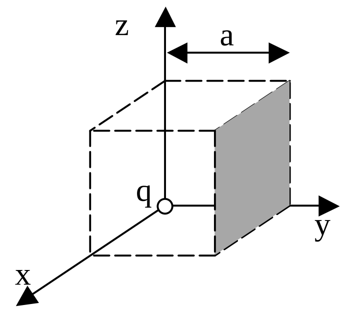
Phys 3310, HW #3, Due at start of class, Wed Jan 28.

Q1. ELECTRIC FLUX. A charge q at the origin is at the back corner of an imaginary cube of edge length *a*.

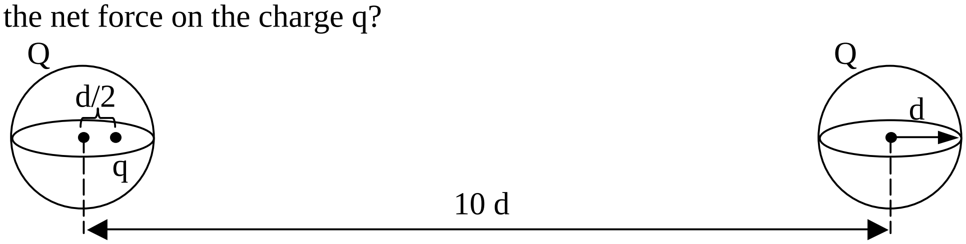


What is the electric flux through the shaded region (the plane at y = *a*)? Work out the answer two ways:
A) By Gauss's Law.
B) By direct integration. (Hint: set up the problem with Cartesian coordinates, not spherical coordinates. Rewrite $\hat{r} = \frac{\vec{r}}{r}$ in terms of Cartesian coordinates and proceed.)

Q2. SPHERICAL CHARGE DISTRIBUTIONS.
For parts A) and B), consider a sphere of radius R, centered on the origin, with a radially symmetric charge distribution $\rho(r)$.
A) What $\rho(r)$ is required for the E-field in the sphere to have the power-law form $E(r) = c\, r^{n}$, where c and n are constants? The case n = -2 is special. How so? Some values of n are unphysical since these would lead to an infinite amount of charge in the sphere. Which values of n are physically allowed?

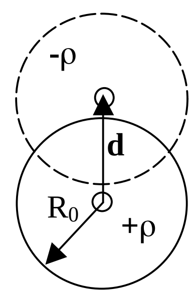
B) What kind of charge distribution is required for the radial E-field inside the sphere to be of constant magnitude; that is, what $\rho(r)$ produces E(r) = constant (inside only)?

C) The following problem is from the 2001 Physics GRE exam. Students were expected to solve the problem in just a few minutes!
Two spherical, nonconducting, and very thin shells of uniformly distributed positive charge Q and radius d are located a distance 10d apart. A positive point charge q is placed inside one of the shells at a distance d/2 from the center, on the ling connecting the centers of the two shells, as shown in the figure. What is the net force on the charge q?



Q3. E FIELD IN HYDROGEN.
A) Find the E-field in a hydrogen atom. Quantum mechanics tells us that the electron is effectively "smeared out", so the electron's resulting contribution to the charge density is $\rho_0(r) = \rho_0 \exp\left(-\frac{2r}{a_0}\right)$ where a_0 is the Bohr radius. (Hint: Don't forget to normalize ρ so the electrons charge is $-e$. This charge density is not uniform, so you must integrate to get enclosed charge. What are the correct limits of this integration? Don't forget there is also a point like proton in the middle of this atom.) Sketch and briefly discuss your result (compare the E field you get to what you'd have from the proton alone)
B) Having done the above, briefly discuss the advantages and disadvantages of using Gauss' Law to find the electric field instead of using Coulomb's Law (Griffiths Eq 2.8). What role does symmetry play?
Note: sketch means sketch - just a rough plot which shows key features (e.g. what's it do near the origin? Near infinity?) We don't want to just *calculate E fields, we want to be able to imagine what they look like, so sketching fields is important. [You can always use a program (like Mathematica) to* check *your sketch, but try on your own first...]*

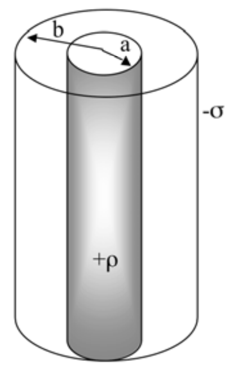
Q4. OVERLAPPING SPHERES
You have two spheres. The first is centered at the origin, has uniform positive charge density $\boldsymbol{\rho}$, and radius R_0. The second has uniform negative charge density $-\boldsymbol{\rho}$, same radius R_0. It is shifted up by a distance **d**.



