

PHYS3310 Fall 08, HW 8 Due in class Wed Oct 22**Q1. BOUND CHARGES I**

Consider a long insulating rod, (a dielectric cylinder), radius a . Suppose that the rod has no free charge but has a *permanent* polarization $\mathbf{P}(s,\theta,z) = C \mathbf{s}$ ($= C s \hat{\mathbf{s}}$), where \mathbf{s} is the usual cylindrical radial vector from the z -axis, and C is a positive constant). Neglect end effects: the cylinder is long. *Note that this rod is NOT a linear dielectric. Only very special materials can have a permanent polarization. Barium titanate, BaTiO_3 , is one such material. A rod with a permanent polarization is called an "electret".*

A) Calculate the bound charges σ_b and ρ_b (on the surface, and interior of the rod respectively). What are the units of C ? Sketch the charge distribution of the rod.

B) Next, use these bound charges (along with Gauss' law) to find the electric field inside and outside the cylinder. (Direction and magnitude)

C) Find the electric displacement field \mathbf{D} inside and outside the cylinder, and verify that "Gauss's Law for \mathbf{D} " (Eqn 4.23, p. 176) works.

Q2. BOUND CHARGES II

Consider now a hollow insulating rod, with inner radius a and outer radius b .

Suppose now that the rod has a different permanent polarization, namely $\mathbf{P}(s,\theta,z) = \frac{C}{s^2} \hat{\mathbf{s}}$ for

$a < s < b$, $C = \text{positive constant}$ (not the same constant as in Q1). *Again, this is not an ordinary linear dielectric. It has a permanent polarization.*

A) We have vacuum for $s < a$ and $s > b$. What does that tell you about \mathbf{P} in those regions? Find the bound charges σ_b and ρ_b (σ_b on the inner AND outer surfaces of the hollow rod, and ρ_b everywhere else. Use these bound charges, along with Gauss' law, to find the electric field everywhere in space. (Direction and magnitude)

B) Use Griffiths' Eq 4.23 (p. 176) to find \mathbf{D} everywhere in space. (This should be quick - are there any *free* charges in this problem?) Use this (simple) result for \mathbf{D} (along with Griffiths basic definition/relation of \mathbf{E} to \mathbf{D} , Eq 4.21) to find \mathbf{E} everywhere in space. *(This should serve as a check for part a, and shows why sometimes thinking about \mathbf{D} fields is easier and faster!)*

Q3. MATH OF BOUND CHARGES

Using Griffiths' formal definition of bound charge distributions, eqns 4.11 and 4.12 (p. 167-8) prove (mathematically) that if you have a dielectric which starts off neutral, and then it gets polarized, that the total bound charge is still exactly zero. Explain how this formal mathematical result makes (simple) physical sense. Look back at all previous questions, does the total charge in those problems satisfy what you found here? *(This problem need not be too long!)*

Q4. LINEAR DIELECTRIC, RECTANGULAR BOUNDARY

You have a large parallel-plate conductor (two big metal sheets, the upper one has free charge density $+\sigma$, the lower one has $-\sigma$) They are a distance d apart. The space between the sheets is half filled with a (linear) dielectric oil.

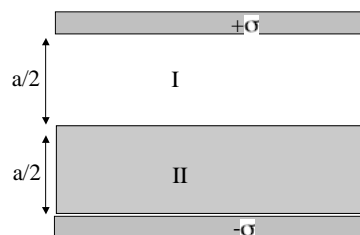
Region I, (filling the top half of the volume) is vacuum.

The lower half, region II, has dielectric constant $\epsilon_r = 3.0$

A) Find the electric displacement ("**D** field") (direction and magnitude) in regions I and II, the electric field **E** in both regions, and the polarization **P** in both regions

B) Find the location and amount of bound charge (surface and volume) everywhere. Given this, go back and compute the E field in the two regions to check your answer for **E** in part a.

C) Find the voltage between the plates. If the plates are large but not infinite (area A), compare the capacitance before and after you added the dielectric oil. How does it compare with what you would get if you filled the *entire* region with that dielectric oil?

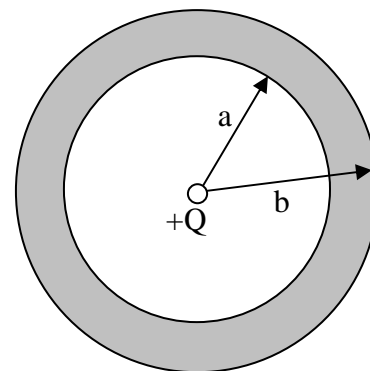
**Q5. LINEAR DIELECTRIC SPHERE I.**

A point charge $+Q$ is at the center of a spherical shell of inner radius a and outer radius b . The shell is made of teflon, which is a linear dielectric material with a dielectric constant $\epsilon_r = 1 + \chi_e = 2.1$. The shell is electrically neutral (it has no free charges).

A) Compute the fields **E**, **D**, and **P** everywhere. Give answers in terms of ϵ_r or χ_e (Don't give numerical answers. We want symbolic answers only.)

B) How is the E-field inside the teflon ($a < r < b$) different from what it would have been if the teflon were not present?

C) Make a sketch showing the E-field everywhere, both inside and outside the teflon. Make another sketch showing the D-field everywhere, both inside and outside the teflon.

**Q6. LINEAR DIELECTRIC SPHERE II**

Do Griffiths problem 4.20, on page 185.

Does your answer come out *larger* or *smaller* than what you would have found for a simple sphere of charge with uniform free charge density ρ (i.e, if you had neglected the effect of the dielectric constant? In this limit, what is ϵ_r ?) Does this result make good physical sense to you? What do you get in the limit of *infinite* dielectric constant? What *physical situation* does that limit remind you of?

Q7. FORCE DUE TO INDUCED POLARIZATION

A) Consider a very long (effectively infinite) half-cylinder of radius r and uniform surface charge density σ . What is the magnitude and direction of the electric field at the center of the cylinder?

B) Consider now a thin, long straight wire with uniform positive charge per length λ at the center of a half-cylinder of inner radius a and outer radius b . The half-cylinder is made of a linear dielectric with susceptibility χ_e . You may assume $\chi_e \ll 1$, that is, this is a weakly polarizable linear dielectric. Compute the bound surface charge densities, σ_a and σ_b , on the inner and outer surfaces and the bound charge density ρ in the interior of the dielectric.

C) Is there a net force per length on the wire due to the presence of the dielectric? If so, compute the force (magnitude and direction). If not, prove that the force is zero.

D) In this situation, can you use $\oint \vec{D} \cdot d\vec{a} = Q_{\text{free, enclosed}}$ to compute \vec{D} everywhere? If you can, do it. If you cannot, explain why not.

