MAGNETIC VECTOR POTENTIAL

Class Activities: Vector Potential

Writing

What is A?

Started with a writing exercise, basically "what is the A field, how is it used" (see my powerpoints for the wording) Gave ~3 minutes for that.

Tutorial

Magnetic Vector Potential due to a Spinning Charged Ring" activity

Oregon State University

Working in small groups students are asked to consider a ring with charge Q, and radius R rotating about its axis with period T and create an integral expression for the vector potential caused by this ring everywhere in space. Students also develop the power series expansion for the potential near the center or far from the ring.

One of Maxwell's equations, $\nabla \times E = 0$ made it useful for us to define a scalar potential V, where $E = -\nabla V$

Similarly, another one of Maxwell's equations makes it useful for us to define the vector potential, **A.** Which one?

A) $\nabla \times E = 0$

$B)$ $\nabla \cdot E = r / e_0$

$C)$ $\nabla \times B = m_0 J$

$D)$ $\nabla \cdot B = 0$

$E)$ something else!

$$\nabla \times \vec{\mathbf{E}} = 0 \rightarrow \boxed{\vec{\mathbf{E}} = -\nabla V}$$

Can add a constant 'c' to V without changing **E**

("Gauge freedom"): ∇constant = 0,

$$\vec{\nabla} \cdot \vec{\mathbf{B}} = 0 \rightarrow \boxed{\vec{\mathbf{B}} = \vec{\nabla} \times \vec{\mathbf{A}}}$$

Can add any vector function '**a**' with ∇x**a**=0 to **A** without changing **B** ("Gauge freedom")

$$\nabla \times (\mathbf{A}+\mathbf{a}) = \nabla \times \mathbf{A} + \underbrace{\cancel{\nabla \times \mathbf{a}}}_{0} = \nabla \times \mathbf{A} = \mathbf{B}$$

5.25

$$\nabla^2 \vec{\mathbf{A}} = -\mu_0 \vec{\mathbf{J}}$$

In Cartesian coordinates, this means:

$$\nabla^2 \mathrm{A}_x = -\mu_0 \mathrm{J}_x$$, etc.

Does it also mean, in spherical coordinates, that $\nabla^2 \mathrm{A}_r = -\mu_0 \mathrm{J}_r$

A) Yes

B) No

5.25
b

$$\vec{\mathbf{A}}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint \frac{\vec{\mathbf{J}}(r')}{\mathscr{r}} \, d\tau'$$

Can you calculate that integral using spherical coordinates?

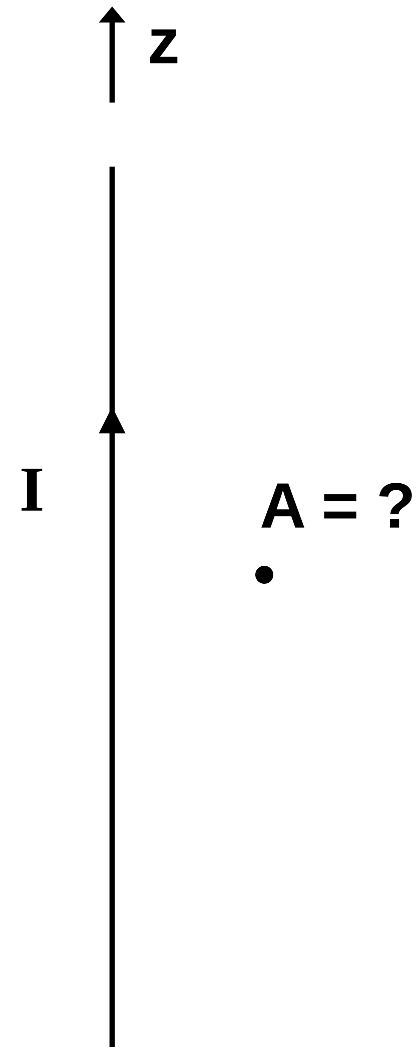
A) Yes, no problem
B) Yes, r' can be in spherical, but **J** still needs to be in Cartesian components
C) No.

