

Magnetostatics: Steady current flow everywhere

$$\vec{\nabla} \cdot \vec{J} = 0$$

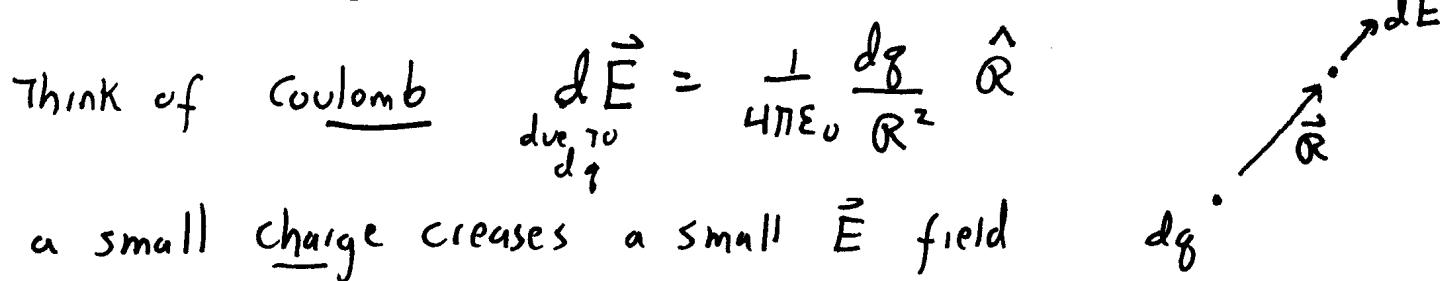
No "individual charges", it's all steady flows,  $\left( \text{no } \frac{\partial}{\partial t} \text{'s} \right)$   
 $\rightarrow$  fuss with

Currents create Magnetic fields:

It's an experimental fact. We have to do exp's to determine direction, magnitude. I cannot derive this!

We will see the formula later as ~~one~~ of Maxwell's eq'n's, but historically, Biot & Savart deduced an equivalent result by careful exp's (following Oersted's original discovery, during a class lecture demo (!) that current  $\Rightarrow \vec{B}$ )

Think of Coulomb  $d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq}{R^2} \hat{R}$



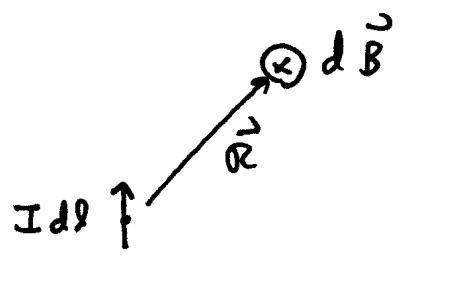
a small charge creates a small  $\vec{E}$  field  $dq$ .

Biot-Savart is similar

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{I} dl}{R^2} \times \hat{R}$$

due to  
chunk of current  $\vec{I} dl$

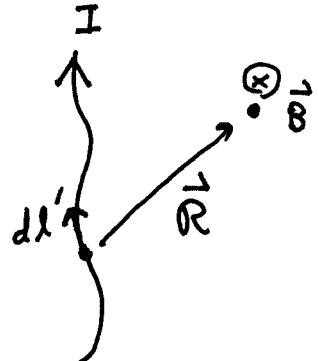
Just like Coulomb!



except for the cross product!

of course, in Magnetostatics there are no isolated chunks of current like this, so really must sum over chunks:

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{I} d\vec{l}'}{R^2} \times \hat{\vec{R}}$$



