

 **INSTRUCTORS MANUAL: TUTORIAL 7**  
***Polarization & Bound Charge***

**Goals:**

1. Use different models to visualize bound charge conceptually (learning goal 2)
2. Visualize polarization and be able to relate it mathematically to different physical parameters in the problem (learning goals 1 and 3)
3. Relate conceptual understanding of bound charge and polarization to mathematical formalism (learning goal 1)
4. Use limiting cases and sketches to understand the final calculated answer (learning goals 5, 7).
5. Communicate reasoning/thought process to group members, LA, and Instructor (learning goal 4)

**This tutorial is based on:**

- Griffiths by inquiry 7
- Written by Darren Tarshis, Steven Pollock, and Stephanie Chasteen, with modifications by Ed Kinney, Michael Dubson, Rachel Pepper and Markus Atkinson.

**Materials needed:**

Charged rod, wash bottle or some other apparatus to create a thin stream, water, oil, basin.

**Tutorial Summary:**

Students use an atomic model to understand bound charge. Students use the concepts of polarization and bound charge to determine the electric field inside a slab of plastic that is between the plates of a capacitor.

**Some reflections on this tutorial:**

**Part 1** The first few questions went quickly. All of the students got the main idea that the dielectric material would separate into a thin layer of positive and negative charge with a neutral region between them. The students had a little difficulty with **question (v)** when the problem began to talk about the polarization. Students had trouble remembering that the polarization points from the negative to the positive charges. Students knew that the charge on the bottom would be equal and opposite the charge on the top, but they had a little trouble getting all of the math to agree with their insights.

**Part 2** The students didn't bother to write down all three versions of the Electric field in **question (i)** feeling that one version was enough and moved on. The students had a lot of trouble with **question (iii)**. Most recognized that they needed to give a relationship between the forces, but were unsure whether to use  $E_{\text{Total}}$  or  $E_{\text{External}}$  in the force equation. In **question (iv)**, most the students had to go back and find  $E_{\text{Induced}}$  in terms of  $\Delta z$  as this one not the one that they had chose to solve for the first time around.

The correct answer is  $E_{\text{tot}} = \frac{E_{\text{ext}}}{1 + (nq^2 / k\epsilon)}$ .

**Part 3** **Question (i)** really cleared things up for the students once they figured out what  $k$  was in this situation. **Question (ii)** led to a lot of good discussion. The students did not, generally, understand why the D field is useful to calculate. The last two questions were really quick for the students.

**Demo** The students were surprised by this demo. While every student knew that the water would be affected by the charged rods and the oil wouldn't, most students thought that a positively charged rod would attract the water while a negatively charged rod would repel it. They were surprised to learn that the water is always attracted to the rod regardless of what sign the charge is on it. This led to a lot of good discussion about torque and randomness of particles.

## Relevant Homework Problems

### Bound charges I

Consider a long insulating rod, (a dielectric cylinder), radius  $a$ . Suppose that the rod has no free charge but has a *permanent* polarization  $\mathbf{P}(s,\theta,z) = C \mathbf{s}$  ( $= C s \hat{\mathbf{s}}$ ), where  $\mathbf{s}$  is the usual cylindrical radial vector from the  $z$ -axis, and  $C$  is a positive constant). Neglect end effects: the cylinder is long.

- A) Calculate the bound charges  $\sigma_b$  and  $\rho_b$  (on the surface, and interior of the rod respectively). What are the units of  $C$ ? Sketch the charge distribution of the rod.
- B) Next, use these bound charges (along with Gauss' law) to find the electric field inside and outside the cylinder. (Direction and magnitude)
- C) Find the electric displacement field  $\mathbf{D}$  inside and outside the cylinder, and verify that "Gauss's Law for  $\mathbf{D}$ " (Eqn 4.23, p. 176) works.

### Bound charges II

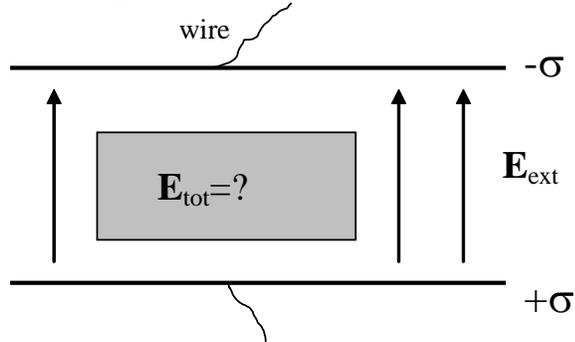
Consider now a hollow insulating rod, with inner radius  $a$  and outer radius  $b$ . Suppose now that the rod has a different permanent polarization, namely  $\mathbf{P}(s,\theta,z) = \frac{C}{s^2} \hat{\mathbf{s}}$  for  $a < s < b$ ,  $C =$  positive constant (not the same constant as in Q1).

