

Transformed E&M I materials

Mathematical materials (Griffiths Chapter 1)

STUDENT DIFFICULTIES

Line and surface integrals

- Students have trouble taking surface and line integrals – setting up the integral, writing the area or line element, and writing the distance from the source. Many of these difficulties may stem from students failing to visualize the problem and account for the physical situation as a sum of little bits. This problem seems to be exacerbated by the differences between the way mathematicians think about integrals (area under a curve) vs. how physicists think about integrals (sum of little bits).
- **Line integrals.** Students don't conceptualize clearly what they are doing. For example, on HW4, students would integrate $\int z dy = zy$ without noticing that $z=0$ along the segment that they were integrating over.
 - Students often do not visualize a particular path when doing a line integral even when they are specifically prompted to do so.
- **Area elements.** We often don't explicitly teach how to write down an area element. There is a good trick that can be taught using the front cover of Griffiths. Why is it that the front cover lists dl and dT but not da ? Because there are three possible area elements that can be listed. So their task is to choose the two components of dl that are relevant for the task at hand. Put those together to make your da .
 - It may not be immediately obvious to students that their area element is wrong, even when the units are incorrect (i.e. $da=drd\phi$)
 - Students may not be familiar with how to derive da or how to check their da once they have one (i.e. integrate over a sphere to get the expression for surface area)

Curvilinear coordinates

- Curvilinear coordinates are also a source of difficulty for students, who have seen them but not used them much. Many continue to have difficulty knowing when to use a particular coordinate system, throughout the course. This is related to the challenge of identifying axes of symmetry.
- Early in the course, many students do spherical integrals in Cartesian coordinates, supposedly because they are not yet comfortable with spherical coordinates.

Delta functions

- **Charge distribution.** Writing a charge distribution as a delta function caused trouble for many students – they did not seem to understand the concept without explicit instruction and even then struggled. They could identify $\delta(x-2)\delta(y-3)\delta(z+2)$, for example, as a point charge, but if given just $\delta(x-2)$ the students in the Traditional class often misinterpreted that as a point charge. On HW4, students calculating the E field and charge distribution arising from and due to a “screened” Coulomb potential that goes as $1/r$, students did not recognize that there would be a delta function at the origin, despite prompting to think about the origin.
- **Dimensionality.** The first example above illustrates that the dimensionality of the delta function is difficult for students. On the post-test, many students misinterpreted two delta functions at r_1 and r_2 as being two concentric spheres with radii r_1 and r_2 (rather than two points at r_1 and r_2). Students sometimes equate a *volume* charge density with a 3D delta function. Asking students to write the *volume* charge density of a *surface* charge also causes confusion. One way of explaining delta functions which was useful for some students was that each subsequent delta function “squishes” the distribution in one dimension. Eg., one delta function “squishes” it into a plane, the second restricts it in one further dimension to make it a line, and the third restricts it in the third dimension to make it a point. The fact that this was a useful idea to students indicates that they are struggling with conceptualizing how additional delta functions affect the dimensionality of the charge distribution.
- **Divergence.** When asked to calculate the divergence of the E field in a region of space containing a delta function charge distribution, students’ lack of connection between divergence and flux (which haven’t yet been stressed in class) came up.
- **Units.** When the *units* of the delta function were stressed (the 1D delta function has units of $1/m$), this was a powerful tool for students to check their conceptualization, and I urge other instructors to emphasize unit analysis of delta functions. For instance, when asked what physical situation is represented by $\rho = c \cdot \delta(x-2)$, many students answered a point charge. The best students, however, were able to argue it correctly with a combination of unit and dimensional analysis.
- **Infinities and idealizations.** Other professors have indicated that students are uncomfortable with the concept of an infinite charge density. The concept of a point charge (density is infinite at the point charge, but total charge is finite) is difficult. Most students continue to confuse delta-function charge densities (where the charge density is infinite at a point) with idealized line, surface, and volume charge densities (where the charge density is smeared out and not infinite at any point). This causes them difficulties later on.

Vector Calculus

- Students at the upper-division level have difficulties attending to both magnitude and direction of a vector at the same time. They often focus on only one aspect at a time, sometimes switching mid-problem.

- Students have considerable difficulty connecting the math of the divergence to the physical situation. They often have difficulty identifying fields with non-zero divergence from diagrams. They also have difficulty identifying where the divergence of E is non-zero (i.e. only where there are charges) and do not seem to utilize the differential form of Gauss' law.
 - Difficulties here seem to be related to students using the common English definition of divergence (i.e. anytime the lines are spreading out) instead of applying a rigorous mathematical approach.