A constant. The magnetic partner of  $\epsilon_0$ , "permeability of free space"

$$\frac{\mu_0}{4\pi} = 10^{-7} \frac{N}{A^2}$$

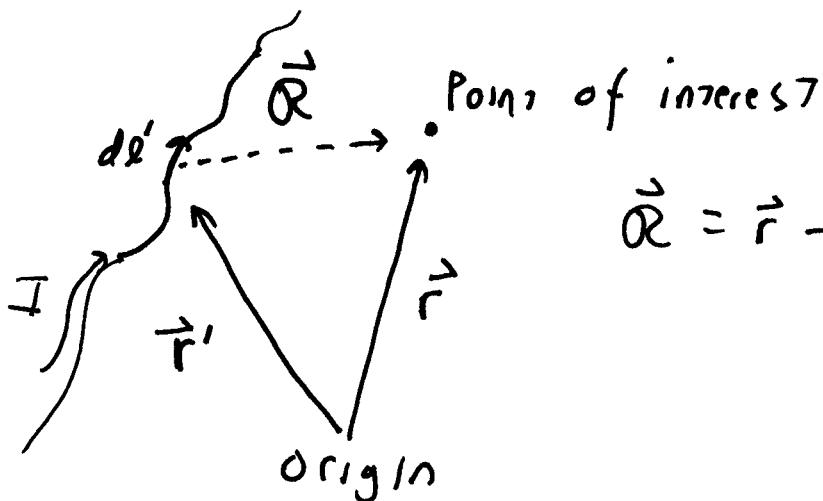
"permeability of free space"

Some authors don't like the vector on  $\vec{I}$ , + shift it

to  $d\vec{l}'$ , so  $I$  is just the amount of current in  $d\vec{l}'$  directions

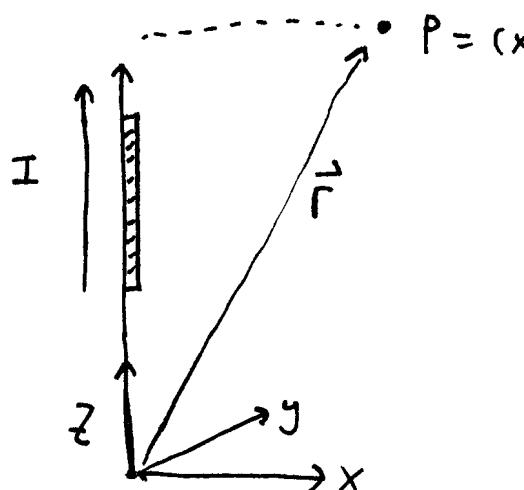
$$\text{so } \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int I \frac{d\vec{l}' \times \hat{\vec{R}}}{R^2}$$

Remember  $\hat{\vec{R}}$ :  
(definition)  $\rightarrow$



$$\hat{\vec{R}} = \vec{r} - \vec{r}'$$

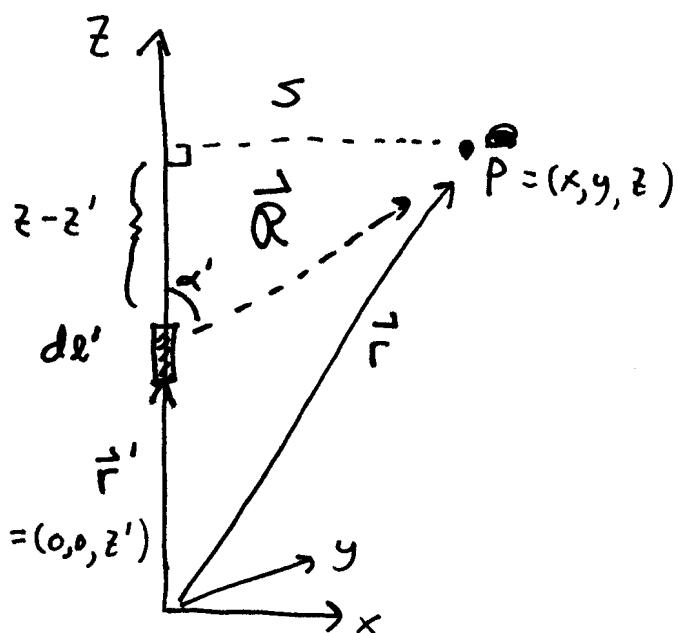
Example 1: Straight current segment  $I \cdot \hat{z}$



Let's figure out  $\vec{B}$  at  $P$  due to this chunk (shown hatched)

