Upper-Division Electricity and Magnetism: Students’ Ideas and Difficulties

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+ working groups

INTEGRATING STEM
Science, Technology, Engineering, and Math Education at Colorado
Why Transform E&M I?

Lecture with clickers

Washington Tutorials

Can our majors learn better from interactive techniques adapted from introductory physics?
What’s special about upper-div?

- More experienced students
- Faculty and student investment & identity
- More complex physics
- Smaller classes
What Changed?

- Faculty collaboration
- Explicit learning goals
- Interactive classroom techniques
- Concept Tests
- Modified Homework
- Homework Help Sessions
- Tutorials (+ pre/post quiz)

Students debate a concept test
Research on student difficulties

Research-based
• Tutorials
• Clicker Questions
• Homeworks
• Class activities
• Consensus learning goals

Research-validated
• CUE instrument
• Interviews and class observations

and in progress:
• pre/post Tutorial assessments

reflective development
Did it Work? Assessments

- **Colorado Upper-Division Electrostatics Assessment (CUE)**
  - Conceptual questions
  - High internal statistical consistency, high inter-rater reliability

- **Traditional** (4 courses) & **Transformed** (8 courses)
  CU and elsewhere (N=462).
Students in transformed courses performed better on the CUE:

![Post-test graph showing comparison between Traditional and Transformed courses](chart.png)
Students in transformed courses performed better on the CUE:

Scores still low ➔ need research on student difficulties
Many opportunities to observe student difficulties in transformed course:

- Interactive classroom techniques
- Concept Tests
- Modified Homework
- Homework Help Sessions
- Tutorials (+ pre/post quiz)

Also, formal student interviews, exams, CUE

Students debate a concept test
Common Mathematical Difficulties:

• Can do calculation – difficulty setting up and interpreting results (e.g. Thompson)

• Don’t connect math to physical situation (e.g. Manogue et al.)

• Do not access all the math tools that instructors expect (e.g. Bing & Redish)
Common Mathematical Difficulties:

- set up and interpret results
- connect math to physical situation
- accessing math tools

Examples:
- Gauss’s Law
  - Precise symmetry arguments
  - Inverse problems
  - Impossible vs. difficult
- Vector Fields
  - Making meaning of div, grad, curl
  - Magnitude and direction
- Potential
Using Gauss’s law is hard

\[ \oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} \]
Lower and upper division students struggle
(Singh AJP 2006)

- 25 question multiple choice test to 541 introductory students and 28 upper-level students

- Introductory students: 49%

- Upper-level: 44% (pre) → 49% (post)

- Graduate students: 75%
We also quantify student difficulties:

- **CUE concept survey**: Do **not** solve, but give “the easiest method you would use to solve the problem” and “why you chose that method (half credit):

\[
\rho(r) = \rho_0 e^{-r^2/a^2}
\]

33% of students do not recognize Gauss’s law as the easiest way to solve. (N=325)
We also quantify student difficulties:

- **CUE concept survey**: Do **not** solve, but give “the easiest method you would use to solve the problem” and “why you chose that method (half credit):

  - 33% of students do not recognize Gauss’s law as the easiest way to solve.  \((N=325)\)

  - 24% of students incorrectly choose Gauss’s law as the easiest way to solve.  \((N=325)\)
Probing student difficulties with interviews

- 4 students interviewed after transformed E&M 1 course
Common Mathematical Difficulties:

• set up and interpret results
• connect math to physical situation
• accessing math tools

Examples:

• Gauss’s Law
  - Precise symmetry arguments
  - Inverse problems
  - Impossible vs. difficult

• Vector Fields
  - Making meaning of div, grad, curl
  - Magnitude and direction

• Potential
Students in interviews struggle to make complete symmetry arguments

- Two types of arguments used by experts:
  - Geometry: invariant to rotations/translations
  - Superposition: Add-up $\mathbf{E}$ from symmetric pieces

- Which do students prefer? Do they use both? Are they proficient?
Students in interviews struggle to make complete symmetry arguments

- Two types of arguments used by experts:
  - Geometry: invariant to rotations/translations
  - Superposition: Add-up $\mathbf{E}$ from symmetric pieces
- Students in interviews seem to predominantly use superposition
  - Even when not applicable
  - May lead to problems assessing novel charge distributions
Common Mathematical Difficulties:

• set up and interpret results
• connect math to physical situation
• accessing math tools

Examples:

