

 **INSTRUCTORS MANUAL: TUTORIAL 8**  
***Spherical Linear Dielectric***

**Goals:**

1. Use different models to visualize bound charge conceptually (learning goal 2)
2. Visualize polarization and be able to relate it mathematically to different physical parameters in the problem (learning goals 1 and 3)
3. Relate conceptual understanding of bound charge and polarization to mathematical formalism (learning goal 1)
4. Use limiting cases and sketches to understand the final calculated answer (learning goals 5, 7).
5. Communicate reasoning/thought process to group members, LA, and Instructor (learning goal 4)
6. Recognize symmetries and be able to take advantage of them in order to choose the appropriate method for solving a problem (learning goal 5).
7. Apply the fundamental theorem for curls (Stoke's Theorem) in specific situations (calculation and computation learning goal).

**This tutorial is based on:**

- Homework problem from Uriel Nauenberg.
- Written by Rachel Pepper and Steven Pollock.

**Materials needed:**

none

**Tutorial Summary:**

Students first predict and then calculate the E-field, D-field, and polarization of a spherical conductor surrounded by a spherical conducting shell with the space in between half air and half dielectric. The E-field is, surprisingly, constant inside the cavity, despite the presence of the dielectric.

### **Some reflections on this tutorial:**

Students seemed to really enjoy this tutorial – thought it could use to be shortened. Luckily there is no “punch line” so not finishing is not a big problem. Ideally students will get through at least parts 1 and 2.

**Part 1:** i) Students may not draw both E and sigma as prompted, so instructor may want to encourage drawing the other one if it is missing. Part ii is good practice with Stokes loops but students often did not know that loops should integrate to zero, and even when they did, needed help from the instructor or LA to go through the correct logical steps to get the answer. In part v some students needed extra prompting to discuss the question of charge distribution.

**Part 2:** In part i. students may often did not think to use Gauss’s law. Also, in part ii. the students need  $\vec{P} = \epsilon_0 \chi_e \vec{E}$ , which they may not remember, and may need to be given. Some students use  $P = \sigma_b \cdot r\text{-hat}$  here, and solve in terms of Q and Q\_eff. The instructor may want to guide students to calculate a P that is valid everywhere inside the dielectric.

Make sure students figure out P both inside the dielectric and the air (same for E and D). Students may spend too long on part iv. without thinking of using P, so the instructor might need to give a bit of guidance. When finding the free charge, students may need encouragement to find it on both the upper and lower parts of the sphere/shell.

Part iv and vii took a long time for some groups. A few groups really got stuck going in circles here and may have needed more help. Some possible hints:

iv. Possible hint: Does part ii. help?

vii. Possible hint: Is Q\_eff uniform? If so, does that help you with sigma free for the upper half.

## Relevant Homework Problems

### Griffiths questions 4.36 (p.199)

#### Cylindrical capacitor, bound charges

CALCULATION (from deGrand)

- (a) Find the capacitance per unit length of a long cylinder with an inner conductor of radius  $a$ , an outer conductor of inner radius  $b$  and outer radius  $c$ , a material with dielectric constant  $K$  in between, and a free charge per unit length  $\lambda$  on each conductor (of opposite signs on either). (b) What and where are the polarization charges?

#### Dielectric series capacitor 1

CALCULATION (deGrand)

The dielectric series capacitor is a parallel plate capacitor of surface area  $A$  and thickness  $d = d_1 + d_2$ , between which a dielectric slab of thickness  $d_1$  and constant  $K_1$  and a dielectric slab of thickness  $d_2$  and constant  $K_2$  are inserted. Assume free charges on the top and bottom of  $\pm\sigma$ . Find (a)  $\vec{D}$  in each slab (b)  $\vec{E}$  in each slab (c)  $\vec{P}$  in each slab (d) the potential between the plates (e) the location and the value of all bound charges (f) the capacitance.

