

In general, inside a dielectric, you may have free charges ρ_f (put there, e.g. on a wire in the plastic, or injected, or rubbed on), and the \vec{E} field from those charges then polarizes the dielectric: adding bound charges to the mix, which superpose & alter the field).

$$\text{So } \rho = \rho_b + \rho_f . \left\{ \begin{array}{l} \text{This } \rho \text{ is real, creates the total,} \\ \text{real } \vec{E} \text{ field.} \\ \uparrow \\ \text{the "placed" ones} \end{array} \right.$$

"response"
of the dielectric

$$\text{So } \vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0 \quad \leftarrow \text{Gauss' law, no exceptions, law of nature}$$

$$= \frac{\rho_b + \rho_f}{\epsilon_0} = - \vec{\nabla} \cdot \vec{P} + \frac{\rho_f}{\epsilon_0}$$

$$\text{so } \vec{\nabla} \cdot \left(\vec{E} + \frac{\vec{P}}{\epsilon_0} \right) = \frac{\rho_f}{\epsilon_0}$$

we Define a field $\boxed{\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}}$

and $\vec{\nabla} \cdot \vec{D} = \rho_F$

This looks like Gauss' law. $\Rightarrow \oint \vec{D} \cdot d\vec{A} = Q_{\text{free, enclosed}}$

(Note units, it's C/m^2 , not units of \vec{E} !)

The "D-field"
or
"Displacement field",
an old word from
Maxwell, (no meaning
now?)

Why \vec{D} ? ρ_F is "externally determined". These are the charges we placed. (The ρ_S is "self determined", it's the response. You don't choose it, + so often don't know it.)

If ρ_F is symmetric, we can use our usual Gauss' law tricks + "read off" $\vec{D}(r)$. If you know \vec{P} , then you can infer \vec{E} at this point.

\vec{D} is a mathematical invention, a tool to help us determine \vec{E} . (often easier to find \vec{D} first) Engineers use it when dealing with fields in media. (We'll see more tricks soon)

Ex: A small charge q is embedded in a rubber ~~sphere~~ (radius R) Find \vec{D} everywhere.

The q creates an \vec{E} field which polarizes the rubber, which in turn modifies the ρ in the sphere, altering \vec{E} in some (as yet) unknown way. So \vec{E} is not easy to find.

But \vec{D} is! $\oint \vec{D} \cdot d\vec{\lambda} = Q_{\text{free, enclosed}}$

This is q , nothing else!!

In the rubber, or out, makes no difference (!!)

$$\oint \vec{D} \cdot d\vec{\lambda} = q \Rightarrow D \cdot 4\pi r^2 = q \leftarrow \begin{array}{l} \text{Now, } \frac{1}{\epsilon_0} \text{ in the} \\ \text{def of } \vec{D} ! \end{array}$$

$$\text{so } \vec{D}(r) = \frac{q}{4\pi r^2} \hat{r}$$

$$\rightarrow \text{Outside, there is no } P, \text{ so } \vec{E} = \frac{\vec{D}}{\epsilon_0} - \vec{P}^0 = \frac{q}{4\pi r^2} \frac{\hat{r}}{\epsilon_0}$$

Just coulomb, the rubber is polarized but neutral, thus has no effect once you're outside.

\rightarrow In the rubber, $\vec{E} = \frac{\vec{D}}{\epsilon_0} - \vec{P}$. If we don't know \vec{P} , we're

still stuck... which will lead us to the next section,

Approximating / Modelling many "Normal" (linear) Dielectrics,

to allow us to figure out \vec{P} , given \vec{E} (or \vec{D} .)

But first, a warning: \vec{D} is not "just like \vec{E} only simpler".

Given P_{free} , you can find \vec{D} if there's nice symmetry.

But if you have complicated boundaries, all bets are off.

P_{free} does not "determine" \vec{D} like P determines \vec{E} ,

because $\vec{D} \times \vec{D}$ is not always zero!

