

# MAXWELL'S EQUATIONS

Maxwell's equations so far...

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

Using Stoke's theorem on Faraday's law gives...

A)  $\iint \mathbf{E} \cdot d\mathbf{A} = -d\Phi_B / dt$

B)  $\iint \mathbf{E} \cdot d\mathbf{A} = -\partial \mathbf{B} / dt$

C)  $\oint \mathbf{E} \cdot d\mathbf{l} = -d\Phi_B / dt$

D)  $\oint \mathbf{E} \cdot d\mathbf{l} = -\partial \mathbf{B} / dt$

E) NONE of the above is correct!

7.3

Look at our full set of Maxwell's equations so far:

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

What is  $\nabla \cdot (\nabla \times \vec{\mathbf{B}})$ ?

- A) zero
- B) non-zero
- C) Could be either
- D) Could be BOTH at the same time
- E) My brain hurts!

7.4

a

Ampere/Maxwell's law:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \partial \mathbf{E} / \partial t$$

A capacitor is charging. (Neglect "edge effects")

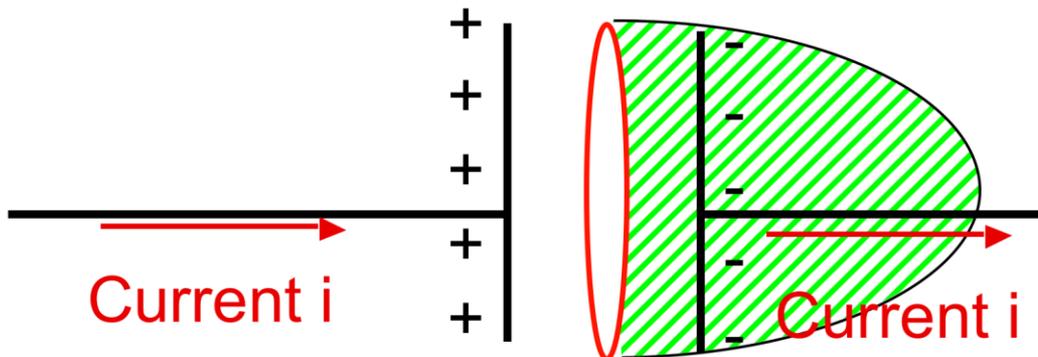
What is  $\iint (\nabla \times \mathbf{B}) \cdot d\mathbf{a}$

over the green hatched area shown?

A) 0

B)  $\mu_0 i$

C) I'm not really so sure



7.4

b

Ampere/Maxwell's law:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \partial \mathbf{E} / \partial t$$

A capacitor is charging. (Neglect "edge effects")

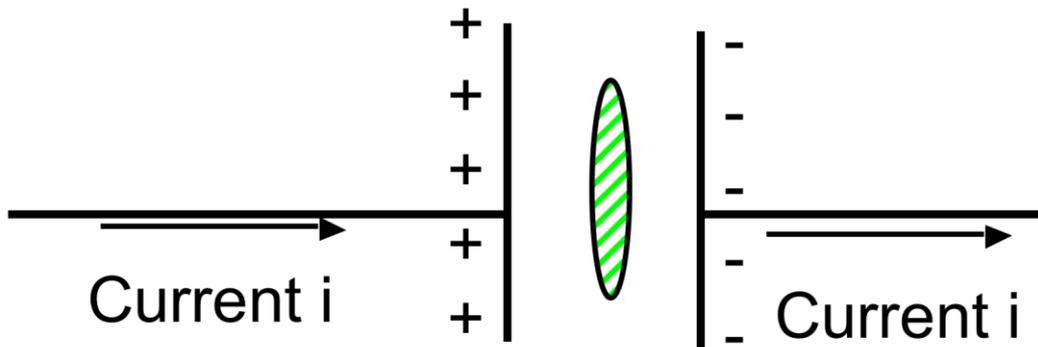
What is  $\iint (\nabla \times \mathbf{B}) \cdot d\mathbf{a}$

over the green hatched area shown?

A) 0

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C) I'm not really so sure



# Maxwell's equations:

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \partial \mathbf{E} / \partial t$$

In vacuum, what is  $\nabla \times (\nabla \times \mathbf{E})$ ?