Divergence and Stokes / Integral and differential

- Students have many difficulties with Divergence and Stokes' Theorems, and translating between integral and differential forms of equations. These difficulties will be detailed in the chapters in which they occur in context of E&M (chapters 2 and 4). Overall, students do not seem to understand the physical significance of the divergence theorem as a way to translate between integral and differential forms. For example, in Griffiths 1.32 students are asked to verify that two equations are equal, using the divergence theorem. Most students did not actually evaluate the surface integrals (to thus verify that they equaled the volume integrals). This is a difficulty that pervades the course. Going between the differential form and integral form of an equation with ease takes students most of the course, and they do not become facile with it unless this is an emphasized topic.

Electric Field, Coulomb's Law (Griffiths Chapter 2)

STUDENT DIFFICULTIES

Setting up integrals

- Script-r notation is difficult for most throughout the course – both remembering what it means (though that gets better quickly with use) and writing it in terms of known variables in the problem (which is always difficult).
 - Students will often revert to the most recent form of script-r that they have seen without addressing the specific geometry of the situation.
 - Students may not see the utility of the script-r notation and thus have trouble manipulating it in different contexts.
- Setting up the integral is challenging. They can calculate the integral once it is written down, but they are still very shaky on translating the physical situation into a mathematical form, such as the correct line, area, or volume differentials.
 - These integrals of continuous charge distributions are a place where student difficulties with connecting the math and the physical situation

become very apparent. See the math resources document for more on these difficulties.

- Students may try to use Gauss' law in situations where Coulomb's law is appropriate. This is particularly common for symmetric shapes (such as a disk or sphere) that have non-uniform charge distributions.

Curvilinear coordinates

- Spherical and cylindrical coordinates are also used for the first time in this section. Students have seen this material before but a refresher is helpful. By the end of the course, some students still didn't know how to recognize when cylindrical coordinates were appropriate.

Charge densities

- Writing charge densities as delta functions is challenging for students (see Chapter 1 notes on student difficulties).
 - Not writing charge densities as delta-functions can lead to students having difficulties knowing which volume/area/line element is appropriate leading to issues of taking a volume integral of a surface charge distribution.
- Students don't grasp the difference between the idealization of a "smeared-out" charge density (like σ) and a point charge. Most said that E goes to infinity as you approach a disk of charge with a charge density σ .

Gauss' Law, divergence and curl of E (Griffiths Chapter 2)

STUDENT DIFFICULTIES

Using Gauss' Law (*)

- A surprising number of students still struggle with the mechanics of applying Gauss' Law. They may forget to include the (EA) term from both sides of a Gaussian pillbox, for example, where E is not zero on one side. They are often off by a factor of "2" in pillbox problems. Some students attempted to use Gaussian pillboxes to solve for E due to a cube with a charge density that depended on z , not recognizing that there would be a nonzero (and varying) E field inside the cube. In another problem, many students calculate that the *flux* of E is zero in a region, and thus wrongly conclude that E itself is zero.
 - Students are often unclear about the distinction between flux and electric field, sometimes using the words interchangeably.
- Students often have difficulty articulating the symmetry arguments necessary to determine when Gauss' law is applicable. They struggle particularly with geometric arguments. Griffiths uses a geometric argument only once and as

such students often use only superposition arguments which are significantly more challenging in certain situations.

Applicability of Gauss' Law (**)

- Most students are familiar with Gauss' Law from previous courses. However, they still have trouble knowing when it's useful. However, once presented with alternative ways of solving for E or V (such as the integral forms) some lose sight of when Gauss' Law is applicable. At this point in the course, most students said they struggled most with knowing how to set up a problem. We stressed that Gauss' law is always *true* but not always *useful* and this seemed to be helpful to students, who scored better than traditionally-taught students on an exam question on when to apply Gauss' Law. On the homework and exams, we observed students calculate E using Coulomb's Law when Gauss' Law was applicable.
- In particular, students (especially in the Traditional course) hadn't seemed to grasp that Gauss' Law is only useful in cases of high symmetry. In many cases, they gained this understanding over the course of the semester, though many still had trouble defining just what type of "symmetry" was necessary (is a cube symmetric in the right way?) When asked how to solve for the potential of a point charge outside of a conducting sphere, many chose Gauss' Law, though they recognized that the sphere would polarize.
 - Even some of these upper-division students still seemed to use Gauss' law by rote, just solving $EA=Q/\epsilon$ without considering symmetry or visualizing the field.
 - It is likely that students know you cannot pull a non-constant function out of an integral as they have completed a multivariable calculus course however they may not apply this knowledge in the context of a Gauss' law problem.