The vector potential **A** due to a long straight wire with current I along the z-axis is in the direction parallel to:

A) $\hat{z}$

B) $\hat{j}$ (azimuthal)

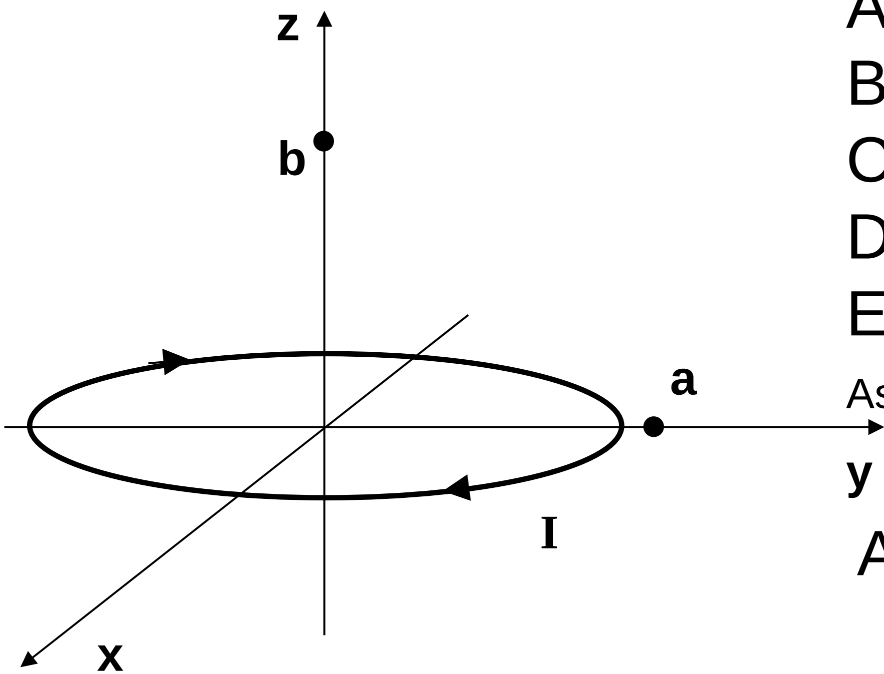
C) $\hat{s}$ (radial)



Assume Coulomb gauge

MD12-4a,b

A circular wire carries current I in the xy plane. What can you say about the vector potential **A** at the points shown?



<u>At point a</u>, the vector potential **A** is:

A) Zero
B) Parallel to x-axis
C) Parallel to y-axis
D) Parallel to z-axis
E) Other/not sure…

Assume Coulomb gauge, and A vanishes at infinity

At point b, the vector potential **A** is:

A) Zero
B) Parallel to x-axis
C) Parallel to y-axis
D) Parallel to z-axis
E) Other/not sure…

AFTER you are done with the front side:

The left figure shows the B field from a long, fat, uniform wire.

What is the physical situation associated with the RIGHT figure?

A) **A** field from a long, fat wire
B) **A** field from a long solenoid pointing to the right
C) **A** field from a long solenoid pointing up the page
D) **A** field from a torus
E) Something else/???

5.27 b

Suppose A is azimuthal, given by

$\vec{\mathbf{A}} = \frac{c}{s}\hat{\phi}$ (in cylindrical coordinates)

What can you say about curl(**A**)?

