

Transformed E&M I materials

Mathematical materials (Griffiths Chapter 1)

STUDENT DIFFICULTIES

Line and surface integrals

- Students have trouble taking surface and line integrals – setting up the integral, writing the area or line element, and writing the distance from the source. Many of these difficulties may stem from students failing to visualize the problem and account for the physical situation as a sum of little bits. This problem seems to be exacerbated by the differences between the way mathematicians think about integrals (area under a curve) vs. how physicists think about integrals (sum of little bits).
- **Line integrals.** Students don't conceptualize clearly what they are doing. For example, on HW4, students would integrate $\int z dy = zy$ without noticing that $z=0$ along the segment that they were integrating over.
 - Students often do not visualize a particular path when doing a line integral even when they are specifically prompted to do so.
- **Area elements.** We often don't explicitly teach how to write down an area element. There is a good trick that can be taught using the front cover of Griffiths. Why is it that the front cover lists $d\mathbf{l}$ and $d\mathbf{T}$ but not $d\mathbf{a}$? Because there are three possible area elements that can be listed. So their task is to choose the two components of $d\mathbf{l}$ that are relevant for the task at hand. Put those together to make your $d\mathbf{a}$.
 - It may not be immediately obvious to students that their area element is wrong, even when the units are incorrect (i.e. $d\mathbf{a}=drd\phi$)
 - Students may not be familiar with how to derive $d\mathbf{a}$ or how to check their $d\mathbf{a}$ once they have one (i.e. integrate over a sphere to get the expression for surface area)

Curvilinear coordinates

- Curvilinear coordinates are also a source of difficulty for students, who have seen them but not used them much. Many continue to have difficulty knowing when to use a particular coordinate system, throughout the course. This is related to the challenge of identifying axes of symmetry.
- Early in the course, many students do spherical integrals in Cartesian coordinates, supposedly because they are not yet comfortable with spherical coordinates.

Delta functions

- **Charge distribution.** Writing a charge distribution as a delta function caused trouble for many students – they did not seem to understand the concept without explicit instruction and even then struggled. They could identify $\delta(x-2)\delta(y-3)\delta(z+2)$, for example, as a point charge, but if given just $\delta(x-2)$ the students in the Traditional class often misinterpreted that as a point charge. On HW4, students calculating the E field and charge distribution arising from and due to a “screened” Coulomb potential that goes as $1/r$, students did not recognize that there would be a delta function at the origin, despite prompting to think about the origin.
- **Dimensionality.** The first example above illustrates that the dimensionality of the delta function is difficult for students. On the post-test, many students misinterpreted two delta functions at r_1 and r_2 as being two concentric spheres with radii r_1 and r_2 (rather than two points at r_1 and r_2). Students sometimes equate a *volume* charge density with a *3D* delta function. Asking students to write the *volume* charge density of a *surface* charge also causes confusion. One way of explaining delta functions which was useful for some students was that each subsequent delta function “squishes” the distribution in one dimension. Eg., one delta function “squishes” it into a plane, the second restricts it in one further dimension to make it a line, and the third restricts it in the third dimension to make it a point. The fact that this was a useful idea to students indicates that they are struggling with conceptualizing how additional delta functions affect the dimensionality of the charge distribution.
- **Divergence.** When asked to calculate the divergence of the E field in a region of space containing a delta function charge distribution, students’ lack of connection between divergence and flux (which haven’t yet been stressed in class) came up.
- **Units.** When the *units* of the delta function were stressed (the 1D delta function has units of $1/m$), this was a powerful tool for students to check their conceptualization, and I urge other instructors to emphasize unit analysis of delta functions. For instance, when asked what physical situation is represented by $\rho = c \cdot \delta(x-2)$, many students answered a point charge. The best students, however, were able to argue it correctly with a combination of unit and dimensional analysis.
- **Infinities and idealizations.** Other professors have indicated that students are uncomfortable with the concept of an infinite charge density. The concept of a point charge (density is infinite at the point charge, but total charge is finite) is difficult. Most students continue to confuse delta-function charge densities (where the charge density is infinite at a point) with idealized line, surface, and volume charge densities (where the charge density is smeared out and not infinite at any point). This causes them difficulties later on.

Vector Calculus

- Students at the upper-division level have difficulties attending to both magnitude and direction of a vector at the same time. They often focus on only one aspect at a time, sometimes switching mid-problem.

- Students have considerable difficulty connecting the math of the divergence to the physical situation. They often have difficulty identifying fields with non-zero divergence from diagrams. They also have difficulty identifying where the divergence of E is non-zero (i.e. only where there are charges) and do not seem to utilize the differential form of Gauss' law.
 - Difficulties here seem to be related to students using the common English definition of divergence (i.e. anytime the lines are spreading out) instead of applying a rigorous mathematical approach.

Divergence and Stokes / Integral and differential

- Students have many difficulties with Divergence and Stokes' Theorems, and translating between integral and differential forms of equations. These difficulties will be detailed in the chapters in which they occur in context of E&M (chapters 2 and 4). Overall, students do not seem to understand the physical significance of the divergence theorem as a way to translate between integral and differential forms. For example, in Griffiths 1.32 students are asked to verify that two equations are equal, using the divergence theorem. Most students did not actually evaluate the surface integrals (to thus verify that they equaled the volume integrals). This is a difficulty that pervades the course. Going between the differential form and integral form of an equation with ease takes students most of the course, and they do not become facile with it unless this is an emphasized topic.