Integral and differential form of Gauss' Law / Divergence Theorem (**)

- Students have difficulty translating between integral and differential forms of Gauss' Law, and struggle with a physical conception of the divergence theorem. A student looking at $\nabla \cdot E = \frac{\rho}{\epsilon_0}$ do not readily recognize this as the familiar Gauss' Law (just in a different form) in an interview setting. By the end of the course, they are better able to do this, but it seems that they may have simply memorized the two forms of the equations (eg., $\nabla \cdot E = \frac{\rho}{\epsilon_0}$ and $\oint E \cdot dA = \frac{Q_{enc}}{\epsilon_0}$).
- Several students translated between those two forms by knowing that ρ needed to be changed to Q, and so both sides must be integrated by volume. This does eventually invoke the Divergence theorem (because the integral of the divergence of E must be changed to the closed integral of the flux of E). However, students generally do not invoke the Divergence theorem as a method of translating a divergence to an integral, rather it's a "trick" to make the two

sides of the above equations match. Using a more familiar physical example (like water flow) to talk about the divergence theorem may be useful.

- Student difficulties here may relate to problems visualizing the divergence in a meaningful way. See the math difficulties resource document for more on student difficulties with divergence.

Potential (Griffiths Chapter 2)

STUDENT DIFFICULTIES

What is V? (***)

- Even the best students don't have a solid conceptualization of what V means – they do not relate it easily to force or work and some have forgotten what work is, how to find it, what its units are. We spend a lot of time *calculating* V and seem to lose sight of the *meaning* of V. Using the “Square of Electricity” (see Activities/Activity Resources in course files) may be one way to address this difficulty. This would make an excellent whiteboard activity.
 - Relating to this, students have difficulty accessing the most effective tool to calculate the potential. They often fall back on calculating E to determine V even when a direct calculation of V is much easier. This may be because, as Griffiths presents E first, they consider it to be more fundamental and are more comfortable with it.
- Only some students use test charges to check whether the sign of the potential works, and most have trouble remembering which direction a test charge would go.
- Some students, when presented with $\int E \cdot dl = ?$ out of context of the chapter did not recognize this as the formula for V though some recognized it after spending some time with it.
- Many students have a hard time which of the many equations for potential will be most effective for calculating the potential.

V arises from a conservative E field, Curl of E (**)

- When looking at the equation $\nabla \times E = 0$, most students do not see a connection to voltage V unless prompted. When attempting to explain what a conservative field means, most draw a loop and say that you have to get back to the same value that you started at, but they are not clear on *what* has to add up to zero around that closed loop (i.e., E dotted with dl).

- Only the very best students are able to connect the curl of E with our ability to define a scalar potential. I was able to “trick” many students by asking them why $\int E \cdot dl = 0$. Most puzzled about it for a long time and did not eventually recognize this as an incorrect statement, though the best students did reply in consternation that they didn’t think that was true.
- When using this form to calculate the potential, student have difficulties determining the limits of integration and connecting them to the physical situation. See the Math Resources doc for more on student difficulties with line integrals.

Reference point (**)

- Many students have trouble choosing a suitable reference point for the potential. On a post-test, many struggled with what to do if the reference point ($V=0$) was set at the center of a charged sphere, rather than at infinity. The ones who were able to correctly reason generally gave work-related arguments, but many failed to do so, underlining the difficulty with relating work and potential.
- Some students have a hard time with the arbitrary nature of V. In addition to having difficulties picking a reference point, there are student who state that ΔV is also arbitrary in sign, depending on where you put zero.

Direct integration to find V

- Students seemed to “forget” Gauss’ Law once given the integral formulation for V, and need to be reminded that Gauss’ Law is still easiest in cases of symmetry (and you can then find V from E).
 - Some students have the opposite problem and (as with calculating E) they will try to use Gauss’ law in situations where there is not enough symmetry (i.e. non uniform charge distributions).

Work and Energy (Griffiths Chapter 2)

STUDENT DIFFICULTIES

None documented

Conductors + Capacitors (Griffiths Chapter 2)

STUDENT DIFFICULTIES

- Didn't realize that E outside a conductor would change when you placed a charge q on the outside of the conductor. Somehow thought that the conductor would also shield from changes on the outside of the conductor? There are many tricky conductor problems which require some deep thought, such as – *What is the distribution of charge on the outside of a conductor when a charge q is placed inside an off-center cavity inside the conductor? (Is it uniform or off-center?). Why?* These questions can make students think about their understanding of conductors.
- Students grasp the essential features of conductors, but may have trouble applying it to real problems (as seen in common errors in Griffiths 2.36). Tapping their conceptions through clicker questions which differ from the simplest conductor problems may be a useful way to help them understand conductors in a deeper way.

Laplace's Equation & Poisson's Equation Uniqueness (Griffiths Chapter 3)

STUDENT DIFFICULTIES

What is Laplace's Equation? (***)

- Students often did not recognize Laplace's equation as a special case of Poisson's equation.
- In student interviews students often confused Laplace's Equation with Poisson's Equation, couldn't remember Laplace's equation (at least two thought that it was $\nabla \cdot E = 0$) and where it was valid (where $\rho=0$).
- Many did not know that we were solving Laplace's equation in these different solution methods, such as separation of variables and method of images. Thus, it's important to emphasize throughout the chapter – not just at the beginning – that each of these methods are one way to solve Laplace's Equation, and what Laplace's equation is.
- At least one student was unsure whether Laplace's Equation was a law of nature or not, and whether it was always true.