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$$

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla \times (-\partial \mathbf{B} / \partial t) = -\partial (\nabla \times \mathbf{B}) / \partial t$$

In vacuum (!!!).....

$$-\nabla^2 \mathbf{E} = -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{E}$$

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

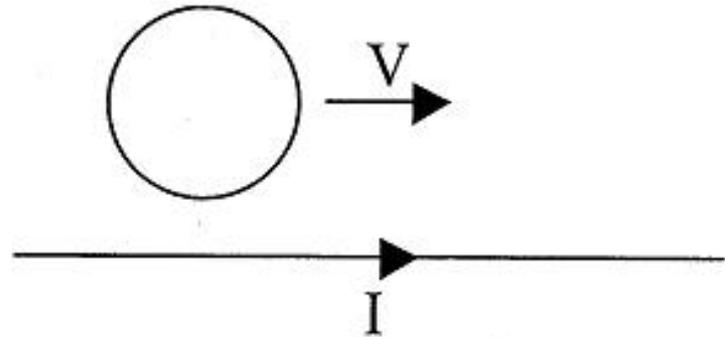
*"It's of no use whatsoever [...] this is just an experiment that proves Maestro Maxwell was right - we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there."* - Heinrich Hertz, 1888

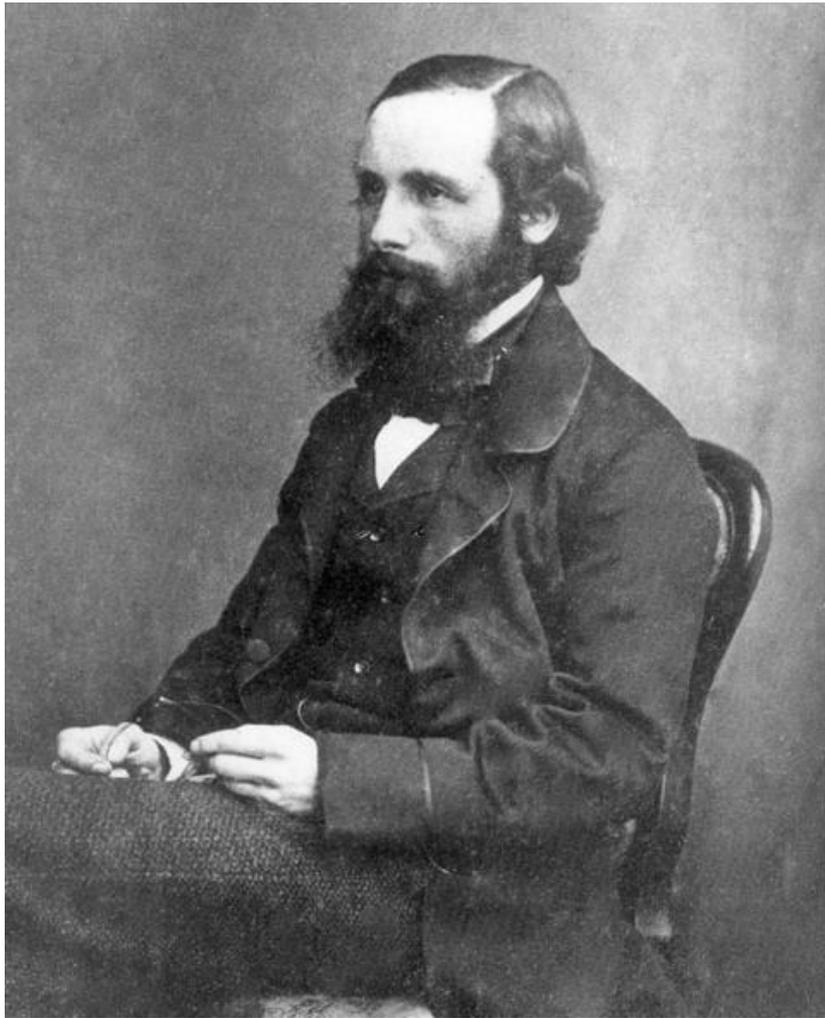
Asked about the ramifications of his discoveries, Hertz replied,  
*"Nothing, I guess."*

Marconi's first wireless radio transmission over large distances (~6 km over water) was in 1897.

A circular loop of wire is moving at constant speed parallel to a long wire with current  $I$ . They are both in the plane of the page. The induced current in the loop is:

- A. Zero
- B. Clockwise
- C. Counter clockwise





## James Clerk Maxwell Scottish 1831-1879

"From a long view of the history of mankind – seen from, say, ten thousand years from now – there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electrodynamics. The American Civil War will pale into provincial insignificance in comparison with this important scientific event of the same decade."

– R.P. Feynman

*"It's of no use whatsoever [...] this is just an experiment that proves Maestro Maxwell was right - we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there."* - Heinrich Hertz, 1888

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