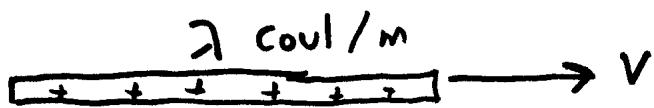


5-7

Currents: Current is the source of \vec{B} fields, (+ also reacts to B fields), so it's very important to clearly define + understand it. It's a measure of the flow-rate of charge. Current counts "how many charges pass by each sec."

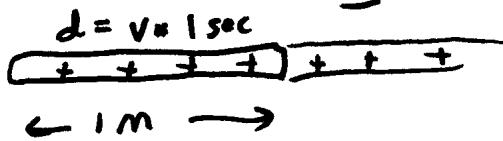
$$|I| = dQ/dt$$

Consider first a line charge (λ coulombs/meter) moving steadily with velocity \vec{v}



↑ How many coul move past this point in 1 sec?

• $v \cdot T = d$, so all the charges in the chunk $v \cdot 1 \text{ sec}$ long "behind"



the point will make it past!

$$\text{That's } Q = \lambda \cdot d = \lambda \cdot v \cdot (1 \text{ sec})$$

$$\text{so charge in one sec} = \frac{Q}{T} = \lambda v.$$

$$\text{so } \vec{I} = \lambda \vec{v}. \quad \leftarrow \text{Griffiths convention is to call } \vec{I} \text{ a vector, (not everyone does this)}$$

Note: If λ is negative, current goes "other way".

$$\text{so } \xleftarrow{v^-} \ominus \text{ is same current as } \oplus \xrightarrow{v^+}, \text{ both are } \vec{I} \rightarrow$$

- I measured in $\frac{\text{Coul}}{\text{sec}} = \text{Amperes}$.
- If the wire has n_L charge carriers length, each of which carries g , then $\lambda \frac{\text{coul}}{\text{m}} = n_L \frac{\text{carriers}}{\text{m}} * g \frac{\text{Coul}}{\text{carrier}}$

so $\vec{I} = n_L g \vec{V}$

Since $F_{Mg} = g \vec{v} \times \vec{B}$ for individual charges,
a current feels a force too (each individual charge
feels a force, so the "current" feels the sum of those)

For a small piece of my wire (above), dL long,
there are $(n_L dL)$ charges, each feeling $g \vec{v} \times \vec{B}$,

$$\text{giving } d\vec{F}_{\text{on chunk}} = n_L (dL) g \vec{v} \times \vec{B}$$

Since \vec{v} is along the wire, and so is $d\vec{l}$,

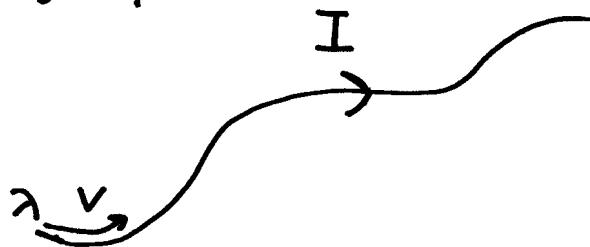
I can write $dL \vec{v} = v d\vec{l} \leftarrow (\text{No } \underline{\text{dots}} \text{ here!})$

$$\text{so } d\vec{F} = n_L g v d\vec{l} \times \vec{B} = I d\vec{l} \times \vec{B}$$

(I is here mag of current, $d\vec{l}$ tells direction)

5-9

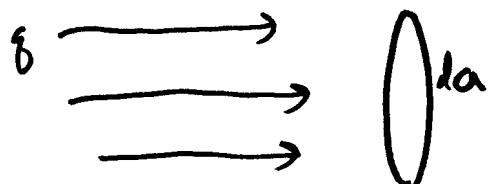
Summary:



$$I = \lambda V \text{ along wire} = n_L g V$$

$$\frac{dF_{\text{chunk}}}{d\vec{s}} = I d\vec{l} \times \vec{B}$$

what if charges move throughout a volume, not just along lines?



We define the current passing our little da as usual,

$$\text{current} = \frac{\text{total charge}}{\text{sec}} = dq/dt$$

If da is tiny, we can think of dI , a tiny current passing through.

$$\text{I will define } \vec{J} = \frac{\text{volume current}}{\text{density}} = \frac{d\vec{I}}{da}$$

(and the direction of \vec{J} will be the direction of $d\vec{I}$)

But to be careful, da is really da_+ , here \rightarrow (see next p)



Given a steady \vec{J}
clearly current through
~~either~~ da or da_{\perp} is same
here, so to uniquely define $\frac{\partial \vec{I}}{\partial a}$

we need to pick da_{\perp} as the area we mean...

$$\text{so } d\vec{I} = \vec{J} da_{\perp} = \underbrace{\vec{J} \cdot d\vec{a}}_{\text{This would also serve to define } \vec{J}!}$$

Just like w. line charges, you can see that in 1 sec,
the total current passing through = total charge in a volume
that extends back (along \vec{J}) by distance $\vec{v} \cdot (1 \text{ sec})$

If we have ρ charges / volume in that region, that means

$\rho [\vec{v} \cdot 1 \text{ sec}] [da_{\perp}]$ charges will pass through, so

$$\frac{d = \vec{v} \cdot 1 \text{ sec}}{da_{\perp}} \Rightarrow da_{\perp} \quad dI = \frac{\rho \vec{v} \cdot 1 \text{ sec} \cdot da_{\perp}}{1 \text{ sec}} = \vec{J} da_{\perp}$$

$$\text{so } \boxed{\vec{J} = \rho \vec{v}}$$

5-11

as before, instead of ρ , we might use

$$\rho = \frac{N}{\text{Volume}} \frac{\text{charge carriers}}{\text{carrier}} \cdot g \frac{\text{coulombs}}{\text{carrier}}$$

so $\vec{J} = Ng \vec{v}$ // Volume current density
↳ number density, per unit volume [Units are $\frac{A}{m^3}$]
= (current passing) area.

so in 3-D situations, for a "chunk" of volume $d\tau$,

$$d\vec{F} = \underline{Nd\tau g} \vec{v} \times \vec{B}$$

this sum of $g \vec{v} \times \vec{B}$ for all $N d\tau$ charges !

