***👓 INSTRUCTORS MANUAL: TUTORIAL REVIEW 2***

***Separation of Variables, Multipole Expansion, Polarization***

**Goals**:

To revisit the topics covered in the previous 4 weeks of tutorials and cement concepts prior to the exam

**Timing:**

This review may fall after tutorial 7, 8, or 9 depending on the semester and the midterm exam schedule.

**Reflections on this tutorial**

**Part 1** This problem took the students a lot of time considering this was the second time they had done this problem. Students had trouble applying the boundary conditions correctly and had difficulty figuring out which direction had waves as a solution and which one had exponentials. Applying Fourier's trick in **question(iii)** was still difficult for some students.

**Part 2** Once again the Taylor series expansion proved to be the real challenge. A lot of students remembered this problem from the previous weeks and knew how everything was supposed to relate to the multi-pole expansion but just could not figure out how to take the Taylor expansion. Similar problems came up again this week. Students could not figure out what x0 and ε were supposed to be. **Question(v)** also proved to be challenging once again. Some students chose by pattern matching and needed instructor guidance to think through their choices.

**Part 3** This section was put at the end as Polarization was covered just one week earlier. Most students did not make it this far having taking a lot of time reviewing the earlier material. One suggestion is to tell the students to do this tutorial in the order they feel is best.

**✯ TUTORIAL REVIEW 2 ✯**

**Separation of Variables; Multipole Expansion; Polarization**

***Please do this tutorial in whatever order you think would help you most. You can start with Part 1, Part 2 or Part 3.***

**Part 1: Laplace’s Equation and Separation of Variables**

Within a very long, rectangular, hollow pipe, there are no electric charges. The walls of this pipe are kept at a known voltage (they are known because in a lab, you can control them).

Three of the walls are grounded:; ; 

The fourth wall maintains a potential that varies with y:  which will be specified later.



In order to find out the voltage inside the pipe, you will need to find a solution to Laplace’s equation:



 i. Inspect the boundary conditions and choose which type of function will be used for each variable (*x*, *y*, and *z*). For each choice, identify whether the separation constant is positive, negative, or zero.

ii. Apply boundary conditions #1-3 (the three grounded walls) and find the general solution to Laplace’s equation which satisfies these 3 boundary conditions.

iii. The fourth wall is maintained at a constant voltage: . Does your solution above satisfy this boundary condition? If not, will a sum of your solutions satisfy the boundary condition? If so, give a formal expression for this sum. If the sum involves coefficients for the individual solution terms, give a formula for the coefficients. Any integrals should be in a form ready for immediate integration (have limits and explicit variables and functions).

**Part 2 – Potential from a Line of Charge**

A uniform line charge density λ extends from the origin to the point (0,0,-d).

Using the script-r technique from earlier in the course, you can find an expression for the potential V(z) for points on the positive z-axis (z>0), Remember: 

Your answer would be: . Don’t take time now to derive this solution (but be sure you can!)

i. If you were now asked to expand the potential, V(z), into a Taylor series for the case z>>d, using the general formula below, what would you use for f(x), x0 and ε? Recall the Taylor expansion about the point : 

In this case the expansion is 

ii. Use this to expand the potential V(z).

iii. For this charge distribution, calculate the monopole and dipole moments. Recall that the dipole moment for a continuous distribution is . Identify which, if any, moments depend on the choice of origin.

iv. A potential can be expanded into the multipole form:

. Write the first two terms of the multipole expansion using the monopole and dipole moments you just calculated. Does the expansion agree with the Taylor series from part 2.i on the *z*-axis? Recall that the numerator “*dipole”* = .

v. Without recalculating the potential using the script-r integral, but instead thinking about the moments calculated in part 2.iii, how would the potential change if the line charge distribution were shifted up by d/2, so it was centered at the origin?

**Part 3 – The Dielectric Sandwich**

An infinite plane of charge with surface charge density σf is between two infinite slabs of neutral linear dielectric (of dielectric constant ε and susceptibility χ), as shown. If the dielectric slabs were not present, the "bare" E-field, due only to the plane of free charge, would have magnitude Eo = σf / 2ε0.

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i. On the figure above make a simple sketch of any bound charge distributions you expect (don’t perform any calculation yet).

ii. Again without calculation, sketch your expectation of the polarization  everywhere, using little arrows.

iii. Does this arrangement of free charge and dielectric allow us to determine  using Gauss’ law for the *D* field? Why or why not?

iv. Assuming that you agree that  can be determined using Gauss’ law, find  everywhere in space.

v. Now find  and  everywhere in space.

vi. How does “outside” the sandwich compare with the “bare” E field (no dielectric slabs present)? Does your answer for  agree with your expectation from part 3.ii?

vii. Calculate the bound charge distributions using  and . Do your distributions agree with your expectation from part 3.i?

viii) What do you expect for the boundary conditions on V and at the dielectric surfaces? How did you decide?

ix) Do your calculations for  and  give a V that satisfies these boundary conditions?