#### Dielectric series capacitor 2

CALCULATION (deGrand)

The dielectric series capacitor is a parallel plate capacitor of width  $w$ , depth  $l$ , and thickness  $d$ , between which a dielectric slab of constant  $K$  is inserted for a width  $s$  (see figure). We put a voltage  $V$  between the plates. Find [4 points each] (a)  $\vec{E}$  in each slab (b)  $\vec{D}$  in each slab (c) the free charge density in each region (d) the bound surface charge density in each region (e) the capacitance. Neglect all fringing fields in your calculations.

#### Question 1. Dielectric sphere

GAUSS' LAW; CALCULATION (U. Nauenberg, solutions available)

A sphere of radius "R" is made up of a dielectric material with dielectric constant  $\epsilon$  and contains a uniform free charge density per unit volume  $\rho_f$ .

Show that the potential at the center is given by:

$$\phi(r = 0) = \frac{4\pi}{3} \frac{2\epsilon + 1}{2\epsilon} \rho_f R^2$$

#### Question 2. Dielectric sphere in E field

CALCULATION; BOUNDARY VALUE (U. Nauenberg, HW8, solutions available)

A solid dielectric sphere of radius "a", with dielectric constant  $\epsilon$ , has a uniform free surface charge/unit area  $\sigma$  is placed in an initially uniform electric field  $E_0$  pointing in the z direction.

- (a) Calculate the potential everywhere.  
 (b) Calculate the E field everywhere.

## TUTORIAL 8: Is the sphere half full or half empty?

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(c) Calculate the distribution of the polarization “P” in the sphere and the surface and volume bound charge distribution.

### Question 3. Electric field of dielectric sphere

CALCULATION (From Pollack and Stump, Electromagnetism, Problem 6.23)

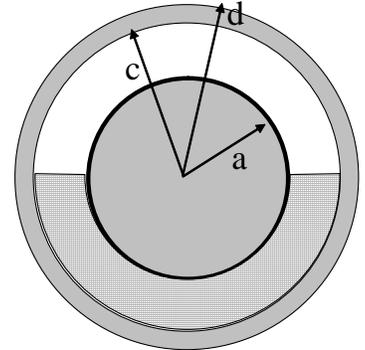
A hollow dielectric sphere, with dielectric constant  $\kappa$ , inner radius  $a$  and outer radius  $b$ , is placed in a uniform applied electric field  $E_0 \hat{z}$ . The presence of the sphere changes the field. Find the field in the three defined regions, i.e.  $r < a$ ,  $a < r < b$  and  $r > b$ . What is the field at the center of the spherical shell? What is the dipole moment of the dielectric medium? [for the last part, you'll find:

$$p_z = \frac{4\pi\epsilon_0 b^3 (a^3 - b^3)(2\kappa + 1)(\kappa + 1)E_0}{2a^3(\kappa - 1)^2 - b^3(2\kappa + 1)(\kappa + 2)}$$

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*Spherical linear dielectric*

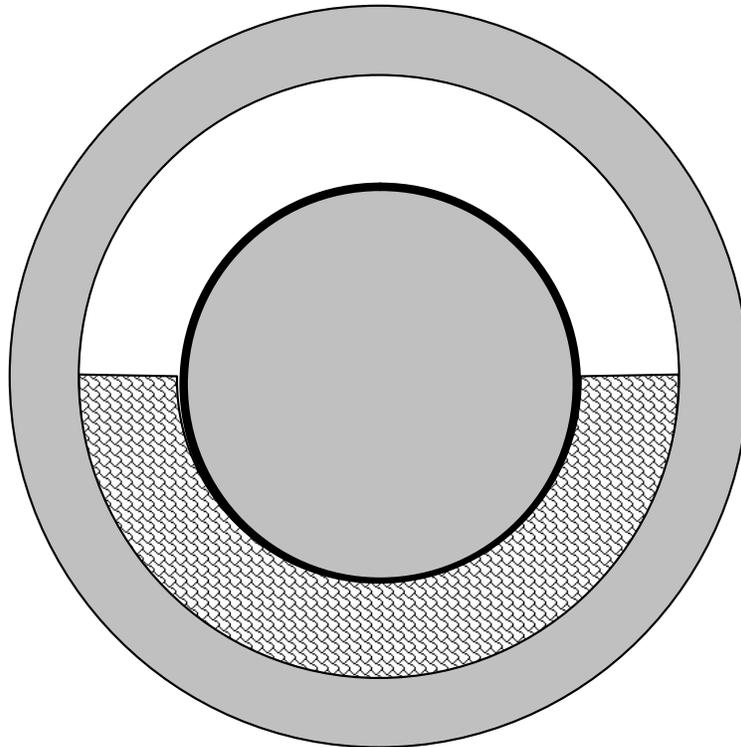
Part 1 – Symmetry and boundary conditions

