Transformed E&M I materials

Multipole Expansion
(Griffiths Chapter 3)

STUDENT DIFFICULTIES

We covered the beginning of Chapter 4 (dipole moments) before Multipole Expansion so that students would have an understanding of a dipole moment before we tackled this tricky expansion, and we recommend this technique for future instructors. It grounds the expansion in something physical (a dipole) rather than being an abstract mathematical tool that they don't understand very well.

The essential message of this portion is that you can treat something that is not a dipole as a dipole, if you are far away. This is also the first time we have seen an expansion, which is going to be an important theme throughout the course. Why are expansions useful physics tools, and when do we want to use them? This is something we want students to come away from this course with, and requires explicit emphasis.

In this section, 1/r is expandable as a series of Legendre polynomials.
Why is this possible? Can we give a conceptual understanding of the math?

A conceptual way of framing this section is: If you get far enough away, the charge distribution looks simpler. Its essential feature comes out as it starts to look more indistinct. It vanishes and how it vanishes with respect to r depends on its particular distribution. It dies away quickly, but how quickly depends on the distribution.

Applicability of Multipole Expansion (***)

- We're often not interested in the monopole term because it's often zero – that is the case in a neutral material. However, many students overgeneralize this and believe that multipole expansion is *only* useful or applicable if the monopole moment is zero.
- Because we generally only do problems involving a dipole moment, many students also overgeneralize to say that the multipole expansion is only when there is a dipole moment.
- Most students recognized, when asked, that multipole expansion and separation of variables should give the same answer, but there is some confusion on when the multipole expansion is useful as opposed to just valid – ie., when you are far away from the charge distribution and thus can neglect higher-order terms.

- Similarly, many students thought that Multipole Expansion is only *valid* (as opposed to useful) when you're far away from the distribution.
- Most students didn't seem comfortable with Multipole Expansion and on the CUE post-test many did not choose to use it when presented with a case that had a dipole moment, falling back instead on direct integration.
- Several students had questions as to how far you have to be until you are "far away" enough to drop leading terms. We were not able to give a hard and fast rule. One student seemed surprised that "100d" (where *d* is the dipole separation) was considered far away.

Exactness of Multipole Expansion (**)

- Most students, even the best, couldn't name a situation when you would want to keep higher-order terms in the multipole expansion (e.g., when you are not far away from the charge distribution).
- Many students thought of the Multipole Expansion as always being an approximate solution, and either didn't think to say, or didn't realize, that it was exact if you keep all the terms. When pressed, most did recognize that it would be exact if you kept all the terms, but it wouldn't be practical to keep all the terms.

Conceptualization of Multipole Expansion (**)

- When this topic was covered in some detail in the Transformed course, students seemed to grasp the technique and when it was useful. In the Traditional course, this appeared to be less the case. One student complained that they never really learned what these multipole moments represent, physically. How is there a quadrupole moment "hiding" inside a charge density, and what does it mean physically?
- I am not sure that the origin or physical basis of Legendre polynomials in the expansion of 1/r is grasped by any student
- Some students didn't seem to understand that the multipole expansion could be rewritten as V(mono) + V(dipole) + V(quadrupole)…. They saw it as a combination of terms that did not have physical meaning.
- At least two people thought that multipole expansion and/or dipole potential was derived from separation of variables.

Dipole moment and choice of origin

- There was some confusion in class about the fact that the dipole moment depends on where we set the origin, and how to translate a drawing of point charges to the abstract dipole moment vector (especially given the fact that it depends on the origin).
- Many students thought that the expression for V_{dip} was an exact form. If you have a dipole, then that is the expression for its potential. They did not recognize that this came from an approximation of being far from the dipole, and most did not remember how we arrived at this formula at all. This was not a universal

difficulty, however, and some realized that as you get close to the charges it no longer looks like a dipole but rather like two separate charges.

- Several students, up until the end of the course, believe that you need both + and – charges in order for a distribution to have a dipole moment.

The expansion

- Many students don't explicitly recognize this method as a rather familiar way of expanding something in powers of 1/r. Also, note that the expansion doesn't depend on theta but rather on theta *prime*, a subtle distinction.