• **Gauss’s Law** (PERC paper and poster)
  - Inverse problems
  - Precise symmetry arguments
  - Impossible vs. difficult

• **Vector Fields**
  - Making meaning of div, grad curl
  - Magnitude and direction

• **Potential**
Common Mathematical Difficulties:

- set up and interpret results
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- accessing math tools

Examples:
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Vector calculus difficulties

- Divergence
- Gradient and Curl
- Line integrals
- Surface and volume integrals
Student troubles with divergence

\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \]
You have a thin spherical shell of uniform negative charge \(-Q\) centered at the origin with no other charge anywhere (i.e. all the charge is concentrated in a hollow shell at \(r=R\)). Where in space (if anywhere) does the divergence of \(E\) vanish?

\[
\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}
\]
Where in space (if anywhere) does the divergence of E vanish?

- Inside: 31%
- Correct - complete: 18%
- Correct - where no charges: 8%
- Origin: 8%
- Infinity: 6%
- Outside: 8%
- Other: 21%

Midterm exam, N = 59
Common Mathematical Difficulties:

- set up and interpret results
- connect math to physical situation
- accessing math tools

Examples:

- Gauss’s Law
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Dealing with Mathematical Difficulties in Lower and Upper Division Physics

AAPT 2010

Univ. of Colorado

Common Mathematical Difficulties:

• set up and interpret results
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• accessing math tools

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• Gauss’s Law
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  – Magnitude and direction
• Potential
Electric potential

\[ V(\vec{r}) \equiv -\int_0^r \vec{E} \cdot d\vec{l} \]

\[ \nabla^2 V = \frac{\rho}{\varepsilon_0} \quad \vec{E} = -\nabla V \]

\[ V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}')}{r'} d\tau' \]
You have a thin spherical shell of uniform negative charge \(-Q\) centered at the origin with no other charge anywhere. What is the sign of \((V(r=0) - V(r=R))\)?

\[
V(\vec{r}) \equiv -\int_{0}^{r} \vec{E} \cdot d\vec{l}
\]

Midterm exam, N = 59
**Common Mathematical Difficulties:**

- set up and interpret results
- connect math to physical situation
- accessing math tools

**Examples:**

- Gauss’s Law
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  - Magnitude and direction
- Potential
Actions to help?

• Transformed course helps:

  Overall Score - Gauss's law with cube

- There is still more to do:
  - Redesign tutorials?
  - Explicitly address in instruction?
  - Your ideas?

Dealing with Mathematical Difficulties in Lower and Upper Division Physics

AAPT 2010

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Summary

We have transformed an upper division class:
- Impact on content learning
- Impact on participation

Students still have many difficulties
- Setting up and interpreting math
- Connecting math to physical geometry
- Using diverse mathematical tools

Transformed course helps, but still more to do
Questions?

• More details:
  • This evening poster (D07)
  • Talk tomorrow morning 10:30 (HE06)
  • PERC poster (Wednesday evening)

• PER course materials for Quantum and E&M
  [http://per.colorado.edu/cts](http://per.colorado.edu/cts)

• Clicker videos at
  [STEMclickers.colorado.edu](http://STEMclickers.colorado.edu)
EXTRA SLIDES
You have a thin spherical shell of uniform negative charge \(-Q\) centered at the origin with no other charge anywhere. What is the sign of \((V(r=0) - V(r=R))\)?

\[
V(r) \equiv -\int_0^r \vec{E} \cdot d\vec{l}
\]

Midterm exam, N = 59
Incorrect inferences about the integrand

- Infer properties of $\mathbf{E}$ from $\oint \mathbf{E} \cdot d\mathbf{a}$

- Unclear distinction between $\mathbf{E}$ and flux

- Similar to previous difficulties reported for integral $= 0$. (Singh, 2006; Chasteen & Wallace 2010)
Incorrect inferences about the integrand

"'Cause if there's Q on the outside, the charge, you know, is making an E-field as well... and therefore it must affect the E field at that point [points to the Gaussian surface] as well. So I'm still... I'm still not really happy with Gauss's law."

- Similar to previous difficulties reported for integral = 0. (Singh, 2006; Chaszteen & Wallace 2010)
We also quantify student difficulties:

- **Midterm exam question:**
  Suppose I evenly fill a cube (length L on a side) with electric charges. I then imagine a larger, closed cubical surface neatly surrounding this cube (length 2L on a side).