- Most did not recognize (at least without some prompting) that Laplace's Equation arises from the differential form of Gauss's Law where there is zero charge.

Uniqueness (*)

- In interviews, many students did not relate Uniqueness theorem to Laplace's equation. They knew that it had something to do with if you find a solution, it's the solution, but didn't remember what "it" was the solution to.
- Students often approach separation of variables problems algorithmically and do not attempt to make sense of the steps. This is particularly problematic when trying to use this technique in novel situations.

Method of Images (Griffiths Chapter 3)

STUDENT DIFFICULTIES

At this point in the course, students have seen a variety of methods for solving the potential in a region. To reduce cognitive load, it is helpful to give as many organizing principles as possible, and highlight important points of each method.

Applicability of Method of Images (*)

- Students struggle a little bit with the idea that you can only use method of images in the region where you have *not* placed the image charge (because the image charge changes V in that region). Griffiths pgs 83 has a good explanation. One student said she thought that Method of Images was "weird" because you're changing the problem that you're solving. However, actually calculating potentials using this method seems to pose little difficulty.
- On a post-test problem where students were given a conducting sphere and a point charge outside that sphere, the majority did not recognize it as a Method of Images problem. Thus, either they do not recognize a conducting *sphere* as appropriate for Method of Images, or they are not abstracting the general principle that Method of Images is useful for *conductors* in general.
- There was also some confusion as to whether the conductor needs to be grounded, but this is a relatively sophisticated point.

What is Method of Images doing? ()**

- Many students didn't understand, in interviews, that Method of Images was solving Laplace's Equation and that we were able to do it because of the Uniqueness Theorem. Only the best students could recall what the uniqueness theorem was. Even after being reminded what the uniqueness theorem was, some did not see its applicability to the Method of Images.

- Several students didn't seem to recognize that matching the *boundary conditions* is what is important in Method of Images. This relates to their lack of understanding that Method of Images is solving Laplace and works because of Uniqueness. I think that clearing up this fundamental difficulty would help them understand the rationale and approach for Method of Images.

Separation of Variations in Cartesian and Spherical (Griffiths Chapter 3)

STUDENT DIFFICULTIES

Applicability of Separation of Variables (*)

- Students are mostly comfortable with the separation of variables technique, having seen it before. They may not understand the motivation for doing separation of variables and when it is useful.
- Students aren't sure if you can always separate a function into XYZ or $R\theta\phi$, if that only works for certain functions.

Successfully completing Separation of Variables (*)**

- Some students get lost in the steps, going about them in an algorithmical way without understanding the purpose of individual steps. One student was helped by looking back over previously worked problems and abstracting out the steps common in each solution (such as exploiting orthogonality), although this might emphasize an algorithmical approach. In the Transformed course, we made the rationale for all the steps and for choosing a particular form of the solution (eg., the sinusoidal solutions in Cartesian separation of variables to match boundary conditions that go to zero in two places) explicit. In the Transformed course, students were able to more successfully complete a separation of variables problem and to put it into the larger context. Students in the Traditional course also did not seem to generalize what they had learned, and were unprepared to do a problem that was not exactly like what they had seen in the book (for instance, knowing whether to apply the oscillatory or exponential solutions to the x or y direction).
- One instructor reported student difficulty in separation of variables as well, despite painstaking examples worked in class, which included class participation and asking students for the next step. He reports that students "could do the boundary conditions OK, and indicate the general solution inside and outside, but following through and solving the problem was difficult... Most could set it up but couldn't get through the algebra for solving for the constants." Instructors teaching E&MII reported that students continued to struggle with separation of variables.

- Writing charge distributions as Legendre polynomials (e.g., pure “ P_0 ” or a combination of P_1 and P_2 , depending) was not too hard for those in the Transformed course but those in the Traditional course really struggled. In the Transformed course, it wasn’t quite clear to students when in the procedure of Separation of Variables this simplification becomes useful (HW7). SJP noted that it worked best to suggest they work the problem as far as they could without the simplification, and then express the charge distribution as Legendre polynomials.