Once again, \vec{v} and \vec{J} point in same direction locally,

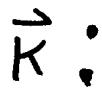
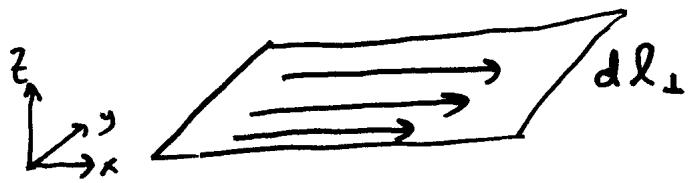
so $Ng \vec{v} = \vec{J}$, and

$$d\vec{F} = (\vec{J} \times \vec{B}) d\tau$$

We jumped from line currents to volume currents.

But many times charges live on surfaces, so we can also define a surface current density

Surface current density \vec{K} :



Here the current passing a line segment dl_{\perp} is defined

$$\text{as } dI = \frac{dQ_{\text{passing}}}{dt} \text{ as usual}$$

$$\text{I will define } \vec{K} = \frac{\text{surface current}}{\text{density}} = \frac{d\vec{I}}{dl_{\perp}}$$

again, direction of \vec{K} = direction of $d\vec{I}$.

(As before, for uniqueness I put dl_{\perp} in the definition)

~~so I could also say~~

This one is slippery! It's a ribbon of current, & K tells how much current passes by a unit length perpendicular to flow!

Just as in prev 2 cases, we can quickly get

$$\vec{K} = \sigma \vec{V} \quad \text{with} \quad \sigma = \frac{\text{coulombs}}{\text{m}^2} = \text{surface charge density}$$

$$(= n_s q \vec{V} \quad \text{with} \quad n_s = \frac{\# \text{ of charge carriers}}{\text{m}^2})$$

Example @ top, could write $\vec{J} = K \delta(z) \hat{x}$. think about this!

Units of \vec{K} = A/m , it's current passing unit length

+ $\vec{F}_{\text{little piece of ribbon}} = (\vec{K} \times \vec{B}) da$, also as before ...

5-13

Conservation of current (+ charge)

Total charge is conserved \leftarrow exptl fact.

which means, if you pick any volume,

total inflow of charge = growth of net charge inside

total outflow of charge = loss of net charge inside.

Since $\vec{J} \cdot d\vec{a} = d\vec{I}$ flowing out through area $d\vec{a}$ (p.10)

total outflow = $\oint \vec{J} \cdot d\vec{a}$ (this is $\frac{\text{coulombs}}{\text{sec}}$, it's rate of loss)

Since $Q_{\text{inside}} = \iiint \rho \cdot d\tau$,

then rate of loss of charge = $-\frac{d}{dt} \iiint \rho d\tau$

so, if ρ decreasing, loss is +.

so $\oint \vec{J} \cdot d\vec{a} = -\iiint -\frac{\partial \rho}{\partial t} d\tau$ \leftarrow Just conservation of charge

↓ Div. theorem

$\iiint (\vec{\nabla} \cdot \vec{J}) d\tau = \iiint -\frac{\partial \rho}{\partial t} d\tau$ \leftarrow True for any volume, remember!

so

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

Continuity equation

5-14 [Summary page.]

Continuity eq'n is basic statement of charge conservation

$$\underbrace{\vec{\nabla} \cdot \vec{J}}_{\text{curl flow of current}} + \frac{\partial \rho}{\partial t} = 0$$

"curl flow of current" + "increase in local charge" must cancel!

and, to summarize

$$\vec{J} = \rho \vec{v} = \frac{\text{volume current density}}{\text{current density}} = \text{Amps passing } a_{\perp}$$

$$\vec{k} = \sigma \vec{v} = \frac{\text{surface current density}}{\text{current density}} = \text{Amps passing } l_{\perp}$$

$$\vec{I} = \lambda \vec{v} = \frac{\text{line current}}{\text{current}} = \text{Amps passing point}$$

$$\text{and } \vec{J} = N_{\text{vol}} g \vec{v}$$

$$\vec{k} = N_{\text{surf}} g \vec{v}$$

$$\vec{I} = N_{\text{linear}} g \vec{v}$$

and, when you need to sum (e.g. finding forces)

$$\iiint \vec{J} d\tau \leftrightarrow \iint \vec{k} da \leftrightarrow \underbrace{\int \vec{I} dl}_{\text{or } I d\vec{l}} \leftrightarrow \underbrace{\sum g_i \vec{v}_i}_{\text{won't use much for now, 'cause }} \downarrow$$

In Magnetostatics, (by definition), charges don't pile up

$$\text{anywhere, } \frac{\partial \rho}{\partial t} = 0 + \boxed{\vec{\nabla} \cdot \vec{J} = 0} \leftarrow \text{STATICS}$$

[Ohm's Law: $\vec{J} \propto \vec{E}$. In conductors, obeying Ohm's law \Rightarrow magnetostatics!]