PHYS3310 HW11, Due start of class Wed April 15

Q0. AMPERE'S LAW - III.

The magnetic field in a region of space centered on the origin has cylindrical symmetry and is given by $\vec{B} = B_0 \hat{\varphi}$ where B_0 is a constant, where $\hat{\varphi}$ is the azimuthal direction in cylindrical coordinates.

A) What is the current density in this region of space?

B) Suppose the current density that you found in part (A) extends out to a radius R and is zero for r > R. What is the magnetic field for r > R?

Q1. VECTOR POTENTIAL I

A) A very long wire is a cylinder of radius R centered on the z-axis. It carries a uniformly distributed current I_0 in the +z direction. Assuming $\nabla \cdot \vec{\mathbf{A}} = 0$ (the Coulomb gauge), and choosing A=0 at the edge of the wire, show that the vector potential *inside the wire* could be given by $A(s) = c\,I_0\left(1 - \frac{s^2}{R^2}\right)$. Find the constant c (including units.)

What is the vector direction of **A**? (Does it make sense in any way to you?) Is your answer unique, or is there any remaining ambiguity in **A**? *(Note that I'm not asking you to derive **A**. Just check that this choice of **A** works.)*

B) What is the vector potential *outside* that wire? (Make sure that it still satisfies $\nabla \cdot \vec{\mathbf{A}} = 0$, and make sure that **A** is continuous at the edge of the wire, consistent with part (a).)

Is your answer unique, or is there any remaining ambiguity in **A(**outside)?

C) Plot or sketch A(s) vs. s, showing the behavior of A both inside and outside the wire.

Q2. VECTOR POTENTIAL II

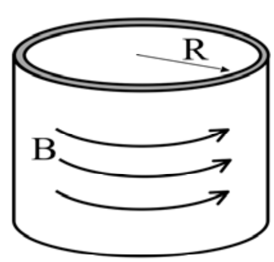
A) Griffiths Fig 5.48 (p. 240) is a nice, and handy, "triangle" summarizing the mathematical connections between **J**, **A**, and **B** (like Fig. 2.35 on p. 87) But there's a missing link, he has nothing for the left arrow from **B** to **A**. Notice that the *equations* defining **A** are really very analogous to the basic Maxwell's equations for **B**:

$$\nabla \cdot \vec{\mathbf{B}} = 0 \qquad \Leftrightarrow \qquad \nabla \cdot \vec{\mathbf{A}} = 0$$

$$\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}} \qquad \Leftrightarrow \qquad \nabla \times \vec{\mathbf{A}} = \vec{\mathbf{B}}$$

So **A** depends on **B** in the same way (mathematically) the **B** depends on **J**. (Think Biot-Savart.) Use this idea to just write down a formula for **A** in terms of **B** to finish off that triangle.

B) We know the B-field everywhere inside and outside an infinite solenoid (which can be thought of as either a solenoid with current per length n I or a cylinder with surface current density K = n I). Use the basic idea from part (a) to quickly and easily write down the vector potential **A** in a situation where **B** looks analogous to that, i.e. $\vec{\mathbf{B}} = C\delta(s-R)\,\hat{\varphi}$, with C constant. (Sketch this **A** for us, please) *(You should be able to just see the answer; no nasty integral needed.) It's kind of cool - think about what's going on here. You have a previously solved problem, where a given **J** led us to some **B**. Now we immediately know what **A** is in a* very different *physical situation, one where **B** happens to look like **J** did in that previous problem.*



To discuss: What physical situation creates such a B field? (This is a little tricky, think carefully.)

Q3. MULTIPOLES - DIPOLE MOMENT

The vector potential of a small current loop (a magnetic dipole) with magnetic moment $\vec{m}$ is $\vec{\mathbf{A}} = \frac{\mu_0}{4\pi}\frac{\vec{\mathbf{m}}\times\hat{\mathbf{r}}}{r^2}$

A) Assume that the magnetic dipole is at the origin and the magnetic moment is aligned with the +z axis. Use the vector potential to compute the B-field in spherical coordinates.

B) Show that your expression for the B-field in part (a) can be written in the coordinate-free form $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right]$

Note: The easiest way to do this one is by assuming the coordinate-free form and then showing *that you get your expression in part (a) if you define your z axis to lie along* ***m****, rather than trying to go the other way around. Coordinate free formulas are nice, because now you can find B for more general situations.*

Q4. FIELDS AND STRENGTHS

A) Find the density ρ of mobile charges in a piece of speaker wire made of gold, assuming each atom contributes one conduction electron. (Look up any necessary physical constants.) Think about the definition of current and then *estimate* the average electron speed in a gold speaker wire carrying an ordinary current. *Your answer will come out quite slow.* If you flip on the stereo, and the speakers are, say, 2 meters away, would there be a noticeable "time lag" before you hear the speaker come on? Why/why not?

B) If you cut open this wire, you'll see that it is really two wires, each insulated, and wrapped close together in a single plastic cylinder (since you need a complete circuit, current has to flow TO and FROM the speaker, right?). Make reasonable guesses for the dimensions involved and estimate the size of the magnetic field in the space between the wires. How large a field to you expect at a distance of 1 meter from the pair of wires?

Q5. MORE MAGNETIC DIPOLES

Do Griffiths Problem 5.56.

Q6. SQUARE LOOP - FAR AWAY

A) In a previous homework, we considered a square current loop (current I running around a wire bent in the shape of a square of side a) sitting flat in the x-y plane, centered at the origin. (You found B at the center). Now redo that problem but find **B**(0,0,z) (magnitude and direction), i.e. find **B** a distance z above the center. (Check yourself by setting z=0 and making sure you get the correct result.)

B) Take your result from part (a) and now take the limit z >> a , finding an approximate simple formula for B_z(0,0, large z). Do the same for a circular current loop of radius a, and compare. (The exact expression is derived in Griffiths, Eq. 5.38, you don't have to re-derive that, just consider the large-z limit) Check both answers by comparing with Griffiths' dipole approximation (Eq. 5.86), looking only along the +z-axis of course.

Notice how simple everything gets far away, and how your two expressions for very different wire shapes differ only by the constant out front, which should go like $m = I \times (\text{area of loop})$

This is the magnetic dipole moment.