

 **INSTRUCTORS MANUAL: TUTORIAL 2**

Goals:

1. Understand what delta functions are, how to integrate them, and use them to describe charge distributions.
2. Describe/interpret/represent a physical situation mathematically (course scale learning goal 1 and 2)
3. Check answer using dimensional analysis (course scale learning goal 7)
4. Invoke sophisticated symmetry arguments to predict an answer (course scale learning goals 5b and 7)
5. Learn how to apply Gauss's Law to a physical situation (course scale learning goals 1, 5c, and 9)
6. Communicate reasoning/thought process to group members/LA/Instructor (course scale learning goal 4)

This tutorial is based on:

- Written by Darren Tarshis, Steven Pollock, and Stephanie Chasteen, with modifications by Ed Kinney, Michael Dubson and Markus Atkinson.

Tutorial Summary:

Students start with basic delta function properties, then sketch a charge density described by a delta function. Next, they describe the charge density of a charged tube (SLAC) using a delta function. They check this answer in various ways. Then, symmetry is used to figure out which way the E-field points before using Gauss's Law to calculate it. As a challenge problem, they figure out where in space there is a divergence.

Homework connections:

- At the end of part 2iii there is a comment, "Note: This is very similar to the E-field produced by *just* the outer shell of the coax cable." If "Electric field of coaxial cable" homework question or one similar to it is assigned, this note can be modified to refer to the homework question by adding at the end "on HW3 Q5," for instance.
- At the end of part 3i something similar to the following can be added "(Note: this is similar to HW3 Q2b)." This would refer to part b of "Delta functions and charge distributions" homework question or a similar question.

Reflections on this tutorial:

The Learning Assistant commented that students appeared to benefit from working through this. There were several comments made by students about how clarifying parts of this tutorial were for them. The tutorial may be a little long. This tutorial introduced the *application* of Gauss's Law prior to being covered in lecture.

Part 1 Students were able to sketch without problems because this was covered in lecture the same day. Students thought that the charge distribution in **question (iii)** described a point charge. The instructor reminded them that x , y , and z all need to be considered, and asked students if there were any constraints on y and z . This clarified that this must be a plane. One student commented while working on this part, "I feel like I understand delta functions so much better now, even though we've used them in other classes."

Part 2 At first, students just "knew" which way E pointed. But when questioned, they couldn't explain why. They argued that it was symmetry "because it's pretty much infinite so there's the same amount of charge on one side as the other." But they lacked a convincing argument. Several students seemed very satisfied after we discussed the symmetry of the situation, and why the E -field could only point in the s -direction. Many students did not know which form of Gauss's Law to use here. To help them decide, they were asked which form of Gauss's Law would lend itself to a closed imaginary surface to integrate over.

Part 3. Determining the volume charge density of the beam line was the most challenging part of the tutorial. Every group asked questions about this, and many were the same. The most common question was, "So the surface charge is kind of 2-dimensional, but volume charge density is 3-D. How do you change something from 2-D to 3-D?" The best way to guide students away from their confusion was to reference part 1, question (iii.). We told them that on that problem, they were given a charge density, and they sketched the physical arrangement (translate math to physics). Now, they are given the physical arrangement, and they need to describe the charge density mathematically. For students concerned with going from 2-D to 3-D, it might be helpful to point out that on part 1, question (iii.), they were given a 3-D charge density, but sketched a 2-D surface of charge. One or two students wondered, "Would the charge distribute itself evenly over the beam line?" We pointed out that it was metal, and the student was still not completely convinced that it would be a uniform charge distribution. He seemed to not understand how charge behaves in a conductor. It was very surprising to see this type of confusion at this level. In **question (iii)** we did have to guide several students to set up the integral properly. But once they integrated, they regained confidence in their answer to question (i) after seeing that the charge-per-area time the area equaled charge.

