

## Ch. 7 - 1

Finishing up rem - Time dependence

Full - Maxwell's Eq's

$\Rightarrow$  ~~Faraday's~~ Faraday's Law

+ Maxwell's addition to Ampere

so far:

1)  $\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$   $\leftarrow$  Gauss

2)  $\vec{\nabla} \times \vec{E} = 0$   $\leftarrow$  Voltage is well defined

3)  $\vec{\nabla} \cdot \vec{B} = 0$   $\leftarrow$  No monopoles

4)  $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$   $\leftarrow$  Ampere.

Experimental observations by Michael Faraday (1830's)

Showed #2 needs fixing if have time-dependent fields

Faraday:  $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$  check units for yourself,  
it works!

or by Stoke's:

$$\oint \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \iint \vec{B} \cdot d\vec{A} = - \frac{d}{dt} \Phi_B^{\text{mag flux}}$$

we call  $\oint \vec{E} \cdot d\ell$  the EMF. Note that early in this term,

$\int_A^B \vec{E} \cdot d\ell = -\Delta V = -(V(B) - V(A))$ , so EMF is "like" voltage around a loop. But if  $\frac{d\Phi_B}{dt} \neq 0$ , then no longer has to be zero,  $E$  fields can do net work around a loop!

The "-" sign in Faraday's Law is "Lenz law",  
the induced EMF opposes the change in magnetic flux.

If that EMF is "allowed" to push charges, the resulting current would oppose the change in  $\Phi_B$ .

We'll do some examples, if time, but first let's round out Maxwell's eqns.

Look @ M.E. so far, and take  $\nabla \cdot (\nabla \times B)$

$$\nabla \cdot (\nabla \times \vec{B}) = \vec{\nabla} \cdot (\mu_0 \vec{J}).$$

This does not have to be zero! You can easily build loops with  $\vec{\nabla} \cdot \vec{J} \neq 0$ . Of course, we showed earlier

$$\vec{\nabla} \cdot \vec{J} = -\partial \rho / \partial t, \quad \text{conservation of charge!}$$

7-3

so  $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = 0$  in magnetostatics ( $\frac{\partial \rho}{\partial t} = 0$ )

But in general,  $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = \mu_0 \vec{\nabla} \cdot \vec{J} \neq 0$ .

But yikes!  $\vec{\nabla} \cdot (\vec{\nabla} \times \underline{\text{anything}}) = 0$  is a math identity!

so, Maxwell's eqns are not consistent if  $\frac{\partial \rho}{\partial t} \neq 0$ !

Maxwell fixed this: Noting  $\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial t} (\epsilon_0 \vec{\nabla} \cdot \vec{E})$

New Ampere's Law

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

Just what you need to ensure  
 $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = 0$ !

New term.

reminds me of Faraday's  $\vec{\nabla} \times \vec{E} = -\partial \vec{B} / \partial t$ !

Change  $\vec{B}$  makes (circulating)  $\vec{E}$  ↗

Maxwell's fixup: changing  $\vec{E}$  makes (circulating)  $\vec{B}$

It's like a current, but with no charge!

Sometimes called "displacement current".

$$J_D = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

7-4

so here is  $\epsilon + \mu$ , full classical field theory,

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

plus  $\vec{F} = q (\vec{E} + \vec{v} \times \vec{B})$

- charges create fields

- fields affect charges.

In media:

$$\vec{\nabla} \cdot \vec{D} = \rho_{\text{free}}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_{\text{free}} + \frac{\partial \vec{D}}{\partial t}$$

← you can check this one for yourself, it's the "new" one...

## 7-5

Where will you go from here? This is all we know about classical  $\vec{E}$  +  $\vec{B}$  fields, the applications + consequences are everything electromagnetic! Toasters, microwaves, computers, ...

- + As you'll see, even light comes directly from these eqns.
- + And, they are consistent with (and even lead to) special relativity!
- Next term, you're likely to learn

Energy + Energy flow : (Like  $\frac{\text{energy}}{\text{Vol}} = \frac{1}{2} \epsilon E^2$ ,

it's also  $\frac{1}{2\mu_0} B^2$ !)

EM waves + light.

Both in vacuum + in matter!

Potentials ( $V + \vec{A}$ ) when there's time dependence!

(+ from moving point charges instead of wires)

EM radiation (waves produced by  $\uparrow$ )

& Relativity. Good stuff!!