Boundary conditions (***)

- Students struggle a great deal with identifying useful boundary conditions for solving a problem, or even with identifying what a boundary condition is. See, for example, the notes on HW7. The term “boundary condition” itself seems to be somewhat opaque to students and could use emphasis. I.e., why do we care what happens at a boundary more than some other region of space? What boundaries are of interest? On the post-test, when asked for boundary conditions at the surface of the sphere, about half the students still gave boundary conditions at infinity, or simply said that $E=0$ on the surface.
- Many students did not recognize that V is always continuous and E is discontinuous at a charge distribution, and that that is a useful boundary condition, though this was less the case in the Transformed course than the Traditional course. Only some students were able to use Gauss’ Law to derive the magnitude of the discontinuity in E at a charge distribution, and often required many hints. This is covered at an odd place in Griffiths (at the end of Chapter 2), and students may need to be directed to that reading when in the middle of Chapter 3. One student stated that V is continuous, but later when solving a problem thought she remembered that $V_{in} - V_{out} = \sigma$. Thus, the physical *meaning* of the continuity or discontinuity of something at a boundary may not be well established.
- Later, in Chapter 4 after they had seen a lot of separation of variables, there was still considerable difficulty. In a “middle” region between free space and an inner enclosed sphere, they didn’t indicate that neither A_l nor B_l would go to zero. At the surface of a sphere with potential V_0 , they indicated that the derivative of V would be discontinuous, indicating they do not understand boundary conditions and where they derive – that the derivative of V is discontinuous because there is a charge σ that creates an E field. Many gave similar boundary conditions for a surface with potential V_0 as for surface with a charge σ .
- Even the best students have great difficulty applying Gauss’ Law (and the fact that $\nabla \times E = 0$) to generate the boundary conditions on E . They do not think, for example, to draw a surface around a surface charge and use Gauss’ Law on that surface. This is often done *for* them and so when asked to *generate* the boundary conditions, they don’t know where to start. They also consistently struggle with whether there should be a factor of two in the final answer. I strongly recommend a whiteboard activity where they are asked to derive the parallel and perpendicular boundary conditions on E .

Legendre Polynomials (*)

- The Legendre Polynomials are intimidating to students, and this is one of the first cases of seeing special functions. One student, who is also a dual math major, understood them well once he realized that they're just another complete orthonormal basis. This fact may be lost on other students, but may be worth emphasizing as it will prepare students for QMI.
- Some were curious as to where the $\sin\theta d\theta$ term comes from in the normalization of the Legendre polynomials.

Orthogonality and Coefficients

- Solving for the coefficients by exploiting orthogonality of the function set (sin/cos or Legendres) creates some difficulty. There seems to be some confusion on when you are "done" with the problem – many stop before solving for the coefficients. Some emphasis could be used on just what the coefficients mean.

Solving Laplace's Equation (Cartesian)

- Students have difficulty recalling the form of the separated solution ($V(x,y,z)=X(x)Y(y)Z(z)$) as well as justifying why that functional dependence is useful/legitimate.
- Once they have plugged in the separated solution into Laplace's equation, many students have difficulty arranging the equation into the form $f(x)+f(y)=0$ so that the PDE can be expressed as two ODE's. Not all students will recognize that this is a necessary step to separate the equation.
- Many students have trouble satisfying the boundary conditions once they have a general solution, particularly in problems that do not directly map onto those the students have seen before. Additionally, many students do not understand the need for the infinite sum in order to match the boundary conditions.

Multipole Expansion (Griffiths Chapter 3)

STUDENT DIFFICULTIES

We covered the beginning of Chapter 4 (dipole moments) before Multipole Expansion so that students would have an understanding of a dipole moment before we tackled this tricky expansion, and we recommend this technique for future instructors. It grounds the expansion in something physical (a dipole) rather than being an abstract mathematical tool that they don't understand very well.

The essential message of this portion is that you can treat something that is not a dipole as a dipole, if you are far away. This is also the first time we have seen an expansion, which is going to be an important theme throughout the course. Why are expansions useful physics tools, and when do we want to use them? This is something we want students to come away from this course with, and requires explicit emphasis.

In this section, $1/r$ is expandable as a series of Legendre polynomials. Why is this possible? Can we give a conceptual understanding of the math?

A conceptual way of framing this section is: If you get far enough away, the charge distribution looks simpler. Its essential feature comes out as it starts to look more indistinct. It vanishes and how it vanishes with respect to r depends on its particular distribution. It dies away quickly, but how quickly depends on the distribution.

Applicability of Multipole Expansion (*)**

- We're often not interested in the monopole term because it's often zero – that is the case in a neutral material. However, many students overgeneralize this and believe that multipole expansion is *only* useful or applicable if the monopole moment is zero.
- Because we generally only do problems involving a dipole moment, many students also overgeneralize to say that the multipole expansion is only when there is a dipole moment.
- Most students recognized, when asked, that multipole expansion and separation of variables should give the same answer, but there is some confusion on when the multipole expansion is useful as opposed to just valid – ie., when you are far away from the charge distribution and thus can neglect higher-order terms.
- Similarly, many students thought that Multipole Expansion is only *valid* (as opposed to useful) when you're far away from the distribution.
- Most students didn't seem comfortable with Multipole Expansion and on the CUE post-test many did not choose to use it when presented with a case that had a dipole moment, falling back instead on direct integration.
- Several students had questions as to how far you have to be until you are "far away" enough to drop leading terms. We were not able to give a hard and fast rule. One student seemed surprised that "100d" (where d is the dipole separation) was considered far away.