Consider a conducting spherical shell of radii  $a$  that is concentric with a conducting sphere of radius  $c$  as shown in the figure. The space between them is filled with a liquid having an electric susceptibility  $\chi_e$ . A total charge of “+Q” is placed in the inner conducting shell and “-Q” in the outer shell.



i. Predict where there would be free and bound charge on all surfaces. Sketch your predictions on the diagram below.

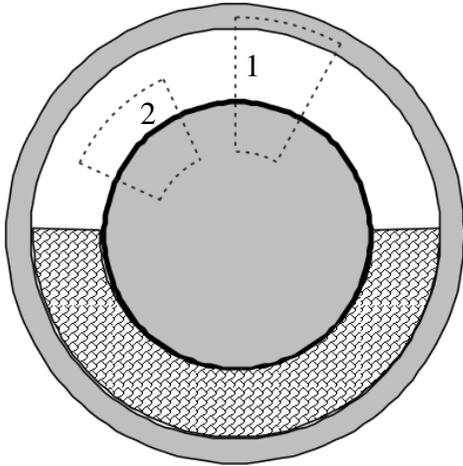
Predict what the total E-field would look like everywhere. Sketch your predictions on the diagram below. Don't worry too much about getting this exactly right, it is just your intuitive guess for now.



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ii. In the empty space between the two conductors, what direction does  $\vec{E}$  point? What variables  $(r, \theta, \phi)$  could  $\vec{E}$  depend on in this region? (You might want to use think about  $\oint \vec{E} \cdot d\vec{\ell}$  for the loops drawn below to help you figure this out).



iii. In the dielectric between the two conductors, what direction does  $\vec{E}$  point? What variables  $(r, \theta, \phi)$  could  $\vec{E}$  depend on in this region?

iv. Is this  $\vec{E}$  the same or different as in part ii? (You might want to think about the boundary condition between the air and the dielectric, or consider drawing more Stokes loops).

v. Imagine some  $Q_{\text{eff}}$  distributed on the surface of the inner conductor which includes both the free charges ( $Q$ ) and some bound charges from the dielectric. How is  $Q_{\text{eff}}$  distributed on the surface of the sphere? Is  $Q_{\text{eff}}$  greater than, less than, or equal to  $Q$ ? How do you know?



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vii. Find  $\sigma_{free}$  everywhere on the inner conductor in terms of  $Q_{eff}$ . Is it safe to assume that  $\sigma_{free}$  is the same on the upper and lower half of the sphere?

viii. Solve for  $Q_{eff}$  in terms of  $Q$  (and other given quantities,  $a$ ,  $c$ ,  $d$ , and  $\chi_e$ ).

ix. Does your answer for  $Q_{eff}$  make sense? Does it match your prediction from Part 1-v.?

Do the limits for large and small  $\chi_e$  make sense?



v. What would change if instead of  $-Q$  on the outer conductor, this conductor was neutral?

vi. Is the sphere half full or half empty?