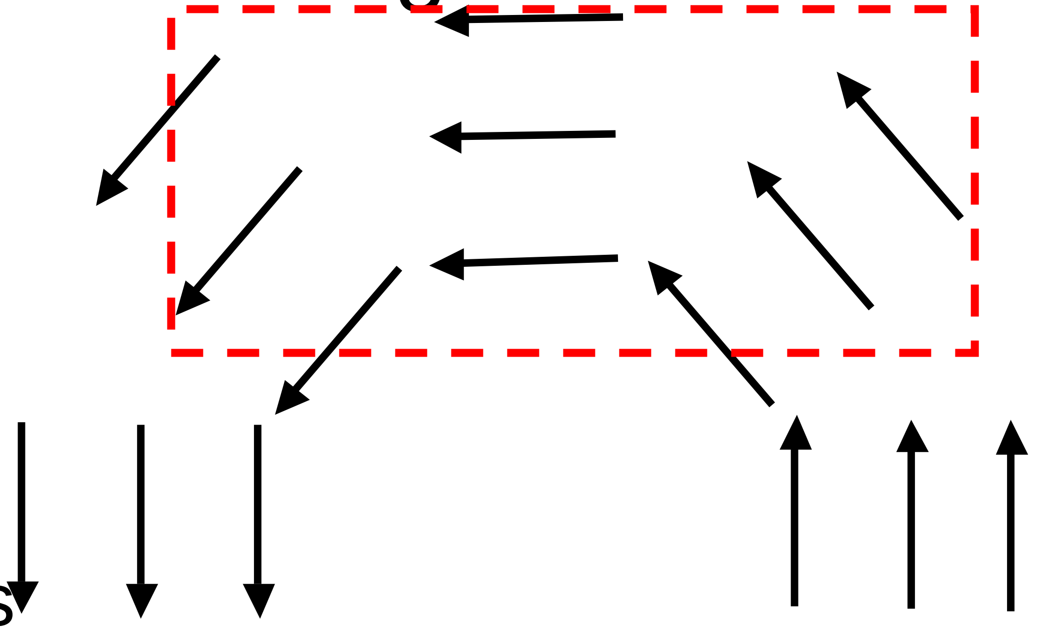
A) curl(**A**) =0 everywhere

B) curl(**A**) = 0 everywhere except at s=0.

C) curl(**A**) is nonzero everywhere

D) ???

5.24 If the arrows represent the vector potential **A** (note that |A| is the same everywhere), is there a nonzero **B** in the dashed region?



A. Yes

B. No

C. Need more information to decide

What is $\oint \vec{\mathbf{A}}(\vec{r}) \cdot d\vec{\mathbf{l}}$

A) The current density **J**

B) The magnetic field **B**

C) The magnetic flux Φ_B

D) It's none of the above, but is something simple and concrete

E) It has no particular physical interpretation at all

5.28 When you are done with p. 1:

Choose all of the following statements that are implied if $\oiint \vec{B}\cdot d\vec{a}=0$ for any/all closed surfaces

(I) $\vec{\nabla}\cdot\vec{B}=0$

(II) $B^{//}_{above}=B^{//}_{below}$

(III) $B^{\perp}_{above}=B^{\perp}_{below}$

A) (I) only

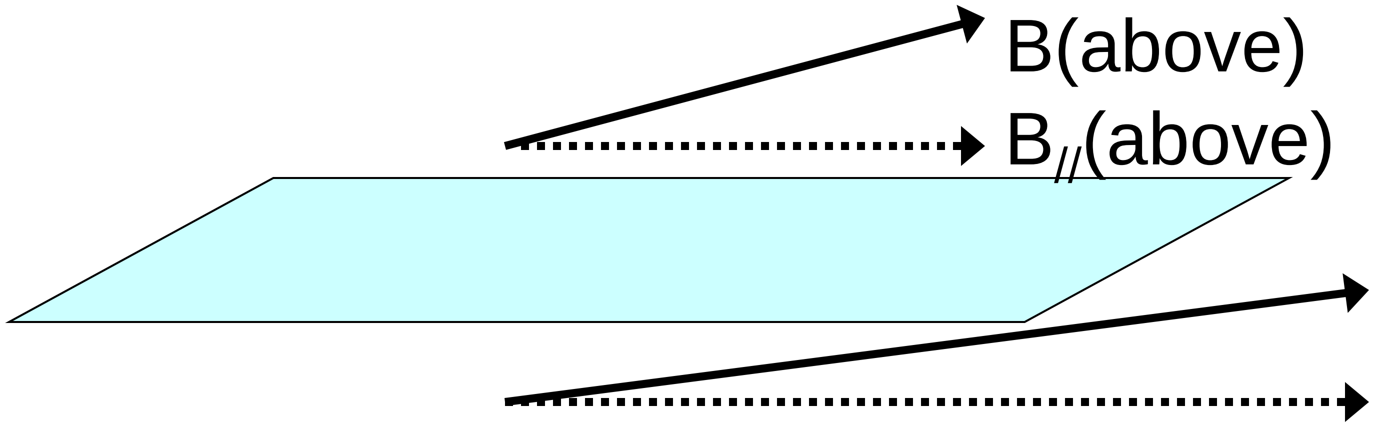
B) (II) only

C) (III) only

D) (I) and (II) only

E) (I) and (III) only

6.11 I have a boundary sheet, and would like to learn about the change (or continuity!) of B(parallel) across the boundary.



Am I going to need to know about

A) $\nabla \times B$

B) $\nabla \cdot B$

C) ???

5.28
b

In general, which of the following are continuous as you move past a boundary?