  Is Gauss' law TRUE in this situation? (Briefly, why or why not?)

  Can one use Gauss' law (written above) to simply compute the value of the electric field at arbitrary points outside the charged cube (Don't try, just tell me if you could, and why/why not?)
Can one use Gauss' law to simply compute the value of the electric field at arbitrary points outside the charged cube?

• Correct: “It cannot be used to simply calculate the E field because over the outer box, E is not constant so the $\int \vec{E} \cdot d\vec{a}$ cannot be replaced with EA.”

• Incorrect – over generalize from highly symmetric charge distributions: “Since the two are the same type of surfaces they will have the same normal vectors you could easily calculate the $|E|$ for both surfaces”

• Incorrect – messy rather than impossible: “I don’t think so. It probably wouldn’t be “simple” because there’s no easy symmetry that allows E to be pulled out of the integral, so it’d be mess. Perhaps someone with crazy math skills could.”
Answers from a recent transformed E&M class

All 4 students interviewed gave this type of explanation for a different question.

Common intro-level difficulty observed by Singh. (AJP 2006)
Indicates?

- Students not yet familiar with solving inverse problems - sometimes not possible to solve
- Students are not thinking through the problem
- Students at the upper division have faith that there is a fancy math trick for every problem
- Your ideas?
Common

- Gauss’s Law
  - Inverse problems
  - Recognizing and exploiting symmetry
  - Impossible vs. difficult
- Vector Calculus
  - Making meaning of \( \text{div,grad,curl} \)
- Sep of Variables
  - ?
- Potential?
Documented difficulties (Singh AJP 2006)

- Lower-division undergraduates
  - Over generalize results from highly symmetric charge distributions
  - Have trouble distinguishing between charge, electric field and electric flux
  - Have difficulty with the superposition principle
  - Have trouble choosing an appropriate Gaussian surface
Documented difficulties

• Lower and upper-division undergraduates
  – Over generalize results from highly symmetric charge distributions – rote use of $EA = \frac{Q_{enc}}{\varepsilon_0}$
  – Have trouble distinguishing between charge, electric field and electric flux (!)
  – Have difficulty with the superposition principle
  – Have trouble choosing an appropriate Gaussian surface
## What do upper-division students think?

<table>
<thead>
<tr>
<th>Code</th>
<th># of responses* (out of 70)</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Positives</strong></td>
<td>64</td>
<td>91%</td>
</tr>
<tr>
<td>Improved mastery</td>
<td>35</td>
<td>50%</td>
</tr>
<tr>
<td>Type of Activity</td>
<td>31</td>
<td>44%</td>
</tr>
</tbody>
</table>
“Clicker questions encourage me to pay attention in class as well as help me to come to firm understanding of material through argument.”

“They were useful because they were challenging but used the knowledge we just learned in the lecture portion. They are a great way to go from hearing the information to actually using the information.”

“It helps a lot to be able to check your understanding of the concepts before moving on to the next, especially when we're going over complex topics that we may not have seen before. Also, discussing the topic with others, as we did when a clicker question was posed, is a great way to develop intuition and stay focused.”
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</tr>
<tr>
<td>Active processing/activity</td>
<td>44</td>
<td>63%</td>
</tr>
<tr>
<td>Discussion with others</td>
<td>20</td>
<td>29%</td>
</tr>
<tr>
<td>Feedback to students</td>
<td>20</td>
<td>29%</td>
</tr>
<tr>
<td>Time/pause to think, OR Immediacy</td>
<td>18</td>
<td>26%</td>
</tr>
<tr>
<td>Engagement</td>
<td>16</td>
<td>23%</td>
</tr>
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</table>
N=4 courses, 66 students

% of students

Types of clicker questions:
- Challenging conceptual
- Recalling a previous fact
- Recalling a recent fact
- Plugging numbers into equation

How useful for learning?

Very useful

Useful

Somewhat useful

Mostly useless

Completely useless

91%

35%

36%

18%
Assessments

Traditional Exam Q’s
Assessments

Traditional Exam Q’s (conceptual focus)

For the E field outside a uniformly charged cube: Is Gauss’ Law true? Is Gauss’ Law useful?
Assessments

Traditional Exam Q's

Brief Electricity & Magnetism Assessment (BEMA)
Students in Transformed course performed significantly better (p<0.05) on all except BEMA and Q5.
Assessments

Students in Transformed course performed significantly better \((p<0.05)\) on all except BEMA and Q5.