This way, we can figure out  $\vec{B}$  from e.g.  $I = \int \vec{f} dt$  by "summing chunk"

This finite chunk is itself a sum (integral) of infinitesimal chunks



$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l}' \times \hat{R}}{R^2}$$

Look @ the figure + convince yourself that  $d\vec{l}' \times \hat{R}$  points in  $\hat{y}$  direction by the right hand rule (RHR)

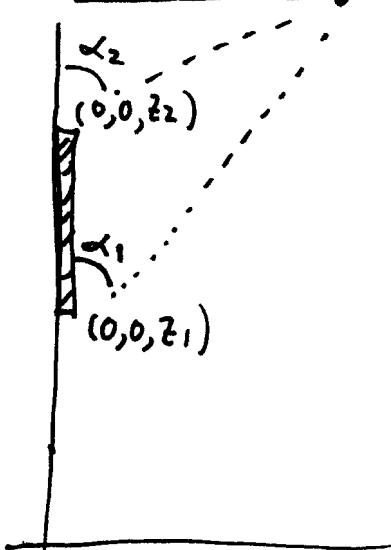
I will use  $S = \text{distance to } P \text{ from } z\text{-axis}$  (cylindrical radial coordinate)

and I have defined an angle  $\alpha'$  in the picture

Here  $d\vec{l}' = dz'$ , and  $|d\vec{l}' \times \hat{R}| = dl' \cdot 1 \cdot \sin\alpha'$

Also, looking at picture,  $R^2 = S^2 + (z - z')^2$ ;  $\sin\alpha' = \frac{S}{R}$

$$\text{So } \vec{B} \text{ at } P = \frac{\mu_0}{4\pi} I \int_{z_1}^{z_2} \frac{dz' \cdot S}{(S^2 + (z - z')^2)^{3/2}}$$



$S$  is a constant in this integral, + MMA gives me the result - (Griffiths works it out)

$$\vec{B} = \frac{\mu_0 I}{4\pi S} \frac{z' - z}{\sqrt{S^2 + (z' - z)^2}} \quad \begin{cases} z' = z_2 \\ z' = z_1 \end{cases}$$

$$\text{I could also observe: } \cos(\alpha_1) = \frac{z - z_1}{\sqrt{S^2 + (z - z_1)^2}}$$

$$\text{So } \vec{B} = \frac{\mu_0 I}{4\pi S} (\cos \alpha_2 + \cos \alpha_1) \text{ into the page (RHR!)}$$

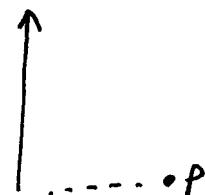
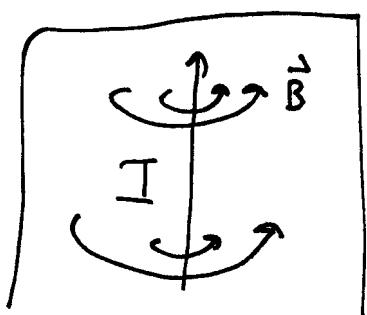
If current is infinite  $\alpha_1 = 0$ ,  $\alpha_2 = \pm \pi$ , and we get  $-1 + 1 = 2$  (in parens)

$$\vec{B} \text{ (long wire)} = \frac{\mu_0 I}{2\pi S} \text{ (RHR sense)}$$

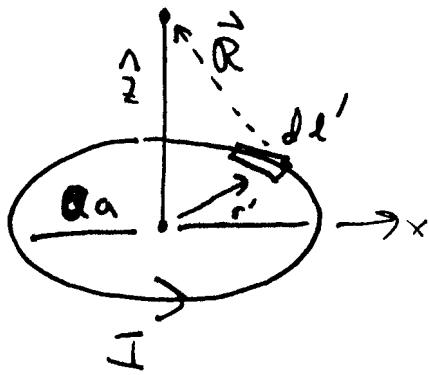
If current is "half infinite, starting across from P

$$\vec{B} = \frac{\mu_0 I}{4\pi S} \quad (\alpha_1 = \pi/2)$$

e.g.



one more example: Ring of current,  $B$  on axis?