A) **A** B) Not all of **A**, just A_{perp}

C) Not all of **A**, just $A_{//}$

D) Nothing is guaranteed to be continuous regarding **A**

DIPOLES, MULTIPOLES

5.29

The formula from Griffiths for a magnetic dipole at the origin is:

$$\vec{\mathbf{A}}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$

Is this the *exact* vector potential for a flat ring of current with **m**=I**a**, or is it approximate?

A) It's exact

B) It's exact if |r| > radius of the ring

C) It's approximate, valid for large r

D) It's approximate, valid for small r

5.30

The leading term in the vector potential multipole expansion involves $\oint d\vec{l}\,'$

What is the magnitude of this integral?

A) R

B) 2 π R

C) 0

D) Something entirely different/it depends!

This is the formula for an ideal magnetic dipole:

$$\vec{\mathbf{B}} = \frac{c}{r^3}(2\cos q\,\hat{r} + \sin q\,\hat{q})$$

What is different in a sketch of a *real* (physical) magnetic dipole (like, a small current loop)?

E-field around electric dipole

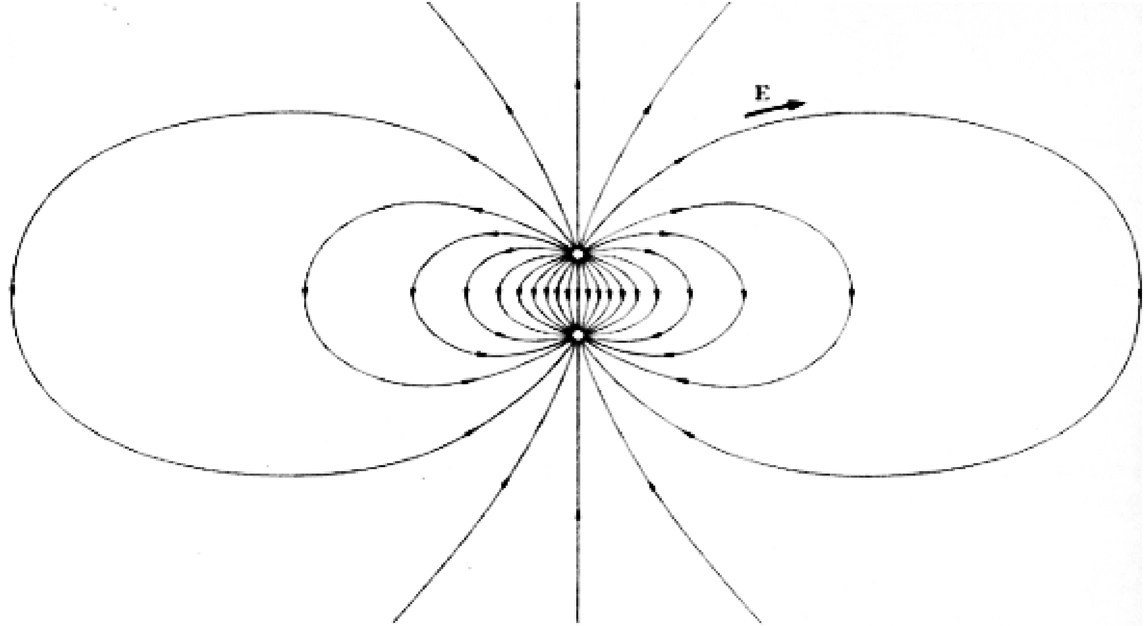
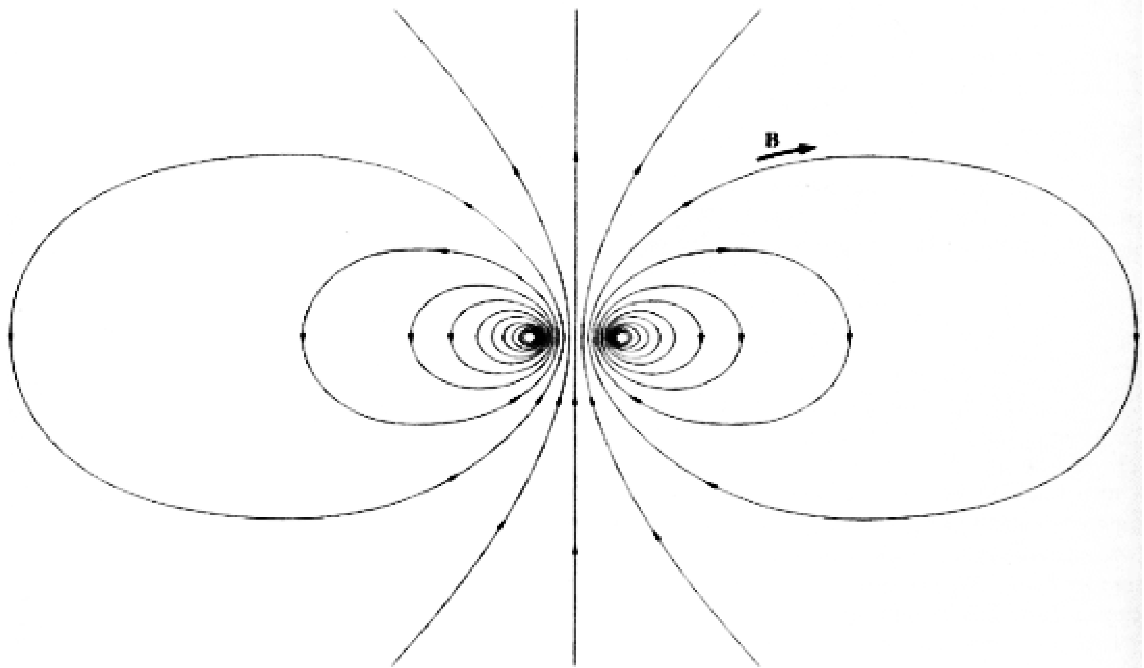