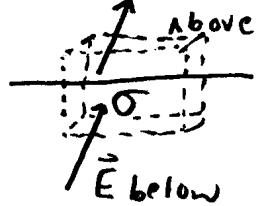
$$\text{It's a subtle point, but } \vec{\nabla} \times \vec{D} = \epsilon_0 \vec{\nabla} \times \vec{E} + \vec{\nabla} \times \vec{P}$$

and $\vec{P} = 0$ in vacuum, so you can easily find situations where $\oint \vec{P} \cdot d\vec{l} \neq 0$. Thus, there is no "potential" for \vec{D} . And, no Coulomb's Law either.

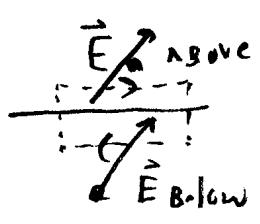
So bottom line: \vec{D} is easy to compute (thus useful) if you have nice symmetry of ρ_f . (Infinite line, spherical, or sheets) But otherwise... \vec{D} may not help us. For this reason, I don't have great physical intuitions about \vec{D} , it's more of a convenient tool to find \vec{E} in certain problems!

One more side comment before we get to the real useful \vec{D} story (the Linear Dielectric story)

We know, from $\oint \vec{E} \cdot d\vec{l} = \frac{P}{\epsilon_0}$, that $E_{\perp}^{\text{above}} \cdot \hat{n} A = \sigma \cdot A / \epsilon_0$



$$E_{\perp}^{\text{above}} - E_{\perp}^{\text{below}} = \sigma / \epsilon_0$$

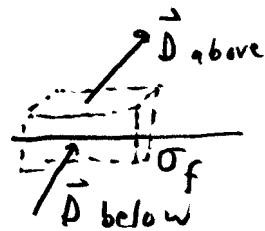
and  $E_{\parallel}^{\text{above}} - E_{\parallel}^{\text{below}} = 0$ from $\oint \vec{E} \cdot d\vec{l} = 0$ on a "skinny loop", as shown

3310 4-15

Those were our usual boundary conditions (always true).

But, when we have dielectrics, we may not know σ (some of it is $\sigma_f \Rightarrow$ known, but some of it is σ_{bound} , not known until we figure out \vec{P} !) So they are true, but we might not be able to use them to figure out \vec{E} .

On the other hand, $\oint \vec{D} \cdot d\vec{\ell} = P_{\text{Free}}$



Says $D_{\perp}^{\text{above}} - D_{\perp}^{\text{below}} = \sigma_F \leftarrow$ Known, most likely

so this might help us deduce D : just like before, in "free space", we sometimes used σ to help us find \vec{E} , like near ∞ sheet, or next to a conductor, ~~or near~~ ...

$\oint \vec{D} \cdot d\vec{\ell} = \oint \vec{P} \cdot d\vec{\ell}$ says $P_{\parallel}^{\text{above}} - P_{\parallel}^{\text{below}} = D_{\parallel}^{\text{above}} - D_{\parallel}^{\text{below}}$

This might help too... but again, only if know \vec{P} ...

So, it's time to finally talk about how to find \vec{P} !

Then, we'll be able to relate ($\vec{D} = \epsilon_0 \vec{E} + \vec{P}$)

\vec{D} , \vec{E} , and \vec{P} in general.

(we'll come back + use these Boundary conditions when solving for Voltage in problems with Dielectric materials!)

A model, an approximate result true for some ordinary substances in ordinary-sized \vec{E} fields:

$$\vec{P} \propto \vec{E}. \quad \text{Seems reasonable! } \vec{E} \text{ will "stretch" dipoles,}$$

(look back e.g. at Griffiths 4.1, where we saw that atoms

$$\text{polarize, and } \vec{P} = \alpha \vec{E}, \text{ so } \vec{P} = N \frac{\text{atoms}}{\text{m}^3} \times \frac{\vec{P}_{\text{atom}}}{\text{atom}}$$

Didn't have to be linear, though

$$= N \alpha \vec{E}.$$

(AND isn't always.)

Careful! \vec{E} here = total field!

$\vec{E}_{\text{external}}$ will polarize material, which superposes onto E_{ext} ,

so \vec{P} is not necessarily given directly from E_{ext} , it's proportional to the total resultant \vec{E} field in the material!

But anyway, $\vec{P} = \epsilon_0 \chi_e \vec{E}$

* For "Linear Dielectrics"
 (Homogeneous, Isotropic)
 the total \vec{E} field

Polarization of Dielectric \rightarrow (Stuck in, to make \vec{E} vanish, a number.)