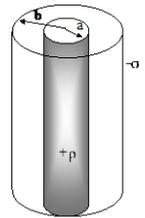
Relevant Homework problems

Delta functions and charge distributions

- a) On the previous homework we had two point charges: $+3q$ located at $x=-D$, and $-q$ located at $x=+D$. Write an expression for the *volume* charge density $\rho(\mathbf{r})$ of this system of charges.
- b) On the previous homework we had another problem with a spherical surface of radius R (Fig 2.11 in Griffiths) which carried a uniform surface charge density σ . Write an expression for the *volume* charge density $\rho(\mathbf{r})$ of this charge distribution.
- c) Suppose a linear charge density is given to you as $\lambda(x) = q_0\delta(x) + 4q_0\delta(x-1)$. Describe in words what this charge distribution looks like. How much total charge do we have? Assuming that q_0 is given in Coulombs, what are the *units* of all other symbols in this equation (including, specifically the symbols λ , x , the delta function itself).

Electric field of coaxial cable

- a) A long coaxial cable carries a uniform volume charge density ρ throughout an inner cylinder (radius a) and a uniform surface charge density σ on the outer cylinder (radius b). The cable is overall electrically neutral. Find \mathbf{E} everywhere in space, and sketch it.

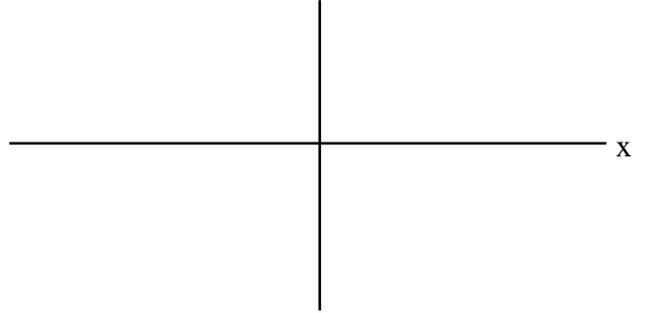


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Part 1 - Delta Joy

i. Sketch: $f(x) = \delta(x - 3)$

(can you *really* sketch this?)



ii. Integrate using $f(x)$ from part (i.):

$$\int_{-10}^{10} b \cdot f(x) \cdot dx =$$

$$\int_{-\infty}^{\infty} b \cdot x \cdot f(x) \cdot dx =$$

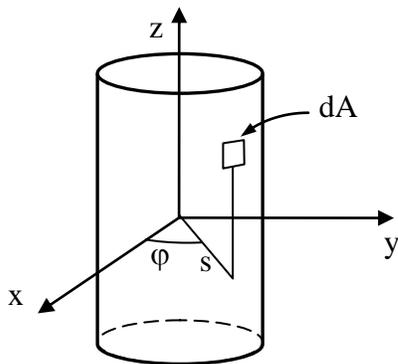
iii. For the volume charge density $\rho(x, y, z) = b \cdot \delta(x - 3)$ what are the units of b ? What is the physical situation represented by this volume charge density? Make a 3-D sketch of the charge distribution.

Part 2 – SLAC Lightning Strike

SLAC (Stanford Linear Accelerator Center) is where quarks (including the charm quark), and the tauon (like a heavier electron) were discovered. Charged particles are accelerated inside a long metal cylindrical pipe, which is 2 miles long and has a radius $R = 6$ cm. All the air is pumped out of this pipe, known as the “beam line.”

One afternoon, the beam line is struck by lightning, which gives it a uniform surface charge density σ . After the lightning strike, Stanford physicists want to start accelerating particles in the beam line, but they are concerned that the charge density might affect the beam particles, causing them to crash into the wall of the pipe and burn a hole through it. Air and dirt would rush into the empty pipe causing months of expensive delay. You will investigate whether the surface charge of the beam line could affect the beam particles.

- i. First, what is the infinitesimal area, dA , of a small patch on a cylindrical shell centered on the z -axis? Assuming you use this dA in a surface integral over a closed surface, give the vector direction of dA .



- ii. What direction does the E -field point at all points in space? Explain in detail how you know.

TUTORIAL 2: SLAC LIGHTNING STRIKE

iii. Use Gauss's Law to find the E-field at all points in space.

(Note: This is very similar to the E-field produced by *just* the outer shell of the coax cable.)

iv. Does the charge σ on the beam line affect the particles being accelerated inside it? Could it affect the electronic equipment outside the tunnel?

