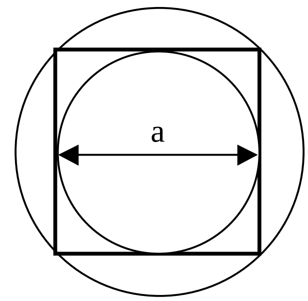
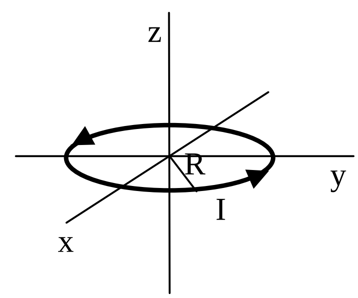
PHYS3310 HW10, Due start of class Wed April 8

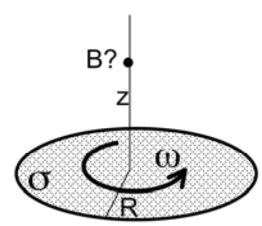
Q1. BIOT-SAVART - SQUARE LOOP
A) Find the magnetic field at the center of a square current loop with current I and edge length *a*.
B) If I had such a loop in my lab and wanted the B field at the center, I might do the above calculation, but if I was planning an experiment and just wanted a *rough estimate* of the B-field, I might "assume a spherical cow": assume the square was really a circle. We've done that problem (**B** at center of a circular loop - it's much simpler than the square.You don't have to rederive it, but do think back to how we got that result, and why it turned out to be a relatively easy application of Biot-Savart.) But what radius circle would you use, to estimate B? Find the B fields for the inscribed and circumscribed circles and then average. How good an approximation does that turn out to be?



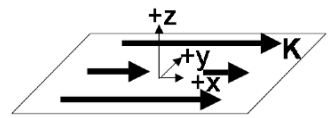
Q2. B FIELD FROM ROTATING DISK
A) Compute the magnetic field **B**(0,0,z) on the axis of a circular ring of radius R carrying a current I . The ring is in the xy plane and is centered on the origin.



B) Last week we had a problem with a CD of radius R with a fixed, constant, uniform surface electric charge density σ everywhere on its top surface. It was spinning at angular velocity $\omega\,\hat{\mathbf{z}}$ about its center (the origin). You found the current density **K** at a distance r from the center. Use that result to find the magnetic field **B**(0,0,z) at any distance z directly above the origin.
Does your answer seem reasonable? Please check its limiting behaviors (e.g. what do you expect if R→0? z→∞? ω→0? Slightly less obvious, but also worth checking/thinking about, what about z→0?)



Q3. SHEET OF CURRENT: BIOT-SAVART VS. AMPERE



Consider a thin infinite sheet with uniform surface current density $K_0\,\hat{\mathbf{x}}$ in the xy plane at z = 0.

A) Use the Biot-Savart law to find **B**(x,y,z) both above and below the sheet, by integration.

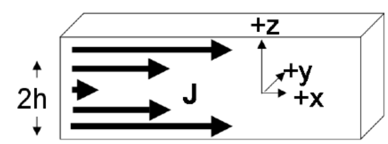
<u>Note:</u> The integral is slightly nasty. Before you start asking Mathematica for help - simplify as much as possible. Set up the integral, be explicit about what curly R is, what da' is, etc, what your integration limits are, etc. Then, make clear mathematical and/or physical arguments based on symmetry to convince yourself of the *direction* of the **B** field (both above and below the sheet), and to argue how **B**(x,y,z) depends (or doesn't) on x and y. (If you know it doesn't depend on x or y, you could e.g. set them to 0... But first you must convince us that's legit!)

B) Now solve the above problem using Ampere's law. (Much easier than part a, isn't it?) Please be explicit about what Amperian loop(s) you are drawing and why. What assumptions (or results from part a) are you making/using?

(Griffiths solves this problem, so don't just copy him, work it out for yourself!)

C) Now let's add a second parallel sheet at z = +a with a current running the other way. $\vec{K} = -K_0\,\hat{\mathbf{x}}$. Use the superposition principle (do NOT start from scratch or use Ampere's law again, this part should be relatively quick) to find **B** *between* the two sheets, and also *outside* (above or below) both sheets.

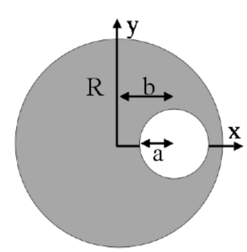
Q4. AMPERE'S LAW-II



Now the *sheet* of current has become a thick SLAB of current.
The slab is *infinite in (both) x and y*, but finite in z.
So we must think about the volume current density **J**, rather than **K**.
The slab has thickness 2h (It runs from z=-h to z=+h)
Let's assume that the current is still flowing in the +x direction, and is uniform in the x and y dimensions. But now **J** depends on height linearly, i.e. $\vec{\mathbf{J}} = J_0 \,|\, z \,|\, \hat{\mathbf{x}}$ inside the slab (but is 0 above or below the slab). Find the **B** field (magnitude and direction) everywhere in space (above, below, and also, most interesting, *inside* the slab!)

Q5. AMPERE and SUPERPOSITION

A) A long (infinite) wire (cylindrical conductor, radius R, whose axis coincides with the z axis) carries a uniformly distributed current I_0 in the +z direction. A long (infinite) cylindrical hole is drilled out of the conductor, parallel to the z axis, (see figure for geometry). The center of the hole is at x = b, and the radius is a. Determine the magnetic field *inside the cylindrical hole.*



B) If this is an ordinary wire carrying ordinary household currents, and the drilled hole has dimensions roughly shown to scale in the figure above, make an order of magnitude estimate for the strength of the B field in that region. How does it compare to the earth's field?