- A) We have vacuum for  $s < a$  and  $s > b$ . What does that tell you about  $\mathbf{P}$  in those regions? Find the bound charges  $\sigma_b$  and  $\rho_b$  ( $\sigma_b$  on the inner AND outer surfaces of the hollow rod, and  $\rho_b$  everywhere else. Use these bound charges, along with Gauss' law, to find the electric field everywhere in space. (Direction and magnitude)
- B) Use Griffiths' Eq 4.23 (p. 176) to find  $\mathbf{D}$  everywhere in space. (This should be quick - are there any *free* charges in this problem?) Use this (simple) result for  $\mathbf{D}$  (along with Griffiths basic definition/relation of  $\mathbf{E}$  to  $\mathbf{D}$ , Eq 4.21) to find  $\mathbf{E}$  everywhere in space.

**★ TUTORIAL 7: SQUINTING CLOSELY AT PLASTIC ★**  
*Polarization & Bound Charge*

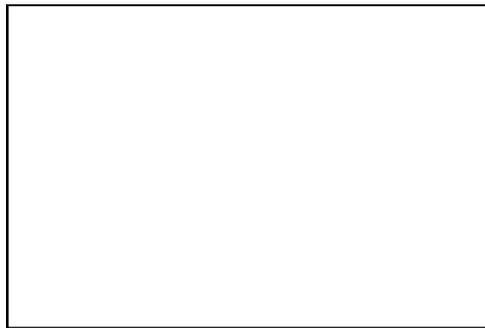
**Part 1 – Polarization and Bound Charge**

A slab of plastic is placed within a charged capacitor. Before inserting the plastic, there is a uniform electric field inside the capacitor,  $E_{ext}$ . We will explore the properties of a dielectric to eventually find the electric field inside the plastic.



i. You can think about the microscopic picture inside the plastic as looking like many negative charges " $-q$ ", each bound to a fixed positive core " $+q$ ", with a spring constant  $k$ . Draw a "zoomed-in" sketch (in the box to the left) of the top few rows of atoms inside the plastic, when the plastic is inside the capacitor.

ii. Unfortunately if we stick too closely to the atomic model, we will have to deal with the details of the quite complicated electric field inside the plastic. It's far easier to go over to our smooth continuous charge distribution model of the matter with a positive charge density  $\rho^+$  and negative charge density  $\rho^-$  occupying the volume of the plastic. Normally they add together to give net charge density of zero, but when the atoms are polarized, the



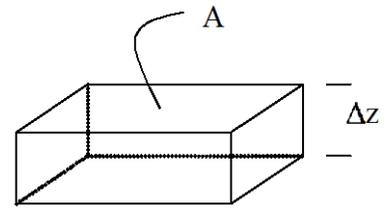
two distributions are shifted by a distance  $\Delta z$ , which is the same stretch of the "springs" of our simple atomic model. Using the continuous model of charge density, again draw the charge distribution in the plastic when it is inside the capacitor. Indicate where the net charge is negative, zero, or positive.

## TUTORIAL 7: SQUINTING CLOSELY AT PLASTIC

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iii. If there are  $n$  atoms per unit volume, each with a  $+q$  and a  $-q$  charge, what values of  $\rho^+$  and  $\rho^-$  should we use in our continuous charge density model? What is the dipole moment,  $\mathbf{p}$ , of a single atom?

iv. In part ii, you probably found, on both the top and bottom of the plastic, a layer of non-zero net charge (if you didn't, why not?) of thickness  $\Delta z$ . Using the continuous model, what is the total charge enclosed in the top volume of height  $\Delta z$ , and area  $A$ ?



v. Using your expression for the charge enclosed, what is the surface-charge density? This is called the bound-charge density,  $\sigma_B$ . Once you have  $\sigma_B$ , look for ways to simplify your expression as much as possible (using identities we've recently covered in class,  $\mathbf{p}$  and  $\mathbf{P}$ ). *Hint: it may help to consider the dipole moment for a single atom from part iii.*

vi. Given the definition of  $\mathbf{P}$ , what is the direction of  $\mathbf{P}$  in this case? Give the direction both inside and outside the plastic.

vii. Now, check to see if your expression for  $\sigma_B$  is consistent with the formula:

$\sigma_B = \bar{P} \cdot \hat{n}$  on the top and bottom of the slab.

viii. What is  $\sigma_B$  on the *side* surface of the plastic slab? Is this consistent with the

formula:  $\sigma_B = \bar{P} \cdot \hat{n}$  ?