"Susceptibility" \uparrow

$\chi_e = 0 \Rightarrow$ material doesn't polarize (e.g. vacuum!)

$\chi_e \rightarrow \infty \Rightarrow$ very polarizable (like, conductor-like!)

(This is sort of like "Ohm's Law" in 1120. Practical, approximate, not a deep law of nature but a handy rule, accurate in many situations.)

For such linear materials,

$$\begin{aligned}\vec{P} &= \epsilon_0 \chi_e \vec{E}, \text{ so } \vec{D} = \epsilon_0 \vec{E} + \vec{P} \\ &= \epsilon_0 \vec{E} + \epsilon_0 \chi_e \vec{E} \\ &= \epsilon_0 (1 + \chi_e) \vec{E}\end{aligned}$$

which means

$$\vec{D} = \epsilon \vec{E} \quad \text{with } \epsilon \equiv \epsilon_0 (1 + \chi_e)$$



"permittivity
of the dielectric"

[Sorry for all the (equivalent!) notations, χ_e , ϵ , and ϵ_r !]

$$= \epsilon_0 \epsilon_r$$



"Dielectric constant"

unless! Typically [1-2 in solids
1.000... in gases.]

[Easy to measure, Known
for most materials]

~~Realize~~ Realize the usefulness now:

If $\vec{D} \cdot d\vec{\lambda} = Q_{\text{free,enc}}$ so, often, you can easily find \vec{D} ,

$$\text{but then } \vec{E} = \frac{\vec{D}}{\epsilon} \leftarrow \text{a simple constant,}$$

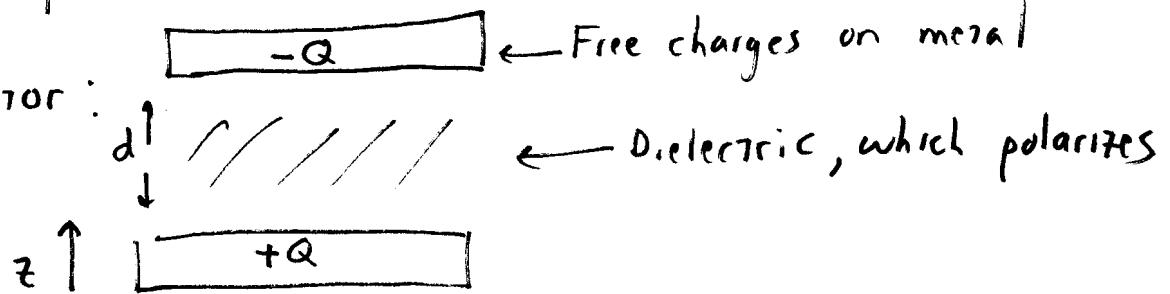
so if you know any one (\vec{D} , \vec{E} , or \vec{P}) the other two are trivial to find if you know the dielectric of the medium

$$\vec{D} = \epsilon \vec{E} \quad (= \epsilon_0 \epsilon_r \vec{E}) (= \epsilon_0 (1 + \chi_e) \vec{E}) \quad \left. \right\}$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E} = \frac{\chi_e}{\epsilon_r} \vec{D} = \left(\frac{\chi_e}{1 + \chi_e} \vec{D} \right) \quad \left. \right\}$$

Examples of uses:

① Capacitor:



In the Dielectric, what's \vec{E}_{tot} ? It's due to the

Capacitor (free) charge (which gives $\vec{E}_{\text{ext}} = \frac{\sigma}{\epsilon_0} \hat{z}$ here) but also it polarizes (!) adding in some bound charges which in turn add to this (superpose on this) field!

Note that $\vec{P} \neq \epsilon_0 \chi_e \vec{E}_{\text{ext}}$, it's $\vec{P} = \epsilon_0 \chi_e \vec{E}_{\text{tot}}$,

and we don't know \vec{E}_{tot} yet!

But check this out:

$$\iint \vec{D} \cdot d\vec{A} = Q_{\text{free}} \underset{\text{enclosed}}{\Rightarrow} \frac{+Q}{D \cdot A} = \sigma \cdot A \Rightarrow \underline{\underline{D = \sigma \epsilon_0 \hat{z}}} \text{ in there!}$$

So $\vec{D} = \sigma / \epsilon_0 \hat{z}$ throughout the Dielectric.

