

**PHYS3310 HW9, Due start of class Wed April 1****Q1. ITERATING FIELDS**

Griffiths solved the problem of a dielectric sphere in a uniform external field in Example 4.7, p.186, using Separation of Variables and applying boundary conditions. There's another approach, which in some ways is perhaps *conceptually* simpler.

A) You've put the sphere into a uniform external field  $\mathbf{E}_0$ , so it would be logical (but wrong) to assume that the polarization of the dielectric would just be simply  $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}_0$ . (Briefly, why is that wrong?) Go ahead and assume it, as a "first approximation": let's call it  $\mathbf{P}_0$ . Now this polarized sphere generates its own *additional* induced E field, call that  $\mathbf{E}_1$ . What is that field? (You don't have to rederive it from scratch; Griffiths has derived the E field in a sphere created by a uniform polarization of that sphere.) Now  $\mathbf{E}_1$  will modify the polarization by an *additional* amount, call it  $\mathbf{P}_1$ . What's that? That in turn will add in a new electric field by an additional amount  $\mathbf{E}_2$ . And so on. The final, total, "real" field will just be  $\mathbf{E}_{\text{tot}} = \mathbf{E}_0 + \mathbf{E}_1 + \mathbf{E}_2 + \dots$

Work it out, and check/compare your answer with Griffiths result at the end of Ex 4.7 (p. 188)

You will need the handy trick for summing the infinite geometric series:  $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ .

If that looks mysterious to you, you should convince yourself that it's correct by multiplying both sides of the equation by  $(1-x)$ .

B) If the sphere was made of Silicon (see Table 4.2 in Griffiths, p.180), compare the "first approximation" for  $\mathbf{P}_0$  with the true result for polarization. How important was it to go through the summation? In what limit, large or small  $\epsilon_r$ , does this summing procedure make a big difference?

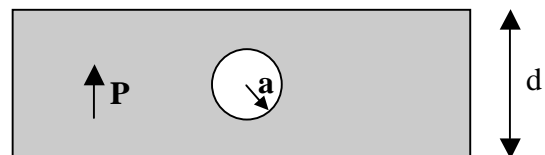
**Q2. SUPERPOSITION**

Consider a large slab (infinite in the  $xy$  directions) of dielectric material which has thickness  $d$ , and has uniform polarization  $\mathbf{P} = k \hat{\mathbf{z}}$ . Assume there are no free charges anywhere (so this polarization  $\mathbf{P}$  is permanent). The polarization is perpendicular to the surfaces of the slab.



A) Compute the electric field everywhere due to the charges arising from the polarization of the slab.

B) Now, assume that at the midplane of the sheet is a spherical hole of radius  $a$ . The inside of the bubble contains vacuum: it has zero polarization. Find the net electric field everywhere *inside* the hole.



*Hint: Carving out a cavity is the same as superimposing an object of the same shape but opposite polarization.*

C) Assume there are still no free charges anywhere. Find the bound surface charge on the surface of the hole. Then, find (or at least describe) the electric field *outside* the slab.

*Hints: Griffiths' Examples 3.9 and 4.2 may be useful.*

### Q3. DERIVING CLAUSIUS-MOSSOTTI

In a linear dielectric, the polarization is proportional to the field:  $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$  (Eqn 1). If the material consists of atoms (or non-polar molecules), the induced dipole moment of each atom is proportional to the field:  $\mathbf{p} = \alpha \mathbf{E}$  (Eqn 2). *Question:* What is the relation between the atomic polarizability  $\alpha$  and the susceptibility  $\chi_e$ ?

If  $n$  = number of atoms per volume, then the polarization  $\mathbf{P} = n\mathbf{p} = n\alpha\mathbf{E}$ . So one's first reaction is to compare equations (1) and (2) and conclude that  $\chi_e = \frac{n\alpha}{\epsilon_0}$ . This is almost correct, if the density is low, but closer inspection reveals a subtle problem. The E-field in equation (1) is the *total macroscopic field* in the medium. But the E-field in equation (2) is the field due to everything, *except* the atom under consideration. Call this field  $\mathbf{E}_{\text{else}}$ . Equation (2) should therefore be rewritten as  $\mathbf{p} = \alpha \mathbf{E}_{\text{else}}$ . Imagine that the space allotted to each atom is a sphere of radius  $a$ . Show that

$$\mathbf{E} = \left(1 - \frac{n\alpha}{3\epsilon_0}\right) \mathbf{E}_{\text{else}}.$$

Use this to conclude that  $\chi_e = \frac{n\alpha/\epsilon_0}{1 - (n\alpha/3\epsilon_0)}$ , and show that this is equivalent to

$$\alpha = \frac{3\epsilon_0}{n} \left( \frac{\epsilon_r - 1}{\epsilon_r + 2} \right). \text{ This last equation is called the Clausius-Mossotti formula.}$$

Under what conditions can the Clausius-Mossotti formula be replaced with the naive expectation,  $\chi_e = \frac{n\alpha}{\epsilon_0}$ ?

