Phys3310 HW7, Due start of class Wed March 4

Q1. DIPOLE MOMENT OF A SPHERE OF CHARGE
The surface charge density on a sphere of radius *R* is $+\sigma_0$ on the entire northern hemisphere, and $-\sigma_0$ on the entire southern hemisphere, where σ_0 is a positive constant. There are no other charges present inside or outside the sphere.

A) *Use the methods of section 3.3.2* (explicitly using separation of variables in spherical coordinates) to find the electrical potential inside and outside this sphere. In principle, you will need an infinite sum of terms, but for this problem, just work out explicitly what the first *nonzero* term is, for both $V(r < R)$, and $V(r > R)$.
Explain physically why the first "zero term" really should be zero. This first non-zero term potential should look familiar. What is its name?
Griffiths solves a generic example problem, but please work through the details on your own. You are welcome to use Griffiths to guide you if/whenever you need it, but in the end, solve the problem yourself and show your work!

B) Compute the dipole moment of this sphere (with the +*z*-axis up through the pole of the positive hemisphere). Begin with the definition of a dipole moment, $\vec{p} = \int \rho\ \vec{r}\, d\tau$, which, in this case, becomes $\vec{p} = \int \sigma \vec{r}\, da\ = \hat{x}\int da\, \sigma\, x\ +\ \hat{y}\int da\, \sigma\, y\ +\ \hat{z}\int da\, \sigma\, z$. Two of the three components are zero. Working in spherical coordinates, show why those components are zero. (Show this explicitly. Simply stating that they are zero "by symmetry" is not enough.) Write your final answer for the dipole moment in terms of the charge Q on the upper hemisphere. Does your answer make sense? How does your answer to part A relate to this part?

Q2. SEPARATION OF VARIABLES - CONCENTRIC SPHERES
Two concentric spherical surfaces have radii of *a* and *b*. If the potential on the inner surface, at $r = a$, is a nonzero constant V_{in} and the potential on the outer surface is given by $V(b,\theta) = V_{out} P_1(\cos\theta)\ =\ V_{out}\cos\theta$, where V_{out} is a constant, find the potential in the region *between* the two surfaces ($a < r < b$). Check explicitly that your final answer gives the correct results for $r = a$ and $r = b$.

Q3. SEPARATION OF VARIABLES - DISK
A disk of radius R has a uniform surface charge density σ_0. Way back in HW2, you found the E-field along the axis of the disk. You can check for yourself by direct integration, (but don't have to); I claim that along the z axis, (i.e. $\theta = 0$),

$$V(r,\theta=0) = \frac{\sigma_0}{2\varepsilon_0}\left(\sqrt{r^2+R^2} - r\right)$$

A) Find the potential *away* from the axis (nonzero θ) , for distances $r > R$, by using the result above and fiddling with the Legendre formula, Griffiths' 3.72 on page 140. You will in principle need an infinite sum of terms, but for this problem, just work out explicitly the first two *non-zero* terms.
Hints: Remember that $P_l(1)$ is always equal to 1(one) for any l ("ell"). It may be useful to write a Taylor-series expansion for $V(r,0)$ for $r >> R$.
B) Griffiths Section 3.4 describes the "multipole expansion". Look at your answer to part A, and compare it to what Griffiths says it *should* look like (generically) on page 148. Discuss: does your answer make some physical sense? Note that there is a "missing term". Why is that?

Q4. MULTIPOLES - POINT CHARGES
You have four point charges. Their charges and locations in Cartesian coordinates are:
A charge –q located at (a,0,0), a second charge –q located at (–a,0,0), a third charge +3q located at (0,0,b), and finally a fourth charge –q located at (0,0,–b)

A) What is the monopole moment and the dipole moment of this distribution of charges? Use the multipole expansion (Griffiths p. 148) to find a first-order formula for $V(r,\theta)$ (in spherical coordinates) valid at points far from the origin. ("First-order" means only the first *non-zero* term is needed.)
B) From your answer in part A, find an approximate expression for the electric field valid at points far from the origin. (Again, express your answer in spherical coordinates; we want $\vec{\mathbf{E}}(r,\theta) = E_r(r,\theta)\hat{\mathbf{r}} + E_\theta(r,\theta)\hat{\theta}$, so you need to compute E_r and E_θ.)
Sketch this (approximate) E field. (Don't worry about what happens near the origin, I just want a sketch of the far-field r >> a,b)

Q5. NON-LINEAR DIELECTRIC
We often assume that the induced dipole moment of an atom is proportional to the external field (at least for small fields). This is not a fundamental law, and it is easy to construct counter-examples, in theory. Suppose the charge density of the electron cloud is proportional to r out to a maximum radius R. (Not a crazy assumption: in QM, the charge density is zero at the origin for non-s states.) In this case, the induced dipole moment p is proportional to what power of the E, in the weak field limit?

Extra Credit: BOUNDARY CONDITIONS**.** In last week's problem 3 (Homework Set 6, problem 3), you had to solve for the voltage inside a square pipe with the voltage fixed on three side of the pipe and the strange-looking condition $\partial V / \partial x = 0$ on the 4th side. What physical situation would produce such boundary conditions: that is, how would an experimentalist arrange things so as to guarantee that $\partial V / \partial x = 0$ on the 4th side? Hint: compare this problem with Griffiths' example 3.4 on pages 132-134.