

This first set is meant to be a review – I'm hoping it's mostly straightforward for you. If not, I strongly urge you to talk with me ASAP, we might need to help you catch up. Future hw sets will involve new ideas, and significantly more time and effort than this!

We grade homeworks for clarity of explanation as much as we do for mere "correctness" of final answer.

**PLEASE READ THE FOLLOWING INSTRUCTIONS (just for this first week)**

For each numbered question, rate your own confidence (A, B, C) in the material with this scale:

**A)** The material is very familiar to me. I was able to answer the question without assistance of a book, human, Internet, etc

**B)** The material was familiar, but in order to answer the question, I needed some assistance (from a book, human, Internet, etc)

**C)** The material was unfamiliar to me. I had to learn it in order to answer the question.

**Write the letter (A, B, or C) with a big circle around the letter, near your answer for EVERY NUMBERED QUESTION up to #14 on this set, please.**

1.) Given a triangle (NOT necessarily a "right triangle") with sides a, b, and c, and an angle ( $\theta$ ) opposite side c. Suppose I tell you a, b, and  $\theta$ : what is c?

2.)  $\int \frac{4x}{(x^2 + a^2)^{3/2}} dx$  (where a is a known constant. Note that it is an indefinite integral)

3.)  $\frac{d}{dx} \int_1^x f(y) dy$  (where  $f(y)$  is some given, known (well behaved) function of y)

4.)  $\frac{d}{dx} \int_0^1 (\ln y + \ln x) dy$

Make a quick sketch, in the x-y plane, of the following vector functions.  
Plot enough different vectors to get a feeling for what this field looks like in the x-y plane.

5.)(a)  $y \hat{\mathbf{x}}$

(b)  $r \hat{\mathbf{r}}$  (The symbol  $\mathbf{r}$  here refers to the usual  $\mathbf{r}$  in spherical coordinates)

(c)  $\frac{x}{\sqrt{x^2 + y^2}} \hat{\mathbf{x}} + \frac{y}{\sqrt{x^2 + y^2}} \hat{\mathbf{y}}$

Also just for this one (part c) - explain in *words* what this plot is showing.

**ARE YOU REMEMBERING to categorize each problem with an A/B/C? (See page 1 for instructions, it's just for this week, but I want you to do it on each problem. Thanks!)**

- 6.) Given the scalar function  $T(x,y,z)$  (e.g. the temperature at any point in the room)  
Which of the three operations (div, grad, or curl) can be sensibly operated on  $T$ ?  
For each which can:  
a) give a formula for the result  
b) explain in words how you would interpret the result.  
c) is the result a vector or scalar?

- 7.) Given an arbitrary vector function  $\mathbf{V}(x,y,z)$  (e.g. the velocity of a flowing liquid)  
Which of the three operations (div, grad, or curl) can be sensibly operated on  $\mathbf{V}$ ?  
For each which can:  
a) give a formula for the result  
b) explain in words how you would interpret the result.  
c) is the result a vector or scalar?

8.) For each of the four vector fields sketched below....

Which of them have nonzero *divergence* somewhere? \_\_\_\_\_

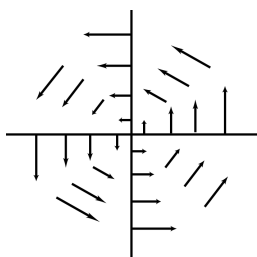
(If the divergence is nonzero *only* at isolated points, which point(s) would that be?)

Which of the following fields have nonzero *curl* somewhere? \_\_\_\_\_

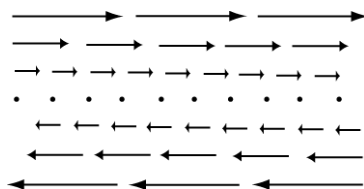
(If the curl is nonzero *only* at isolated points, which point(s) would that be?)

(A brief explanation of your answers below each figure would be welcome)

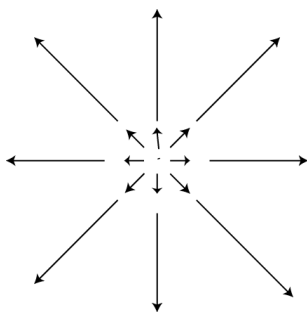
**A.**



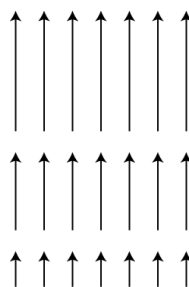
**B.**



**C.**



**D.**



9.) Given vectors **A** and **B**. ("Given" means you know the components, or alternatively, the length and angle of the vectors)

Define the dot product mathematically in two very different looking ways.  
(Hint: one way should involve the components, the other, the length/angles)

Give a brief physical interpretation of what the dot product means or tells you (you can give a concrete example if you like)

10.) Define the vector cross product mathematically in two very different looking ways.  
(Hint: one way should involve the components, the other, the length/angles)

Give a brief physical interpretation of what the cross product means or tells you (you can give a concrete example if you like)

11.) Compute the gradient of the following two scalar fields.

a)  $e^x \cos(y)$

b)  $\cos(x^2 + y^2 + z^2)$

12.) Compute the divergence and curl of  $\hat{i}(x^2 + yz) + \hat{j}(y^2 + zx) + \hat{k}(z^2 + xy)$ .

13.) Evaluate the line integral  $\int (y^2 dx - 2x^2 dy)$  along the parabola  $y = x^2$  from the point (0,0) to the point (2,4).

14.) In Phys 1120, one of the early chapters was on Gauss' law, one of the most fundamental laws of electricity. It looks like this:  $\oint_S \vec{E} \cdot \hat{n} \, dA = q(\text{enclosed})/\epsilon_0$ ,

where  $\vec{E}$  is the electric field,  $S$  is a closed surface,  $\hat{n}$  is a unit vector which points everywhere *outward* from the surface.

Suppose I fill a cube (length  $L$  on a side) uniformly with electric charge. I then imagine a larger, closed cubical surface symmetrically surrounding this cube (length  $2L$  on a side)

A) Is Gauss' law TRUE in this situation? (Briefly, why or why not?)

B) Can one *use* Gauss' law to easily compute the value of the electric field at arbitrary points outside the charged cube (Don't try, just tell me if you *could*, and why/why not?)

C) What exactly is  $\vec{E}$ , the electric field? (Define it, and explain how you think about it, first mathematically *and* then in words? Please define any new technical words you introduce into your definition.)

The next questions will **not** be graded:

15.) Have you ever learned about orthogonality of functions in a math or physics class? If so, tell me briefly what you remember about the orthogonality of sin and/or cos.

16.) Have you ever learned about the delta function (also called the "Dirac delta function",  $\delta(x)$ , in either a math or a physics class)? If yes, tell me briefly what you remember.

If you can, evaluate  $\int_{-\infty}^{\infty} (x - 6) \delta(x - 6) dx$ .