

 **INSTRUCTORS MANUAL: TUTORIAL 3**

Goals:

1. Gain confidence with divergence and curl, and understand what they mean physically (Learning Goal 9)
2. See a relationship between mathematical representation of E-field and graphical (Learning Goal 2)
3. Understand what charge distribution causes an electric field (Learning Goal 1)
4. To decide if a mathematically plausible field is physically possible as well. (Learning Goal 1)
5. Path independence of voltage difference (Learning Goal 5c)
6. How to choose a “good” voltage reference point (Learning Goal 5c)
7. See the connection between calculating potential difference and measuring it with a voltmeter. (Learning Goal 1)
8. Apply physics knowledge to a real-world example.
9. Estimate order of magnitude of a real-world problem (Learning Goal 5a)

This tutorial is based on:

- Oregon State University “Measuring Voltage” activity – Corinne Manogue
- Written by Darren Tarshis, Steven Pollock, and Stephanie Chasteen, with modifications by Ed Kinney, Michael Dubson and Markus Atkinson.

Materials needed:

Jacob’s ladder demonstration (could watch a video of a Jacob’s ladder if demo is not available – youtube.com has several examples).

Special notes:

After the students work on the tutorial for 45 minutes, they should shift their attention to the challenge problem. They are then shown a Jacob’s ladder demonstration.

Reflections on this Tutorial

Part 1 The students had no trouble determining the field outside in **question (i)**, but the internal field gave them some trouble. Students were quick to realize that Gauss's Law should work, but the problem came when determining the enclosed charge. In **questions (iii) and (iv)** most students remembered that the curl of E was zero but not the expression for the divergence of E . Most students tried to directly calculate it from their expression for the E -field; very few students invoked Maxwell's Equation. The students had a lot of trouble with the path integrals in **question (v) and (vi)**. When infinity was the reference point, the students had little trouble getting V outside the sphere. When finding V inside the sphere, most students only integrated over the field of the sphere lying within that point, forgetting that they needed to integrate to the edge of the sphere from infinity. Similar problems happened when the origin was the reference point.

Part 2 Most students had no trouble drawing the field. But again, the path integrals were troublesome for the students. Most students were aware that the answer doesn't depend on which path is chosen so they didn't bother to calculate the second path and just said it is the same. It was very hard to motivate them to attempt the second path as they were bored of doing path integrals at this point.

Part III – Jacob's Ladder

After calculating voltage on the tutorial, we want students to estimate how large of a potential difference would actually be required to see sparks (15,000 V). More importantly, they should see that what they've just been working on can be applied to the real world.

The voltage estimate was not too difficult for the students, and it was nice that as a check, the voltage was printed on the Jacob's ladder. Almost everyone knew about the step-up transformer. Some of the students knew why the spark rises, but used very technical terms to explain it. In this situation, the instructor can ask students to describe it as they would to a 4th grader. This is good practice in articulating scientific explanations, which is an important skill and is also a good check whether students understood the important concept. Not every student understood why the spark rises. One student gave a great reason why it makes sound: the spark heats the air, and pressure is proportional to temperature, so the change in pressure is what you hear as sound.

Relevant Homework Problems

Divergence and Curl

Consider an electric field $\mathbf{E} = c \frac{\vec{\mathbf{r}}}{r^2}$ (Please note the numerator is not $\hat{\mathbf{r}}$: this is NOT the usual E field from a point charge at the origin, which would give $c' \frac{\vec{\mathbf{r}}}{r^3}$, right?!)

- a) - Calculate the divergence *and* the curl of this \mathbf{E} field.
- Explicitly test your answer for the divergence by using the divergence theorem. (Is there a delta function at the origin like there was for a point charge field, or not?)
- Explicitly test your answer for the curl by using the formula given in Griffiths problem 1.60b, page 56.
- b) What are the units of c ? What charge distribution would you need to produce an E field like this? Describe it in words as well as formulas. (Is it physically realizable?)

Allowed E fields

Which of the following two static E-fields is physically *impossible*. Why?

i) $\mathbf{E} = c(2x\hat{\mathbf{i}} - x\hat{\mathbf{j}} + y\hat{\mathbf{k}})$

ii) $\mathbf{E} = c(2x\hat{\mathbf{i}} + z\hat{\mathbf{j}} + y\hat{\mathbf{k}})$

where c is a constant (with appropriate units)

For the one which IS possible, find the potential $V(r)$, using the *origin* as your reference point (i.e. setting $V(0)=0$)

- Check your answer by explicitly computing the gradient of V .

★ TUTORIAL 3: ELECTRIC POTENTIAL ★

Part 1 – Electric Fields and Potentials

A. The Uniformly Charged (Solid) Sphere

i. Derive the electric field from a uniformly charged **sphere** with total charge Q and radius R , centered at the origin. Make a sketch of the magnitude of the field as a function of the radial distance r for $\theta = \pi/4$ and $\phi = \pi/6$.

ii. Make a vector field sketch of the electric field.

iii. Considering all space, where is the curl of \mathbf{E} equal to zero for this charge distribution?

What does this say about $\oint \vec{E} \cdot d\vec{L}$?

iv. Considering all space, where is $\vec{\nabla} \cdot \vec{E} = 0$ for this charge distribution?

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v. Using the definition of voltage (potential), $V = -\int \vec{E} \cdot d\vec{L}$, find V for this distribution, in all regions of space ($r < R$ and $r > R$) with the **choice of zero voltage at $r = \text{infinity}$** . Sketch V as a function of r . Compare your sketches of $E(r)$ and $V(r)$; how should they be related?

vi. Now using **the origin** as the point of zero voltage, sketch this V as a function of r .

vii. Explain how the choice of zero voltage at the origin changes the electric field.

Part 2 –Electric Fields

i. Sketch the field vectors for the field $\vec{E} = y \hat{x} + x \hat{y}$.

ii. What charge distribution would give you the \vec{E} from part (i.)? Is this E-field physically plausible?

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iii. Calculate for the electric field in part (i.) the potential difference you would expect to measure between two arbitrary points $(x_1, y_1, 0)$ and $(x_2, y_2, 0)$ for this electric field.

Choose two different paths to integrate over, and compare these answers:

- 1) a segment from x_1 to x_2 at y_1 followed by a segment from y_1 to y_2 at x_2 ,
- 2) a segment from y_1 to y_2 at x_1 followed by a segment from x_1 to x_2 at y_2 .

iv. Are the two path integrals from part (iii.) equal? Should they be?

(Note that when you do a line integral $\int \vec{E} \cdot d\vec{l}$ this is over a *specific* path! It doesn't matter which path, but you must choose a path.)

Part 3 – Jacob’s ladder

A Jacob’s ladder is a continuous spark that rises between two conductors. (The dielectric strength of air is about three million Newtons per Coulomb). We will all watch a real Jacob’s ladder then each group will choose one of the questions below and take 5 minutes to discuss the question and find an answer. One representative from each group will present the results of the group to the class.

- 1) Estimate the voltage required for a Jacob’s ladder to spark. Since it’s plugged into the wall (120V AC), how is the voltage you estimated possible?
- 2) What is the potential difference between the two conductors when they are closest together? What about when they are farthest apart?
- 3) Why does the spark initially start at the bottom?
- 4) Why does the spark rise?
- 5) Why do you hear it?