Transformed E&M I materials

Separation of Variations in Cartesian and Spherical
(Griffiths Chapter 3)

STUDENT DIFFICULTIES

Applicability of Separation of Variables (*)

- Students are mostly comfortable with the separation of variables technique, having seen it before. They may not understand the motivation for doing separation of variables and when it is useful.
- Students aren't sure if you can always separate a function into XYZ or $R\theta\phi$, if that only works for certain functions.

Successfully completing Separation of Variables (***)

- Some students get lost in the steps, going about them in an algorithmical way without understanding the purpose of individual steps. One student was helped by looking back over previously worked problems and abstracting out the steps common in each solution (such as exploiting orthogonality), although this might emphasize an algorithmical approach. In the Transformed course, we made the rationale for all the steps and for choosing a particular form of the solution (eg., the sinusoidal solutions in Cartesian separation of variables to match boundary conditions that go to zero in two places) explicit. In the Transformed course, students were able to more successfully complete a separation of variables problem and to put it into the larger context. Students in the Traditional course also did not seem to generalize what they had learned, and were unprepared to do a problem that was not exactly like what they had seen in the book (for instance, knowing whether to apply the oscillatory or exponential solutions to the x or y direction).
- One instructor reported student difficulty in separation of variables as well, despite painstaking examples worked in class, which included class participation and asking students for the next step. He reports that students "could do the boundary conditions OK, and indicate the general solution inside and outside, but following through and solving the problem was difficult… Most could set it up but couldn't get through the algebra for solving for the constants." Instructors teaching E&MII reported that students continued to struggle with separation of variables.
- Writing charge distributions as Legendre polynomials (e.g., pure "P_0" or a combination of P_1 and P_2, depending) was not too hard for those in the Transformed course but those in the Traditional course really struggled. In the Transformed course, it wasn't quite clear to students when in the procedure of

Separation of Variables this simplification becomes useful (HW7). SJP noted that it worked best to suggest they work the problem as far as they could without the simplification, and then express the charge distribution as Legendre polynomials.

Boundary conditions (***)

- Students struggle a great deal with identifying useful boundary conditions for solving a problem, or even with identifying what a boundary condition is. See, for example, the notes on HW7. The term "boundary condition" itself seems to be somewhat opaque to students and could use emphasis. Ie., why do we care what happens at a boundary more than some other region of space? What boundaries are of interest? On the post-test, when asked for boundary conditions at the surface of the sphere, about half the students still gave boundary conditions at infinity, or simply said that E=0 on the surface.
- Many students did not recognize that V is always continuous and E is discontinuous at a charge distribution, and that that is a useful boundary condition, though this was less the case in the Transformed course than the Traditional course. Only some students were able to use Gauss' Law to derive the magnitude of the discontinuity in E at a charge distribution, and often required many hints. This is covered at an odd place in Griffiths (at the end of Chapter 2), and students may need to be directed to that reading when in the middle of Chapter 3. One student stated that V is continuous, but later when solving a problem thought she remembered that $V_{in} - V_{out} = \sigma$. Thus, the physical *meaning* of the continuity or discontinuity of something at a boundary may not be well established.
- Later, in Chapter 4 after they had seen a lot of separation of variables, there was still considerable difficulty. In a "middle" region between free space and an inner enclosed sphere, they didn't indicate that nether A_l nor B_l would go to zero. At the surface of a sphere with potential V_0, they indicated that the derivative of V would be discontinuous, indicating they do not understand boundary conditions and where they derive – that the derivative of V is discontinuous because there is a charge sigma that creates an E field. Many gave similar boundary conditions for a surface with potential V_o as for surface with a charge sigma.
- Even the best students have great difficulty applying Gauss' Law (and the fact that $\nabla \times E = 0$) to generate the boundary conditions on E. They do not think, for example, to draw a surface around a surface charge and use Gauss' Law on that surface. This is often done *for* them and so when asked to *generate* the boundary conditions, they don't know where to start. They also consistently struggle with whether there should be a factor of two in the final answer. I strongly recommend a whiteboard activity where they are asked to derive the parallel and perpendicular boundary conditions on E.

Legendre Polynomials (*)

- The Legendre Polynomials are intimidating to students, and this is one of the first cases of seeing special functions. One student, who is also a dual math major, understood them well once he realized that they're just another complete orthonormal basis. This fact may be lost on other students, but may be worth emphasizing as it will prepare students for QMI.

- Some were curious as to where the $\sin\theta d\theta$ term comes from in the normalization of the Legendre polynomials.

Orthogonality and Coefficients

- Solving for the coefficients by exploiting orthogonality of the function set (sin/cos or Legendres) creates some difficulty. There seems to be some confusion on when you are "done" with the problem – many stop before solving for the coefficients.   Some emphasis could be used on just what the coefficients mean.

Solving Laplace's Equation (Cartesian)

- Students have difficulty recalling the form of the separated solution (V(x,y,z)=X(x)Y(y)Z(z)) as well as justifying why that functional dependence is useful/legitimate.
- Once they have plugged in the separated solution into Laplace's equation, many students have difficulty arranging the equation into the form f(x)+f(y)=0 so that the PDE can be expressed as two ODE's. Not all students will recognize that this is a necessary step to separate the equation.
- Many students have trouble satisfying the boundary conditions once they have a general solution, particularly in problems that do not directly map onto those the students have seen before. Additionally, many students do not understand the need for the infinite sum in order to match the boundary conditions.