## Part 2 – Electric Fields in Dielectric Materials (or Insulators)

i. What is the magnitude of the induced electric field,  $\mathbf{E}_{\text{ind}}$ , inside the plastic slab?

Express it in three ways:

1. in terms of  $\sigma_B$ , the bound charge

Now use this expression to find  $\mathbf{E}_{\text{ind}}$

2. in terms of  $\mathbf{P}$ , the polarization

3. in terms of  $\Delta z$ , the distance the atom “stretches”

In each case, are any other “givens” (i.e.  $\mathbf{E}_{\text{ext}}$ ,  $n$ ,  $\epsilon_0$ ,  $q$ ) needed?

ii. What is the magnitude of the total electric field inside the plastic slab,  $|\mathbf{E}_{\text{tot}}|$ , in terms of the magnitude of the induced field,  $|\mathbf{E}_{\text{ind}}|$ , and the magnitude of the external field of the capacitor,  $|\mathbf{E}_{\text{ext}}|$ ?

iii. Given a spring constant  $k$ , find  $\Delta z$  by once again returning to the atomic model.

iv. Now you have everything you need to write down  $\mathbf{E}_{\text{tot}}$ , the electric field inside the plastic. Make sure you express it using only the “givens” ( $\mathbf{E}_{\text{ext}}$ ,  $n$ ,  $\epsilon_0$ ,  $q$ ,  $k$ ).

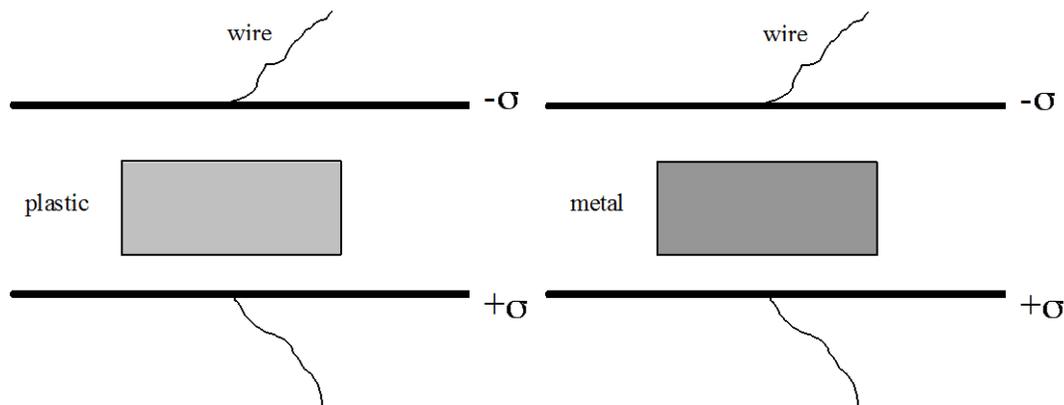
### Part 3 – Making Sense of the Answer

It’s a good idea to always use a limiting case to check if your answer makes sense.

i. What physical situation would you model as an “ultra” strong spring? What is  $k$  in that situation? What is  $\mathbf{E}_{\text{tot}}$ ? Does that make sense? How about for an “ultra” weak spring? Does your limit for  $\mathbf{E}_{\text{tot}}$  make sense?

ii. Griffiths' equation 4.21 states:  $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ . In this tutorial, we've had three  $\mathbf{E}$ 's. Which one does Griffiths mean in this equation? Is this consistent with saying that “ $\mathbf{D}$  arises from free charge”?

iii. Sketch the electric field strength and any bound charges everywhere inside the capacitor plates for: 1. the plastic slab; 2. same size chunk of metal



iv. In a linear dielectric (such as plastic), can  $|\mathbf{E}_{\text{ind}}|$  ever be larger than  $|\mathbf{E}_{\text{ext}}|$ ? Check your answer to Part II (iv).

**Demo:** Water has a dielectric constant so large it acts almost like a conductor. If we apply a non-uniform electric field to either an oil stream (“normal” dielectric) or a water stream (conductor-like), which feels a bigger force? Is this consistent with your answer to iii?