Phys 3310, HW #4, Due in class Wed February 4.

Q1. NONUNIFORM SURFACE CHARGE
Consider an insulating sphere of radius R with a *non-uniform* surface charge density
$\sigma = \sigma_0 \sin^2(\theta)\ \cos^2(\phi)$.

A) Find the total charge on this sphere.
B) Describe briefly in words and pictures what this charge distribution looks like.
C) Briefly but clearly, describe a *procedure* to find **E**(x,0,0) for x > R.
You do not need to come up with a final closed-form answer! We just want a discussion, with formulas, of how you would proceed. Get as far as you reasonably can, but stop when the going gets too nasty, and discuss what you would do next if you really *needed* to know this E field in, say, a laboratory/experimental situation.

Q2. DIVERGENCE AND CURL
Consider an electric field **E** = $c\dfrac{\vec{\mathbf{r}}}{r^2}$. Please note the numerator is not $\hat{\mathbf{r}}$: this is NOT the usual E field from a point charge at the origin, which would be $c'\dfrac{\vec{\mathbf{r}}}{r^3}$.

A) Calculate the divergence *and* the curl of this **E** field.
B) Explicitly test your answer for the divergence by using the divergence theorem.
Is there a delta function at the origin like there was for a point charge field, or not?
Explicitly test your answer for the curl by using Stokes' theorem. If you perform a volume, surface, or path integral, be sure to make clear what volume, surface, or path you have chosen.
C) What are the units of the constant c? What charge distribution would you need to produce an E field like this? Describe it in words as well as formulas. Is such a charge distribution physically realizable?

Q3. ALLOWED E-FIELDS
A) Which one of the following two static E-fields is physically *impossible* (NOT possible). Why?

$\vec{\mathbf{E}}_1 = c(2x\,\hat{\mathbf{i}} - x\,\hat{\mathbf{j}} + y\,\hat{\mathbf{k}})$ $\qquad\qquad$ $\vec{\mathbf{E}}_2 = c(2x\,\hat{\mathbf{i}} + z\,\hat{\mathbf{j}} + y\,\hat{\mathbf{k}})$

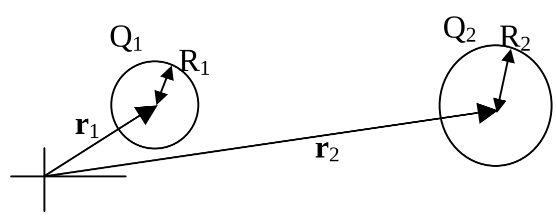
Here, c is a constant (with appropriate units)
B) For the E-field which IS possible, compute the potential, the voltage $V(\vec{r}) = V(x,y,z)$, by integrating along a path using the *origin* as your reference point [i.e. setting V(0)=0] Check your answer by explicitly computing the E-field from your voltage.
Note: Specify clearly your chosen path of integration from (0,0,0) to (x, y, z). The answer is path-independent, but proper choice of path will make the computation much easier.

Q4. FINDING VOLTAGE FROM CHARGE DISTRIBUTION
A) Derive a formula for the electrostatic voltage (the potential) V(z) everywhere along the symmetry-axis of a charged ring of radius a. The ring is in the xy plane and centered on the origin, with uniform linear charge density λ around the ring. Please use the method of direct integration (Griffiths 2.30, on p. 85) to do this, and set your reference point to be $V(\infty)=0$.
B) Use your result for V(0,0,z) to find the z-component of the E field anywhere along the z-axis. Check that your expression for E(0, 0, z) is correct by explicitly computing the voltage V(0, 0, z) using the definition of voltage (Griffiths, p.78, 2.21).
C) Make sketches of E vs. z and V vs. z for points on the z-axis (both $z<0$ and $z>0$).
D) How does V(z) behave as $z\rightarrow\infty$? (Don't just say it goes to 0. HOW does it go to zero? Does your answer make physical sense to you? Explain, briefly.)

Q5. CALCULATING VOLTAGE FROM E-FIELD
A) Use Gauss's law to show that the electric field a distance s from an infinite line of uniform charge density λ is $\vec{\mathbf{E}} = \frac{\lambda}{2\pi\varepsilon_0}\frac{\hat{\mathbf{s}}}{s}$.
B) Find the voltage difference between any two points near the line. Check your answer by explicitly taking the gradient of V to compute **E**.

C) Briefly discuss the question of "reference point". Where can you set V=0 in this problem? Can you use $s = \infty$, or $s = 0$, as the reference point, V(s)=0? Explain, briefly.
D) Suppose a lightening stroke briefly created a long line of charge with a charge per length of say, 1 C/100 m. You see the stroke and a couple of second later you hear it. Estimate the voltage difference caused by the lightning across an object the size of your heart, where you are standing. Does the answer surprise you?

Q6. SHELLS OF CHARGE.
Two charged droplets of toner ink behave as two *shells* of uniform surface charge density. (Toner is an insulating material, not metal.) Shell 1 has total charge Q_1 , radius R_1, with its center at position $\mathbf{r}_1$. Shell 2 has Q_2 , R_2, and $\mathbf{r}_2$. Write expressions for the voltage V(**r**) everywhere: outside both spheres, inside sphere 1, and inside sphere 2. (Hint: use curly r notation, but define your symbols carefully.) Is the voltage inside shell 1 constant?



Q7. SCREENED POTENTIAL

Consider the “screened Coulomb potential” of a point charge of charge q that arises, for example, in plasma physics: $V(\mathbf{r}) = \dfrac{q}{4\pi\varepsilon_0}\dfrac{\exp(-r/\lambda)}{r}$, where λ is a constant (called the screening length).

A) Determine the E-field $\mathbf{E}(\mathbf{r})$ associated with this voltage.

B) Find the charge distribution $\rho(\mathbf{r})$ that produces this voltage. Think carefully about what happens at the origin. What does this charge distribution look like very close to the origin?

Sketch this function $\rho(\mathbf{r})$ in a manner that clearly describes its characteristics (What’s the best way of representing this three-dimensional charge distribution? Explain clearly what you’re plotting.)

C) Show by explicit calculation over $\rho(\mathbf{r})$ that the net charge represented by this distribution is zero (!) (If you don't get zero, think again about what happens at r = 0).