Exactness of Multipole Expansion ()**

- Most students, even the best, couldn't name a situation when you would want to keep higher-order terms in the multipole expansion (e.g., when you are not far away from the charge distribution).
- Many students thought of the Multipole Expansion as always being an approximate solution, and either didn't think to say, or didn't realize, that it was exact if you keep all the terms. When pressed, most did recognize that it would be exact if you kept all the terms, but it wouldn't be practical to keep all the terms.

Conceptualization of Multipole Expansion (**)

- When this topic was covered in some detail in the Transformed course, students seemed to grasp the technique and when it was useful. In the Traditional course, this appeared to be less the case. One student complained that they never really learned what these multipole moments represent, physically. How is there a quadrupole moment “hiding” inside a charge density, and what does it mean physically?
- I am not sure that the origin or physical basis of Legendre polynomials in the expansion of $1/r$ is grasped by any student
- Some students didn't seem to understand that the multipole expansion could be rewritten as $V(\text{mono}) + V(\text{dipole}) + V(\text{quadrupole})\dots$. They saw it as a combination of terms that did not have physical meaning.
- At least two people thought that multipole expansion and/or dipole potential was derived from separation of variables.

Dipole moment and choice of origin

- There was some confusion in class about the fact that the dipole moment depends on where we set the origin, and how to translate a drawing of point charges to the abstract dipole moment vector (especially given the fact that it depends on the origin).
- Many students thought that the expression for V_{dip} was an exact form. If you have a dipole, then that is the expression for its potential. They did not recognize that this came from an approximation of being far from the dipole, and most did not remember how we arrived at this formula at all. This was not a universal difficulty, however, and some realized that as you get close to the charges it no longer looks like a dipole but rather like two separate charges.
- Several students, up until the end of the course, believe that you need both + and - charges in order for a distribution to have a dipole moment.

The expansion

- Many students don't explicitly recognize this method as a rather familiar way of expanding something in powers of $1/r$. Also, note that the expansion doesn't depend on θ but rather on θ *prime*, a subtle distinction.

Polarization and Fields of Polarized Objects

Linear Dielectrics

(Griffiths Chapter 4)

STUDENT DIFFICULTIES

What is a dielectric? (**)

- Some students are not quite clear on what a dielectric is (or isn't). What would be a nonlinear dielectric? Can you embed a charge in a dielectric, versus a conductor? Are semiconductors dielectrics?

Bound surface versus volume charge

- In the Traditional course, several students didn't differentiate between rho-bound and sigma-bound in the homework, suggesting that they don't have a firm grasp on what those numbers represent. Questioning of one student elicited memory of the instructors' description of what sigma-bound is (the dangling positives/negatives from the dipoles), but there was a general sense that students were calculating things that they didn't fully understand. A few weeks later, they also said they didn't have a strong concept of bound charge. In the Transformed course, the students were forced to grapple with the concepts in clicker questions, and they showed much more of an understanding of what it was through interviews and homework.

Initial E vs. Produced E (*)

- In several homework problems, students struggled with the difference between the E field that creates a polarization, and the E field created *by* the polarization, which sum together iteratively to create the final result. This requires a little care, but also can represent a difficulty in understanding just what polarization of a linear dielectric is; i.e., the response of a substance to an external electric field (not to its own electric field).

Electric Displacement (Griffiths Chapter 4)

STUDENT DIFFICULTIES

Notes

Just what “D” represents is a useful discussion. In the Transformed course, we paid some attention to the fact that the curl of D is not always zero, and that is partly what differentiates it from an “E for bound charge.” This was helpful for a small fraction of students who were ready to consider that concept. The D-field is unchanged by the presence or absence of the dielectric only in situations with sufficient symmetry that we can use Gauss’ Law in integral form (see Griffiths 4.3.2) and the curl of D is zero. Thus D is most useful in situations of high symmetry.

What is “D”? (*)**

- Most students don’t grasp what D is. They get the sense that it’s “like E, but not really.”
- Whether D is in the same direction as E (and P) is a bit tricky and students make many mistakes on the homework on this.

Calculating D (*)

- In general students are able to recognize to use the Gauss’ law for D to find E for a dielectric and can set up the right Gaussian surface.
- A useful discussion question was “when can you calculate D and when can’t you?”

Boundary Value Problems with Dielectrics (Griffiths Chapter 4)

STUDENT DIFFICULTIES

None documented

Magnetic Fields and Forces (Lorentz Force Law)

(Griffiths Chapter 5)

STUDENT DIFFICULTIES

None documented

Current Density
(Griffiths Chapter 5)

STUDENT DIFFICULTIES

Notes

The idea of volume and surface currents J and K , and how they relate to I is difficult. It is slightly difficult to visualize J flowing through an area A and that integral gives you the total current, but it's very difficult to visualize K flowing past a line element to give you $I=2\pi R*K$. Prof. Pollock used the visual analogy of the Mississippi which seemed to stick with many students:

I motivated J by thinking about the "flow of the Mississippi" compared to "flow of Boulder Creek", and characterizing flow as total current (Mississippi clearly vastly bigger) but what about "water flowing at me through this circle I am making with my fingers". Then perhaps Boulder Creek even wins - so there's some OTHER quantity to characterize flow, which motivates our definition of "current density" as current/area. (Then rotate the circle to show them that it's really perpendicular area needed to DEFINE this current density).