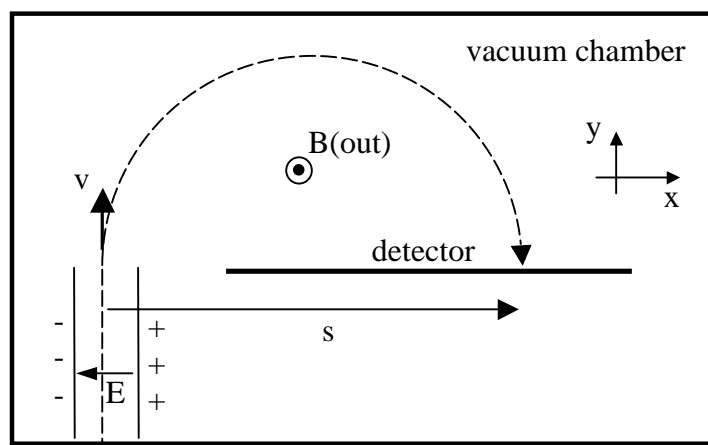
### Q4. APPLYING CLAUSIUS-MOSSOTTI

Use the Clausius-Mossotti equation (which you just derived in the previous problem) to determine the polarizability of atoms in air (which is largely  $\text{N}_2$ ). Table 2 in the book gives dielectric constants for  $\text{N}_2$  and “air” as separate entries but the numbers are basically identical. Combine this result with the simple classical calculation we did in lecture (Example 1 in chapter 4) to *estimate* a typical radius of an air molecule.

**Q5. CURRENT DENSITIES:**

- A) A solid cylindrical straight wire of radius  $a$  has a current  $I$  flowing down it. If that current is uniformly distributed over the outer surface of the wire (none is flowing through the "volume" of the wire; it's all surface charge), what is the surface current density  $\mathbf{K}$ ?
- B) Suppose that current *does* flow throughout the volume of the wire, in such a way that the volume current density  $\mathbf{J}$  grows quadratically with distance from the central axis, what then is the formula for  $\mathbf{J}$  everywhere in the wire?
- C) A CD has been rubbed so that it has a fixed, constant, uniform surface electric charge density  $\sigma$  everywhere on its top surface. It is spinning at angular velocity  $\omega$  about its center (which is at the origin). What is the surface current density  $\mathbf{K}$  at a distance  $r$  from the center?
- D) A sphere (radius  $R$ , total charge  $Q$  uniformly distributed throughout the volume) is spinning at angular velocity  $\omega$  about its center (which is at the origin) What is the volume current density  $\mathbf{J}$  at any point  $(r, \theta, \phi)$  in the sphere?
- E) A very thin plastic ring has a constant linear charge density, and total charge  $Q$ . The ring has radius  $R$  and it is spinning at angular velocity  $\omega$  about an axis thru its center (which is at the origin) and perpendicular to the plane of the ring. What is the current  $I$ , in terms of given quantities?

**Q6.** In a mass spectrometer, a particle of known charge  $q$  and unknown mass  $m$  enters a vacuum chamber filled with constant uniform magnetic field  $\mathbf{B} = B\hat{z}$ . It passes between the closely spaced plates of a capacitor with electric field  $\mathbf{E} = -E\hat{x}$  (and with the same B-field as elsewhere in the chamber) and emerges with velocity  $\mathbf{v} = v\hat{y}$ . Its semi-circular trajectory then carries it to a point on the detector a distance  $s$  from the exit of the capacitor, as shown.



A) The capacitor acts as a *velocity selector*. Explain how it works and relate the speed  $v$  of the emerging particle to  $E$  and  $B$ .

B) How is the mass  $m$  related to the known quantities  $q$ ,  $E$ ,  $B$ , and  $s$ ? (Gravity can be ignored since it is very weak compared to the forces due to  $E$  and  $B$ .) Given the geometry shown, will this spectrometer work for  $+$  charges only,  $-$  charges only, or both  $+$  or  $-$  charges?

**Q7. MAGNETIC FIELD:**

An infinitely long wire has been bent into a right angle turn, as shown in the figure. The curved part is a perfect quarter circle of radius  $R$ . Point  $P$  is exactly at the center of that quarter circle. A steady current  $I$  flows through this wire.

Find the magnetic field at point  $P$  (magnitude and direction).

