Phys 3310, HW #6, Due in class Wed February 25th.

Q1. UNIQUENESS THEOREM

In Griffiths section 3.1.5 is a proof of the uniqueness theorem for solutions to Laplace's equation given *V* on the boundary of some volume in space. For the homework, prove that the electric field (not the potential) is uniquely determined in a volume with given charge density $\rho(\vec{r})$ if the derivative of *V* normal to the surface at the boundary, $\partial V / \partial n \equiv \hat{n} \cdot \nabla V$, is given. As usual, start by assuming two solutions V_1 and V_2. You will probably find it useful to use Green's identity, which is stated in Griffiths' problem 1.60 (c) on p. 56 (Hint: you can use the identity in the case that *T*=*U*). *The first type of boundary condition, on the *value* of V, is called the Dirichlet type and the second type of boundary condition, on the *derivative* of V, is called the Neumann type. These names are often used as a shorthand way to indicate which type of boundary condition is given.*

Q2. METHOD OF IMAGES - spherical

Take a look at Griffiths' Fig 3.12, which shows a grounded metal sphere with a charge *q* outside it. He argues (leading up to Eq. 3.17) that there is a simple "method of images" trick available here - you just have to put the right charge (*q'*) at the right spot (*b*, inside the radius of the sphere). Your task:

A) Solve Griffiths' problem 3.7a (p. 126)

(which shows WHY this particular "image trick" works for a spherical conductor)

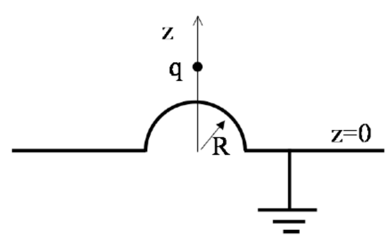
B) Solve Griffiths' problem 3.7b

C) Now let's apply this result to a novel situation:

Imagine a grounded infinite conducting plane in the x-y plane, that has a (conducting) hemispherical bump (radius *R*) in it, centered at the origin, as shown.

A charge *q* sits a distance "*a*" above the plane, *i.e.*, at the point (0,0,a)

I claim that you can find the voltage *V* anywhere in the plane above the conductor using the method of images, with *three* image charges.



Where should they be? (Explain your reasoning - you need to ensure the boundary condition *V*=0 on the entire conductor.) Is it now easy for you to construct a formula for *V* at any point above the plane?

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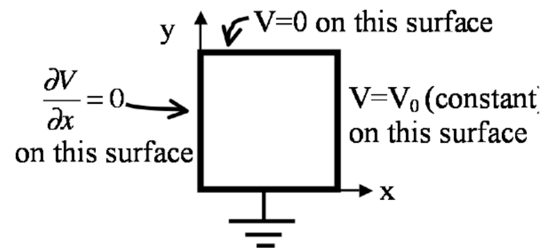
Q3. SEPARATION OF VARIABLES - CARTESIAN 2-D

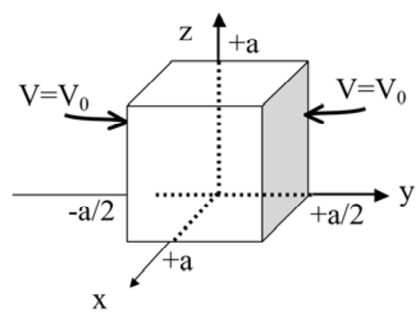
A square rectangular pipe (sides of length *a*) runs parallel to the *z*-axis (from -∞ to +∞)
The 4 sides are maintained with boundary conditions given in the figure. (Each of the 4 sides is insulated from the others at the corners)



A) Find the potential $V(x,y,z)$ at all points in this pipe.
B) Sketch the E-field lines and equipotential contours inside the pipe. (Also, state in words what the boundary condition on the left wall means - what does it tell you? Is the left wall a conductor?)
C) Find the charge density $\sigma(x,y=0,z)$ everywhere on the bottom conducting wall ($y=0$).
D) Extra credit! – Use Mathematica or a similar program to make a 3D plot of the voltage $V(x,y,z)$ at a given z (*i.e.*, plot V as a function of x and y). If you have a series solution, state how many terms you kept in the series.

Q4. SEPARATION OF VARIABLES - CARTESIAN 3-D

You have a cubical box (sides all of length *a*) made of 6 metal plates, which are insulated from each other.
The left wall is located at $y = -a/2$,the right wall is at $y = +a/2$.
Both left and right walls are held at constant potential $V=V_0$.
All four other walls are grounded.
(Note that I've set up the geometry so the cube runs from $x = 0$ to $x = a$, and from $z = 0$ to $z = a$, but from $y = -a/2$ to $y = +a/2$
This should actually make the math work out a little easier!)



Find the potential $V(x,y,z)$ everywhere inside the box.
(Also, is V=0 at the center of this cube? Is E=0 there? Why, or why not?)

Q5. SEPARATION OF VARIABLES - SPHERICAL

The potential on the surface of a sphere (radius R) is given by $V=V_0\cos(2\theta)$.
(Assume $V(r=\infty)=0$, as usual. Also, assume there is no charge inside or outside, it's ALL on the surface!)
i) Find the potential inside and outside this sphere.
(Hint: Can you express $\cos(2\theta)$ as a simple linear combination of some Legendre polynomials?)
ii) Find the charge density $\sigma(\theta)$ on the sphere.