

Magnetostatics

New topic. Until 1800's, different distinct force of ~~nature~~ ^{nature!}

Lodestones attract + repel. Is it electric? No.

→ Think of how you convince yourself Kitchen magnets are not electrical in nature!

→ Magnets don't attract or repel charges!

→ Magnets don't fade away

→ Charged rods never repel magnets

Compass needles are simple indicators of presence (+ direction) + even strength of magnetic field. (1600 Gilbert: Earth is a big lodestone 90 yrs before Newton realizes earth gravitates!)

1820 Oersted observed that currents

produces magnetic effects on compasses.

$$\vec{F} (= m \vec{a}) = q \vec{E} + q \vec{v} \times \vec{B}$$

↑
on q

Spent 8 weeks
learning about
this!

Magnetic field

Purely from
exp'.
"Lorentz
force Law"

$$\text{Units: } 1 \text{ Tesla} = \frac{1 \text{ N}}{\text{C} \cdot \text{m/s}}$$

$$\text{Big!} = 10^4 \text{ gauss}$$

(Earth field $\approx \frac{1}{2}$ gauss)

Order of investigation:

1) General features; sources of \vec{B} (still static/steady, no time dep. yet!)

2) Consequences of $\vec{F} = q\vec{v} \times \vec{B}$

3) Current = source of \vec{B} fields
 — Current
 — Surface current
 — Volume current

4) Biot-Savart: "Like Coulomb's law for magnetism"

(How current creates \vec{B}).

5) Ampere's Law: "Like Gauss' Law: How currents $\Rightarrow \vec{B}$ fields"

1) \vec{B} fields are both very different + also very similar/analogous to \vec{E} . Look for parallels + connections, some are very deep!

Statics:

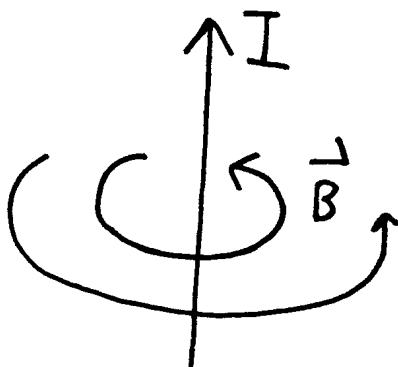
$$\nabla \cdot \vec{E} = \rho/\epsilon_0 \quad \leftarrow \rho \text{ is "source of } \vec{E} \text{" which diverges}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} \quad \leftarrow \vec{J} = \text{"current is "source of } \vec{B} \text{" whichcurls.}$$

$$4\pi \cdot 10^{-7} \text{ N/A}^2 \text{ with } 1 \text{ A} = 1 \text{ C/sec.}$$

(Exact! Defines the Amp \Rightarrow the Coulomb!)

$$\nabla \cdot \vec{B} = 0 \quad \leftarrow \text{No magnetic monopoles}$$



5-3

$$2) \text{ Since } \vec{F} = q \vec{v} \times \vec{B}, \quad \vec{F} \cdot d\vec{l} = q (\vec{v} \times \vec{B}) \cdot (\vec{v} dt) = 0.$$

$\uparrow \perp \text{ to } \vec{v}$!

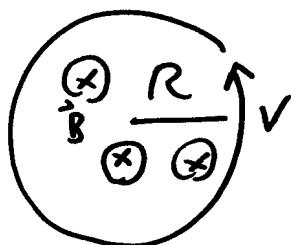
\vec{B} fields do not work. (They "bend" trajectories) (we'll return to this)

Cyclotrons : $\vec{F} = \frac{d\vec{p}}{dt} = q \vec{v} \times \vec{B}$. Suppose $\vec{B} = B_0 \hat{z}$

- So $F_z = 0$, any motion in z -direction is unaffected, "drifts" (so let's set $v_z = 0$.)
- No work $\Rightarrow |\vec{p}|$ won't change. Force is \perp to motion, constant speed \Rightarrow uniform circular motion.

$$\vec{F} = m \vec{a} \Rightarrow q v B = m v^2 / R$$

Circular motion with



$$R = \frac{mv}{qB} = \frac{|\vec{p}|}{qB}$$

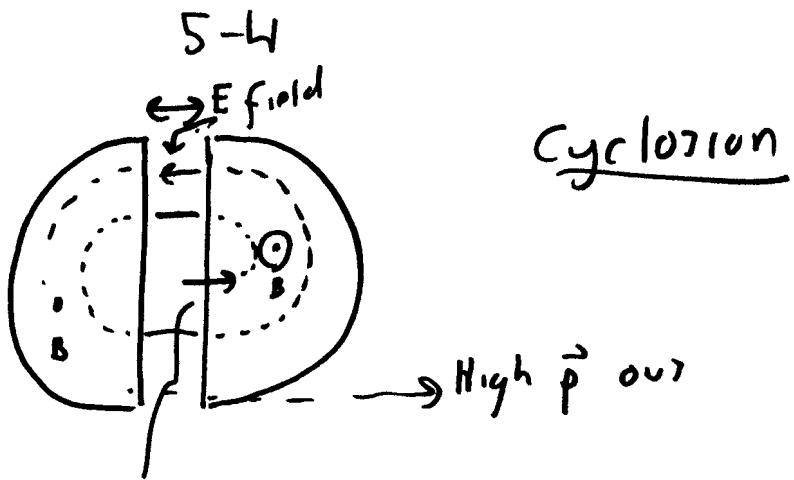
Frequency = $\frac{\text{cycles}}{\text{sec}} = \frac{1}{(\text{sec/revolution})}$ But $v \cdot T = 2\pi R \Rightarrow T = 2\pi R / v$

so $f_{\text{cyclotron}} = \frac{v}{2\pi R} = \frac{qBR/m}{2\pi R} = \frac{qB}{2\pi m}$

Indep. of radius or initial speed!