Common difficulties

Volume and surface current (*)**

- After many weeks of dealing with surface and volume currents, even some of the best students did not realize that they had to integrate over the volume current J to get the current I . Some attempted to add K and J to get I , or to multiply $J*A$ to get I , even if J was not uniform. More students in the Transformed class seemed to understand J and K via their units, but students in both classes struggled.

- Most student had a fuzzy understanding of the distinction between I and J and K, and were not able to articulate their ideas clearly. They have trouble understanding a density that is *flowing* rather than static (as in charge density).
- Even among the best students, many saw K as “ I/L ” rather than “ dI/dl ” and this caused difficulties in calculations (and similarly for J) since it was then not obvious to them that $I = \int K \cdot dl$.

Biot-Savart Law (Griffiths Chapter 5)

STUDENT DIFFICULTIES

Setting up the integral (**)

- The same problems from Coulomb’s Law appear again here. Putting together the pieces of Biot-Savart is a bit of a challenge – many can understand *why* you would use a particular dl , or formulation of script- r , but have difficulty coming up with that formulation on their own.

Cylindrical coordinates (**)

- Magnetism often involves rotational symmetry about the z-axis, resulting in the use of cylindrical coordinates. These are a little more unfamiliar to students, though they can use them well once prompted. I noticed a general tendency to forget to use cylindrical coordinates or not notice rotational symmetry, or to forget to use operators (such as del) in cylindrical coordinates instead of cartesian.

Divergence and Curl of B (Ampere’s Law) (Griffiths Chapter 5)

STUDENT DIFFICULTIES

Amperian loops (*)

- Drawing Amperian loops is a little difficult. It seems hard for students to visualize where B will be constant along a loop (suggesting a particular geometry for the loop), as well as where to place the loop once they have figured out the right

geometry. I.e., should a rectangular loop be placed symmetrically about the boundary, or have one side lie along the boundary? Giving students challenging (non-standard) Ampere's Law problems seems to help them struggle with it. Those in the Transformed course seemed more comfortable.

Boundary conditions on B (***)

- Even the best students have great difficulty applying Ampere's Law (and the fact that $\nabla \cdot B = 0$) to generate the boundary conditions on B. They do not think, for example, to draw a loop around a surface current and use Ampere's Law around that loop. This is often done *for* them and so when asked to *generate* the boundary conditions, they don't know where to start.
- Even though are familiar with using Gauss' Law to generate the conditions on the perpendicular part of E, they do not generalize this result to be able to derive the conditions on the perpendicular component of B using $\nabla \cdot B = 0$. At least some think that Stokes' Theorem is for B, and Divergence Theorem/Gauss' Law is only used for E. This is true even of many of the best students.
- As with the boundary conditions for E, students also struggle with whether there should be a negative in the final equation ($B_{above}^{\parallel} - B_{below}^{\parallel} = \mu_0 K$) because they interpret B^{\parallel} as a vector instead of a component, and so do not include the negative sign where appropriate.
- I strongly recommend a whiteboard activity where they are asked to derive the parallel and perpendicular boundary conditions on B.

Integral and differential forms (***)

- Students struggle with understanding the equivalence of the integral and differential form of Ampere's Law using Stokes' Law. This is a major and persistent difficulty.
- I saw several students translate from the differential form by looking at the right hand side of the differential equation (i.e. "I") and recognizing you need to integrate over volume to get the right hand side of the integral form (i.e., "J"). They invoked Stokes' along the way, but it was not the driving force of the calculation. Their shaky understanding of Stokes' was evident when they were asked to translate $\nabla \times E = 0$ to integral form. Without the right-hand side as a guide, they were stuck as to whether to do the integral over volume or area. They did not easily recognize that if a vector field has no curl, then the integral of that field around a closed loop is zero.

Stokes' Theorem (***)

- Students struggle with understanding and using Stokes' theorem. A physical interpretation of Stokes' theorem is useful. However, many students will draw a closed loop and realize that the circles inside the loop add up to something around the whole loop, but they are not clear on *what* is being added up. Are they bound current loops representing atoms, for example? Students have much more difficulty *visualizing* Stokes' theorem than the divergence theorem.

- Students can understand a physical interpretation of Stokes' theorem, but making the connection to understand how this applies to Ampere's Law is difficult.