Here,  $R = \sqrt{a^2 + z^2} = \text{constant}$ ,  
so that's nice & easy!

Unfortunately  $d\vec{l} \times \hat{R}$  points at  
a crazy angle, • But if we sum  
over all  $dl'$ 's, only the vertical (z) component of those will  
survive! (convince yourself!)

$d\vec{l} \times \hat{R} = dl'$ , but the vertical component is

$$dl' \cos \alpha \cancel{= dl'} \frac{a}{\sqrt{z^2 + a^2}}$$

see picture!

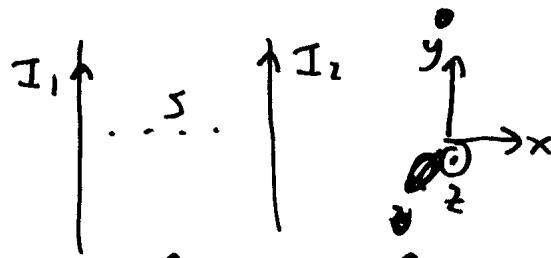
$$\text{so } B_z = \frac{\mu_0}{4\pi} I \int \frac{dl' \times \hat{R}}{R^2} = \frac{\mu_0 I}{4\pi} \cdot \frac{1}{a^2 + z^2} \cdot \frac{a}{\sqrt{a^2 + z^2}} \cdot \int dl'$$

$$= 2\pi a$$

$$B_z(0,0,z) = \frac{\mu_0 I}{2\pi} \frac{a}{(a^2 + z^2)^{3/2}}$$

To wrap up:

- ~~Parallel wires~~



$$\mathbf{B}_{\text{due to } I_2} = \frac{\mu_0 I_2}{2\pi s} \text{ into page } (-\hat{z} \text{ direction})$$

$$F_{\text{on } I_2 \text{ due to } I_1} = B_{\text{of } I_1} = \int I_1 d\vec{l}_1 \cdot \frac{\vec{B}}{-\hat{x} \text{ direction}} = I_1 \cdot \frac{\mu_0 I_2}{2\pi s} \int d\vec{l}_1 (-\hat{x})$$

so  $\frac{F_{\text{on } I_2}}{\text{unit length}} = \frac{\mu_0 I_1 I_2}{2\pi s}$  (towards  $I_1$  if both are parallel)  
away " anti-parallel "

- If current is spread out, Biot-Savart becomes

$$\int d\vec{B} = \iint \frac{\mu_0}{4\pi} \frac{\vec{K} \times \hat{\alpha}}{R^2} da' \quad \text{for surface currents}$$

$$= \iiint \frac{\mu_0}{4\pi} \frac{\vec{J} \times \hat{R}}{R^2} dt' \quad \text{" volume currents"}$$

- Don't try to use Biot-Savart to find  $\vec{B}$  from one single moving charge; that's not magnetostatics, wait till next semester!

- Superposition principle holds for  $\vec{B}$  just like  $\vec{E}$

5.21

$$\cdot \vec{B} = ?$$

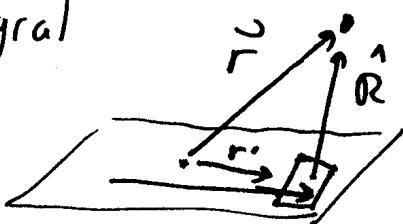
One more example: Sheet of current  $\vec{K}$ .  
(infinite in extent)



This is more common than you think, solenoids  
solar wind

...

we could set up the integral  
breaking surface into patches  
 $dA$  with current



$$\frac{\mu_0}{4\pi} \int \left( \vec{K} \times \hat{x} \right) \times \vec{R} \, dA'$$

But there will be a much easier way, coming soon! So let's hold off.

But by symmetry, convince yourself  $\vec{B}$  must point towards you above, + away (below) the sheet in the figure above.

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