Fig. 10.8 (*a*) The electric field of a pair of equal and opposite charges. Far away it becomes the field of an electric dipole.

B-field around magnetic dipole (current loop)

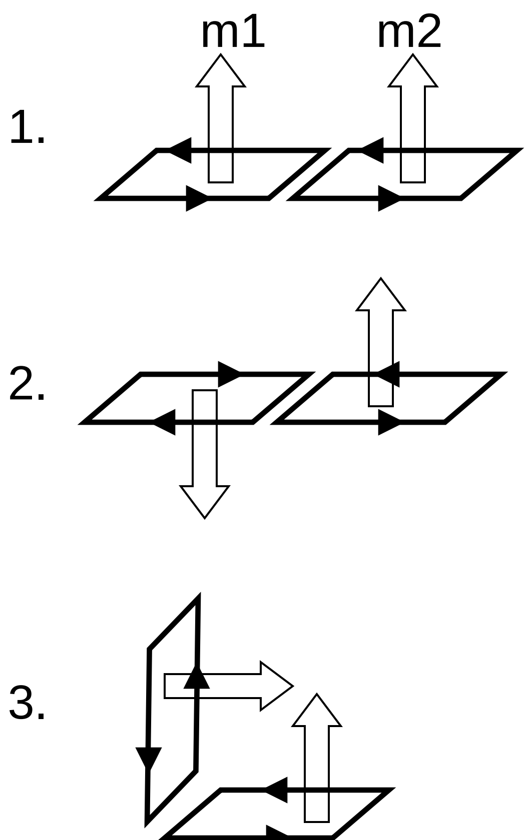


(*b*) The magnetic field of a current ring. Far away it becomes the field of a magnetic dipole.

From Purcell,
Electricity and Magnetism

MD12-5

Two magnetic dipoles **m**1 and **m**2 (equal in magnitude) are oriented in three different ways.



Which ways produce a dipole field at large distances?

A) None of these

B) All three

C) 1 only

D) 1 and 2 only

E) 1 and 3 only

MD12-7

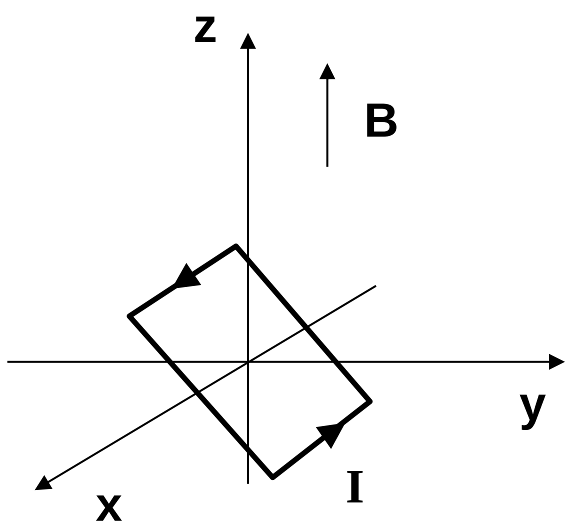
The force on a segment of wire L is $\vec{F} = I\,\vec{L} \times \vec{B}$

