Phys3310 HW5, Due start of class Wed Feb 11

Q0. For 5 pts credit, please go to the link below and fill out the survey questions, giving us feedback about how the course is going for you.
http://www.colorado.edu/physics/phys3310/phys3310_sp09/particip_1.html

Q1. ENERGY OF POINT CHARGE DISTRIBUTION
A) Imagine a small square, of edge length *a*, with four positive point charges +q, one on each corner. Calculate the total stored energy of this system (i.e. the amount of work required to assemble it).
B) Calculate how much work it takes to neutralize these charges by bringing in one more negative point charge, –4q, from far away and placing it right at the *center* of this square.
C) Is the total PE of the 5 charge system (four +q's and –4q in the center) positive or negative. Argue qualitatively why the total PE has this sign.
(When studying crystal structures, it is sometimes convenient to model them as rectangular grids of charged ions, this problem forms the starting point for such a model)

Q2. ENERGY OF A CONTINUOUS CHARGE DISTRIBUTION
A) Find the total electrostatic energy stored in a uniformly charged sphere of radius R and total charge Q. (Note: this is uniform throughout the whole volume; it's not a shell.) Express your answer in terms of Q and R. There are many different ways to do this; I want you to use two different methods so you can check yourself.
i) figure out E(r) and then use Griffiths Eq. 2.45 (being careful to integrate over *all space*, not just where the charge is!)
ii) figure out V(r) and then use Griff. 2.43 (what region do you need to integrate this over?)
B) Sketch V(r) vs r and directly below your V(r) plot, sketch E(r) vs. r. Indicate clearly the functional forms both inside and outside the sphere. Your plots of V(r) and E(r) should line up vertically so that their relationship is clear. How are V(r) and E(r) related?
C) According to Einstein, a static electron has a rest mass energy $E = mc^2$. It is tempting (though wrong!) to imagine that the electron is NOT really a point charge, but instead is a tiny sphere, with uniform charge density, whose total electrostatic energy (found above) EQUALS this mass energy of the electron. In other words, the idea would be that the rest energy of the electron is purely electromagnetic.
What radius would an electron have to have, for this to work out? Give formula and numerical answer. *Don't take your result too literally; a correct model must take quantum mechanics into account.*
D) Compute the electrostatic energy of an atomic nucleus with Z protons and N neutrons, or in other words, A = N + Z nucleons. Use an approximation for the nuclear radius of $R = (1.2 \times 10^{-15}\ \text{m})\, A^{1/3}$. You can assume that the nucleus is a uniformly charged sphere. Convert your answer, which is likely in Joules, to the energy units of MeV. (Give your result in units of MeV times $Z^2/A^{1/3}$) Now use this result to estimate the *change* of electrostatic energy when a uranium nucleus undergoes fission into two roughly equal-sized nuclei. Go online and check what the experimental answer is. What does this tell you about the source of the energy released in a nuclear explosion? Is it mostly electrostatic?

Q3. GAUSS' LAW AND CONDUCTORS: Griffiths 2.35 (p. 101)

Q4. GAUSS' LAW AND CAVITIES: Griffiths 2.36 (p. 101)
Please be sure to explain your reasoning on all parts, and add to this problem the following:
In part a, *sketch* the charge distribution.
In part c, *sketch* the E fields (everywhere in the problem, in the cavities and outside the big sphere)
f) Back to the case of no charge q_c present. Suppose we move q_a a little off to one side, so it was no longer at the *center* of its little cavity? Make a sketch indicating the charge densities and the E-fields, showing clearly which would change from the on-center answers. Is there a net force on q_a now?
I really like this problem, lots of good physics in it. Try to think it through carefully, make physical sense of all your answers, don't just take someone else's word for it!

Q5. COAX CAPACITORS
In HW 3 you found the E field everywhere in and around a coaxial cable. We can slightly modify that problem, making it more realistic by letting the inner cylinder be a conductor (a wire). The inner conducting cylinder has radius *a* and the outer conducting cylindrical shell has inner radius b. It is physically easy to set up any fixed potential difference ΔV between the inner and outer conductors. In practice, the cable is always electrically neutral.
A) Assuming charge per length $+\lambda$ and $-\lambda$ on the inner and out cylinders, derive a formula for the voltage difference ΔV between the cylinders.
B) Assuming infinite cylinders, find the energy stored per length (W/L) inside this capacitor. Once again, let's do it two ways so we can check:
i) Integrate the energy density stored in the E field . Write your answer in terms of ΔV
ii) Find the capacitance per length (C/L) of this system, and then use stored energy $W = \frac{1}{2} C (\Delta V)^2$
Based on your answers above, *where* in space would you say this energy is physically stored?
C) Estimate the capacitance per meter of the coaxial cable that the cable company uses to send TV signals into homes.
D) This is also an excellent model for "axons", which are long cylindrical cells (basically coax cables) carrying nerve impulses in your body and brain. Estimate the *capacitance* (in SI metric units, Farads) of your sciatic nerve.
Assumptions - the sciatic nerve is the longest in your body, it has a diameter of roughly 1 micron, and a length of perhaps 1 m. Note that axons generally have a value of b which is very close to a (i.e. the gap is extremely tiny, b-a is about 1 nanometer.) so you can simplify your expression using $\ln(1+\varepsilon)\approx\varepsilon$

Q6. STATIC CHARGE SHOCK.

Suppose you walk across your rug on a dry winter's day, and see a centimeter-long blue spark jump from your finger to the door-knob. Let's estimate the energy involved and the number of electrons that passed between you and the knob.
A) Make a very rough estimate of the capacitance between you and the door knob, just before the spark jumped. State your assumptions clearly. HINT: *very rough*.
B) Knowing the breakdown voltage of air, estimate the energy stored in the charged human/door knob capacitor just before the spark jumped.
C) Estimate the charge on you, just before the spark. Give your answer in coulombs and in number of electrons.