Like I said, if you know Q_{free} , + have symmetry,

\vec{D} is quick-n-easy with Gauss' law.

~~QUESTION~~ But now $\vec{E}_{\text{tot}} = \vec{D}/\epsilon_0 \epsilon_r$ (always true in Dielectrics)

So $\vec{E}_{\text{tot}} = \frac{Q}{\lambda \epsilon_0 \epsilon_r} \hat{z}$. (It's $\frac{\vec{E}_{\text{ext}}}{\epsilon_r}$ in this case!)

The dielectric weakened the \vec{E} field, by $\frac{1}{\epsilon_r}$. This is common!

Why? It polarized!

$$\begin{aligned} & \begin{array}{c} \boxed{-Q/\lambda \text{ or } -\sigma} \\ \vdots \\ \boxed{+\sigma_B} \\ \vdots \\ \boxed{-\sigma_A} \\ \vdots \\ \boxed{+Q/\lambda \text{ or } +\sigma} \end{array} \quad \vec{P} \uparrow \vec{E}_{\text{tot}} = \epsilon_0 \chi_e \vec{E}_{\text{tot}} \\ & = \epsilon_0 \chi_e \frac{\sigma}{\epsilon_0 \epsilon_r} \hat{z} \\ & = \frac{\chi_e}{1+\chi_e} \sigma \hat{z} \end{aligned}$$

$$\sigma_B = \vec{P} \cdot \hat{n} = + \frac{\chi_e}{1+\chi_e} \sigma \text{ at top + bottom.}$$

This partly cancels the $\mp \sigma$ on capac!

$$\text{Effectively, } \sigma_{\text{tot}} = \sigma - \sigma_B = \sigma \left(1 - \frac{\chi_e}{1+\chi_e}\right) = \frac{\sigma}{1+\chi_e} = \frac{\sigma}{\epsilon_r} !$$

So \vec{E}_{tot} resulting from σ_{tot} is suppressed by $\frac{1}{\epsilon_r}$.

$$\text{Now } |\Delta V| = \left| \int \vec{E} \cdot d\vec{l} \right| = \frac{\sigma}{\epsilon_0 \epsilon_r} \cdot d$$

This is $\frac{1}{\epsilon_r} \times$ what we get w/o Dielectric

$$\text{So } C = Q/\Delta V \text{ is } \epsilon_r \times \underline{\text{bigger}} .$$

← Faraday
first studied this!

\Rightarrow Capacitors always have dielectrics in them...

\rightarrow Bigger capacitance

\rightarrow weaker \vec{E} for given $V \Rightarrow$ less likely to break down

\rightarrow Can get $\epsilon_r = 10,000$ for some materials! (Barium Titanate)

\rightarrow Stored energy = $\frac{1}{2} C \Delta V^2$ is thus also bigger by $\epsilon_r \rightarrow$ (nice!)

Going back to my example on p. 13, q in rubber ball.

we found $\vec{D} = \frac{q}{4\pi r^2} \hat{r}$ in rubber. If rubber is "linear",

$$\text{then } \vec{E}_{\text{in}} = \frac{\vec{D}}{\epsilon} = \frac{q}{4\pi \epsilon_0 r^2} \hat{r} \cdot \left(\frac{1}{\epsilon_r} \right) \quad \begin{array}{l} \text{"Screening"} \\ \text{but not} \\ \text{"Shielding"} \end{array}$$

again, just like normal, but down by factor ϵ_r . (↑
like conductors)

Here again, we had nice symmetry \Rightarrow could find \vec{D} .

(Remember our warning: without symmetry, don't know \vec{D} ,
and can't assume $\vec{E} = \text{"normal } \vec{E} \text{"/} \epsilon_r \dots$)

What about finding V (+ thus \vec{E}) the old ch. 3 way?

In linear dielectric, $\rho_B = -\vec{\nabla} \cdot \vec{P} \propto -\vec{\nabla} \cdot \vec{D} = \rho_{\text{free}}$

so, if don't have ρ_f (no "embedded Q's"), then

$\rho_B = \rho_f = 0$, $\nabla^2 V = 0$, + we can use old tricks!