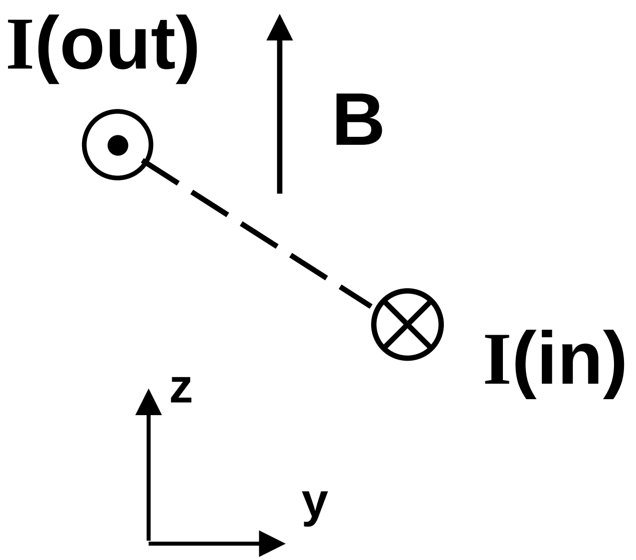
A current-carrying wire loop is in a constant magnetic field **B** = B **z_hat** as shown.

What is the direction of the torque on the loop?

A) Zero B) +x C) +y D) +z

E) None of these



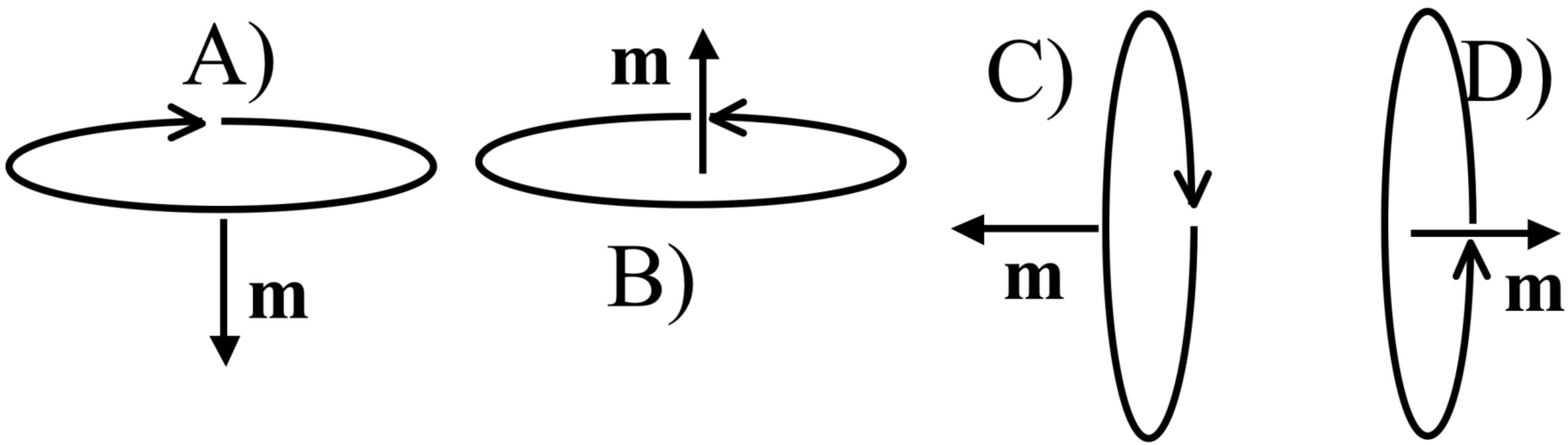


6.1

Griffiths argues that the torque *on* a magnetic dipole in a B field is:

$$\vec{\tau} = \vec{\mathbf{m}} \times \vec{\mathbf{B}}$$

How will a small current loop line up if the B field points uniformly up the page?



6.2

Griffiths argues that the force *on* a magnetic dipole in a B field is: $\vec{\mathbf{F}} = \vec{\nabla}(\vec{\mathbf{m}} \bullet \vec{\mathbf{B}})$

If the dipole **m** points in the z direction, what can you say about **B** if I tell you the force is in the x direction?

A) **B** simply points in the x direction
B) Bz must depend on x
C) Bz must depend on z
D) Bx must depend on x
E) Bx must depend on z