanged

D) The answer depends on what else is "held fixed" (V? Q?)

4.12 b

If we push this dielectric inside the *isolated* capacitor, will it be drawn into the capacitor or repelled?



A. It gets sucked into the capacitor
B. It gets pushed out from the capacitor
C. I just don’t know.