***👓 INSTRUCTORS MANUAL: TUTORIAL 10***

**Goals**:

1. Understand the origin of the vector potential (mathematically)
2. Be able to visualize and sketch the vector potential
3. Understand and apply a high-level mathematical analogy (between vector potential and Maxwell’s equations)
4. Connect physical representations to the real-world
5. Calculate vector potential

**This tutorial is based on:**

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**Tutorial Summary**:

Students first summarize the mathematical origin of the vector potential. Then, using a mathematical analogy, and their familiarity with Ampere’s Law, they visualize and sketch **A**. Finally, they predict **A** for a surface current, then calculate **A** (or guess and check a solution).

**Reflections on this tutorial:**

While this tutorial appears to be an effective activity for gaining familiarity with vector potential through sketching various situations, it doesn’t really address the “slipperiness” of **A**. For example, students see that **J** and **A** are in the same direction and start to assume this is always true. However, one student noticed (mathematically) that **A** can also have other components when he was calculating **A** in the final part.

**Part 1**  Most students knew where **A** comes from in **question (i).** Only one student was seen attempting to write a formula using a delta function in **question (ii).** Students are heavily guided in **question (iii),** and so have few problems here. A few students thought that **A** would also be a delta function just like **J**. Since this is a high-level mathematical analogy, guidance is appropriate. In **question (iv)** we had to push each student to say “magnetic flux.” The sketches in **question (iv)** are not trivial for students. A few students again thought that **A** would just be on the surface of the toroid like **J** is and they had to be pushed to recognize that **A** is a dipole field.

**Part 2.** Most students didn’t know what a Tokamak was in **question (i),** but were able to figure out that it had to be the toroid by a process of elimination. In **question (ii)** sketches were in the right direction, but none of the students expected **A** to increase linearly with z; some were disturbed by this. Some students get confused by the absolute values and thought that **A** switched direction as it crossed **J**. Upon questioning they realized that through symmetry arguments it shouldn’t change direction . In **question (iii)**, one student realized (mathematically) that although **A** pointed in the same direction as J in the examples above, it was not necessarily so here.

**Relevant Homework Problems**

**Vector potential II**

A) Griffiths Fig 5.48 (p. 240) is a nice, and handy, "triangle" summarizing the mathematical connections between **J**, **A**, and **B** (like Fig. 2.35 on p. 87) But there's a missing link, he has nothing for the left arrow from **B** to **A**. Notice that the *equations* defining **A** are really very analogous to the basic Maxwell's equations for **B**:



So **A** depends on **B** in the same way (mathematically) the **B** depends on **J**. (Think Biot-Savart.) Use this idea to just write down a formula for **A** in terms of **B** to finish off that triangle.

B) We know the B-field everywhere inside and outside an infinite solenoid (which can be thought of as either a solenoid with current per length n I or a cylinder with surface current density K = n I ). Use the basic idea from part (a) to quickly and easily write down the vector potential **A** in a situation where **B** looks analogous to that, i.e. , with C constant. (Sketch this **A** for us, please)  *(You should be able to just see the answer; no nasty integral needed.)*  *It's kind of cool - think about what's going on here. You have a previously solved problem, where a given* ***J*** *led us to some* ***B****. Now we immediately know what* ***A*** *is in a* very different *physical situation, one where* ***B*** *happens to look like* ***J*** *did in that previous problem.*

**✯ TUTORIAL 10: VISUALIZING VECTOR POTENTIAL ✯**

Part 1 – Sketching Vector Potential

One of Maxwell’s equations,, made it useful for us to define a scalar potential V, where . Similarly, another one of Maxwell’s equations makes it useful for us to define the vector potential, **A**.

1. Which of Maxwell equations makes it useful for us to define **A**? Explain.
2. What current density **J** would create the **B**-field (uniform within a cylindrical volume) in Figure 1 below? Can you write an explicit mathematical formula for it?



1. Notice that the equations defining **A** are mathematically analogous to Maxwell's equations for **B**: 

(Coulomb gauge)

Sketch **B** in Figure 2 (note this is a “cylindrical” volume with uniform **J**). Then, using the mathematical similarities above, sketch **A** in Figure 3:

*Side view:*

*Side view:*









Figure 2: Given **J**, sketch the **B** field.

Figure 3: Given **B**, sketch the **A** field.

s

|B|

s

|A|

1. One way to check your previous answer (conceptually) is using an Ampere’s Law analogy. Ampere’s Law tells you that the **J**-flux (or Iencl) is equal to . What is a similar relationship between the vector potential and magnetic field?

Try using this “Ampere’s Law analogy” to check your sketch of **A**.

1. A toroidal wire coil looks like a doughnut wrapped with wire, as shown in Fig. 4. On the “blank” toroid, indicate the direction of **J**, then sketch the **B** and **A** fields.

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Part 2 – Calculating Vector Potential

In a common homework question, you are asked to calculate the magnetic field produced by a uniform surface current: . The answer you may have calculated is:





1. Which of the following practical devices can be modeled by the currents or fields shown in figures 2 through 5?

A. Tokamak B. Solenoid C. Ribbon conductor D. Wire of finite diameter

1. Sketch your best guess of what **A** looks like for the uniform surface current. Which components (x, y, or z) does **A** have (it might help to look at relationship between **A**, **B**, and **J** in the two examples in Part 1)? Which variables (x, y, or z) does **A** depend on?

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1. Using your assumption for which components **A** has, and which variables **A** depends on, calculate (or guess) what **A** is. Check your answer by calculating the **B** field from your vector potential **A**.
2. Sketch vector potential **A** you calculated (or guessed) in part iii. Does your new sketch of **A** agree with your guess from part ii?

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