Curl of B

- If students have difficulty with curl (which most do), this is where it will show up. Students struggle with the idea that if there were magnetic charge, then B would have a divergence like E. When checking to see if a B field could exist, many check for curl but do not check for divergence.

Magnetic monopoles

- When considering the hypothetical existence of magnetic monopoles, students do not automatically assume that monopoles would create a nonzero $\nabla \cdot B$. They don't know what a monopole would look like (our best illustration is of an isolated north pole), and don't assume the B field would be radial. One student was concerned that two monopoles with radial B fields can't be easily combined to give a circumferential B field.

Magnetic Vector Potential (Griffiths Chapter 5)

STUDENT DIFFICULTIES

Notes

- How does A make our lives any easier? It does not have a direct connection to energy, like V does. It's useful for separation of variables and multipole expansions.
- When do we use A to get B and when can we get B directly? Biot-Savart is hard to use, and there are few problems that are solvable using Ampere's law, so A is a mathematical shortcut to get to the physics. We have a lot of tricks for calculating V , and we can use those same tricks for A , just in 3 dimensions. A will also be useful in radiation and modern physics, and has application to Schroedinger equation.
- We have a few articles on the vector potential available.

Common difficulties

What is A ? (*)**

- There is a general consensus that A is an opaque quantity – students don't know why it's useful or what it represents. When do we use A and when is it easier to use B ?

- The analogy between B and A, and J and B is very tricky for students.

Vector calculus (*)**

- Vector calculus problems arise at this point in the course because of some of the mathematical formalism required.
- On the homework in the Traditional course they were asked to do some vector calculus operations in order to verify the curl of $A = B$ and some other relations (#5.27). Many people did poorly on this, and did not give reasoning for most of their mathematical steps. There was an overall impression that they did not understand the vector calculus and could not make the connection between the mathematical operations and the physics of the operations.
- There was also a tendency to be sloppy about whether a particular vector operator was operating on a particular vector (eg., ∇ not operating on primed coordinates), but this was not observed in the Transformed course where most knew that a non-primed operator operating on primed coordinates would be zero.
- In the Transformed course, vector calculus problems also cropped up around this time, such as uncertainty about how to do the integral of a vector where there are three components inside the integral. Another question that came up several times was when can you move a derivative outside of an integral (HW10, Q2).

Magnetic dipoles (*)**

- By the end of the course, most students don't see a current loop as a magnetic dipole, and when asked the best method to find B of a current loop when far away, most answer by direct integration.

Magnetization – diamagnetic, paramagnets, ferromagnets (Griffiths Chapter 5)

STUDENT DIFFICULTIES

Notes

Superconductivity provides interesting fodder for discussion, as does the applicability of chemistry (e.g., unpaired electrons) to determining the magnetic nature of a material.

Common difficulties

None observed

Field of Magnetized Object (Bound Current) (Griffiths Chapter 5)

STUDENT DIFFICULTIES

Notes

One student asked a very good question – when we are calculating A inside an object (eg., #6.8), then why do we not use a surface current K ? When you are sitting at any point inside the object, there are “dangling” surface dipoles which are not cancelled by the ones above them (since you’re not including those in the enclosed current).

Common difficulties

Bound vs. free current

- Many students struggled to recognize that $\int B \cdot dl = \text{total } I_{\text{enclosed}}$, where $\text{total } I_{\text{enc}} = I_{\text{free}} + I_{\text{bound}}$, which includes both free and bound current.
- I see evidence that the physical interpretation of bound charge didn’t stick with some students; they are now having trouble with bound current.
- There was some difficulty in knowing (a) what the direction of B was if you have bound currents and (b) how to calculate B using bound currents.
- Griffiths mentions briefly that if there is no free current flowing through the linear medium, then the bound volume current is zero. However, this isn’t obvious and in one problem (6.16) I was not able to reason through the physical situation.
- When calculating K_b (such as problem 6.16) you need to be careful of the surface you are using. If the surface is that of a cavity inside a material, then the normal to that cavity actually points inwards, and this can trip students up.

Magnetism M

- When calculating the magnetism for a thick wire from $M = \chi_m H$ (Griffiths 6.17), the question arose whether to use B_{in} or B_{out} to calculate M . They figured out that it was B_{in} because you’re looking at the material, but this seems to suggest a slight uneasiness in going between the physical situation and the math.
- In 6.16 the magnetism M is in the ϕ direction, and so we tried to figure out whether the bound current J_b was zero or not by our physical intuition. It turns out that when you calculate $\nabla \times M$ in cylindrical coordinates, you get zero, but it’s not obvious to me why this is. It “looks” curly.

Auxiliary Field H (including Boundary Value Problems) (Griffiths Chapter 6)

STUDENT DIFFICULTIES

What is H? (**)

- Students understand that H is used “like B” for free current, but they often don’t understand why. Instructors often tell students that H is used by engineers, but many students need to explicitly struggle with H in order to understand that H depends on what we *know* (the *free* current).