A) Show that the field in the region of overlap of the two spheres is constant, and find its value. (Please check that the units are correct)
[Hint: You will first want to figure out what the E field is in a *single* sphere with uniform charge density $\boldsymbol{\rho}$. It is definitely not zero, nor is it uniform. Another possible hint: $\vec{r}_1 - \vec{r}_2 = \vec{d}$ for suitable $\vec{r}_1$ and $\vec{r}_2$.]

B) In the limit that |**d**| becomes small compared to R_0, discuss in words and make a sketch of what the resulting (total, physical) charge distribution in space really *looks* like (so that later in the course when we encounter such a charge distribution, we will know where it came from and what the E field looks like inside!)

Q5. COAXIAL CABLE.
A) A long cylindrical cable carries a uniform positive charge density $+\rho$ throughout an inner cylinder (radius a) and a uniform negative surface charge density $-\sigma$ on the outer cylinder (radius b). The cable is overall electrically neutral. Find **E** everywhere in space, and sketch it.
(Note: this is not a normal coaxial cable. A real coaxial cable has a metal inner cylinder and the charges on the inner conductor reside on its surface only, not throughout its volume.)



B) Making any reasonable guess as to physical dimensions for this cable like one that might attach your stereo to your TVs - what would be the maximum static surface charge density on the outer conductor before a spark occurs? (This is *estimation* - I don't care if you're off by a factor of 2, or even 10, but would like to know the rough order of magnitude of the answer!) From where to where do you expect a spark to go, if it *does* break down?

Q6. DELTA FUNCTIONS

a) Calculate $\int_{-1}^{1} \left|x-c\right|^2 \delta(2x)\,dx$, with the constant c = 3.

b) Calculate $\iiint_V |\vec{\mathbf{r}}-\vec{\mathbf{c}}|^2 \delta^3(2\vec{\mathbf{r}})\,d\tau$, where the volume V is a sphere of radius 1 centered on the origin, and the constant vector **c**=(3,4,0)

c) Evaluate the integral $\int_V \left(1+e^{-r}\right)\left(\nabla\cdot\left(\frac{\hat{r}}{r^2}\right)\right)d\tau$ where volume *V* is a sphere of radius R centered at the origin, by two different methods, as Griffiths does in Ex 1.16, page 51

The delta function, both 1-D and 3-D appears throughout this course. Physically, it represents a highly localized source (like a point charge).

Q7. δ FUNCTIONS AND CHARGE DISTRIBUTIONS

A) On the previous homework we had two point charges: +3q at x = –D, and –q at x = +D.
Write an expression for the *volume* charge density ρ(**r**) of this system of charges.

B) On the previous homework we had another problem with a spherical surface of radius *R* (Fig 2.11 in Griffiths) which carried a uniform surface charge density σ. Write an expression for the *volume* charge density ρ(**r**) of this charge distribution. (Hint: use spherical coordinates, and be sure that your total integrated charge comes out right.)
(Verify explicitly that the units of your final expression are correct)

C) Suppose a linear charge density is given to you as $\lambda(x) = q_0\delta(x) + 4q_0\delta(x-1)$
Describe in words what this charge distribution looks like. How much total charge do we have? Assuming that q_0 is given in Coulombs, what are the *units* of all other symbols in this equation (including, specifically the symbols λ, x, the delta function itself, the number written as "4" in front of the second term, and the number written as "1" inside the last δ function.

It can also be tricky to translate between the math and the physics in problems like this, especially since the idea of an infinite charge density is not intuitive. Practice doing this translation is important (and not just for delta functions!)