If $v_z \neq 0$, superpose "z drift" \Rightarrow helix





inject low \vec{p} electron, fixed time $T_{1/2} = \frac{1}{2f} = \frac{qB}{4\pi m}$ to go

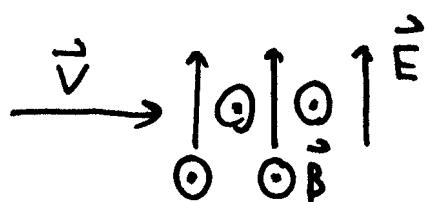
"half circle", then turn on \vec{E} to accel across gap (capac!)

Capac switches sign every $\frac{qB}{4\pi m}$ sec, (steady freq) \Rightarrow
electrons accel each time in \vec{E} field,
can build up KE, ~~&~~ \vec{B} field "contains" the electrons.

Lots of uses, particle accelerators
nowadays, medical beams!

works up to relativistic energies, then need other tricks

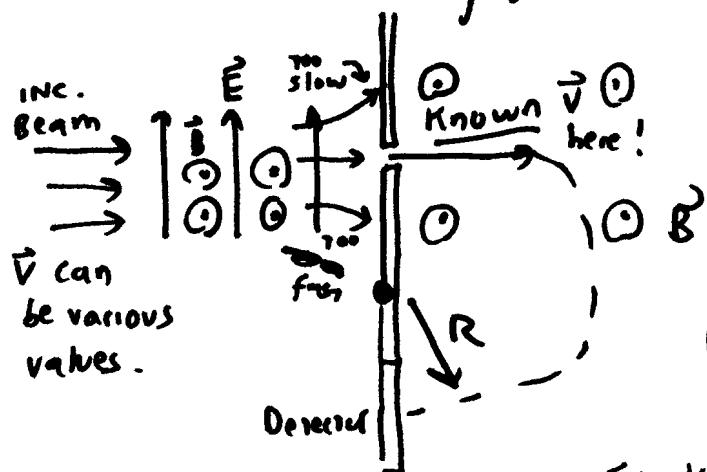
E.g. Fermilab has fixed R , increases \vec{B} as speed up,
gets protons up to $\text{TeV} = 10^{12} \text{ eV}$
($R \approx 2 \text{ Km}$)

Mass spectrometer

velocity selector. ~~$\vec{F} = qE$~~ up
 qvB down

If $qE = qvB$, i.e. $v = E/B$, then

make it through, in straight line...



If know q (usually e , or $-e$)
Veloc. selector tells you v

$$R = \frac{|p|}{qB} \text{ tells you } m\vec{v},$$

so know $m \Rightarrow \underline{\text{identify atoms}}$.

what if release θ from rest in the "velocity selector".

Something different!

$$\begin{aligned} \vec{E} &= E\hat{z} & \Rightarrow F_x = 0 \text{ so } v_x = 0 \\ \vec{B} &= B\hat{x} & \Rightarrow \vec{v} = (0, v_y, v_z) \end{aligned}$$

$$\vec{F} = m(0, \dot{v}_y, \dot{v}_z) = q(0, 0, E) + q\vec{v} \times \vec{B}$$

$$= (0, qv_z B, qE - qv_y B)$$

$$\begin{aligned} & v_y \hat{y} \times B \hat{x} + v_z \hat{z} \times B \hat{x} \\ & = v_y B (-\hat{z}) + v_z B \hat{y} \end{aligned}$$

$$so m\dot{v}_y = qv_z B \Rightarrow m\ddot{v}_y = qB\dot{v}_z \quad \text{Plug this into next one:}$$

$$m\dot{v}_z = qE - qv_y B \Rightarrow m(\dot{v}_y / qB) = qE - qBv_y$$

5-6

$$\text{thus, } \ddot{v}_y = \frac{q^2 B}{m^2} (E - BV_y)$$

$$\text{this is of the form } \ddot{x} = a - bx \quad (a = \frac{q^2 B}{m^2} E, b = \frac{q^2 B^2}{m^2})$$

The general sol'n of this ↑ is

$$x = C_1 \overset{\sin}{\cancel{\cos}}(\sqrt{b}t) + C_2 \overset{\cos}{\cancel{\sin}}(\sqrt{b}t) + a/b$$

(Proof: check it! ∵ it has 2 undetermined coeffs...)

But here $x = V_y = dy/dt$, so integrating gives

$$y = C_1 \overset{\cos}{\cancel{\sin}}(\sqrt{b}t) + C_2 \sin(\sqrt{b}t) + \frac{a}{b}t + C_3$$

with $\sqrt{b} = qB/m$ and $\frac{a}{b} = E/B$, see above!

then $\dot{V}_z = \frac{qE}{m} - \frac{qB}{m} V_y$ can be integrated, to get

~~$$V_z = \frac{qE}{m} t - \frac{qB}{m} \left(\frac{C_1}{\cancel{b}} \cos \sqrt{b}t + C_2 (\sqrt{b}t + \frac{E}{b}t) + C_3 \right)$$~~

Easier!
See prev page → $V_z = \frac{m \dot{V}_y}{qB} = \frac{m}{qB} \left(-\frac{q^2 B^2}{m^2} (C_1 \cos \sqrt{b}t + C_2 \sin \sqrt{b}t) \right)$

$$= -\frac{qB}{m} (C_1 \cos \sqrt{b}t + C_2 \sin \sqrt{b}t)$$

$$\text{so } z = \int V_z dt = -C_1 \sin \sqrt{b}t + C_2 \cos \sqrt{b}t + C_4$$

(See Griffiths for interp!)