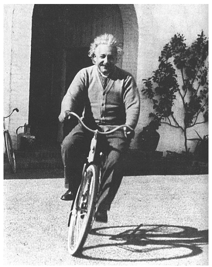
PH300 Modern Physics SP11



"I sometimes ask myself how it came about that I was the one to develop the theory of relativity. The reason, I think, is that a normal adult never stops to think about problems of space and time. These are things which he has thought about as a child. But my intellectual development was retarded, as a result of which I began to wonder about space and time only when I had already grown up."

- Albert Einstein

1/27 Day 5:
Questions?
Time Dilation
Length Contraction

Next Week:
Spacetime
Relativistic Momentum & Energy

1

Last time:
- Galilean relativity
- Michelson-Morley Experiment & Postulates of SR

Today:
- Time dilation, length contraction

Reminder:
HW02 due, beginning of class; HW03 assigned

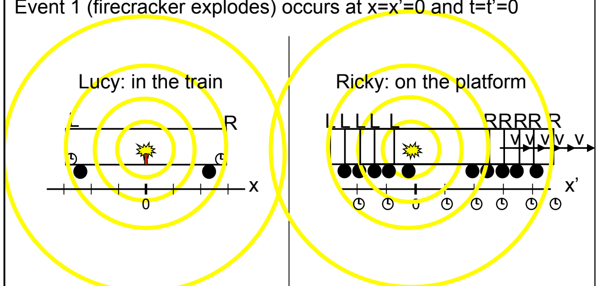
Next week:
Spacetime, relativistic momentum & energy
➔ $E=mc^2$!!

Exam I – Thursday, Feb. 10

2

Recall from last time:

Events are recorded by local observers with synchronized clocks.
Event 1 (firecracker explodes) occurs at x=x'=0 and t=t'=0



Event L: (x=-3, t=3s)	Event R: (x=+3, t=3s)	Event L': (x'=-2, t'=2s)	Event R': (x'=+5, t'=4s)

Lucy says: 'Simultaneous!' Ricky says: 'Not simultaneous!'

Proper Time

…refers to the time measured by a clock in an inertial frame where it is at rest.

Example: Any given clock never moves with respect to itself. It keeps proper time for itself in its own rest frame.

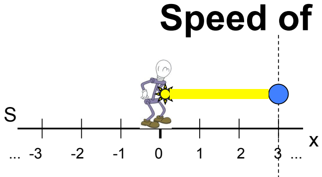
Any observer moving with respect to this clock sees it run slow (i.e., time intervals are longer). This is time dilation.

Same location

Mathematically: Event 1: (x_1,y_1,z_1,t_1)
Event 2: (x_1,y_1,z_1,t_2)

→ Proper time is the shortest time that can be recorded between two events. $\Delta\tau = t_2 - t_1$

Speed of light

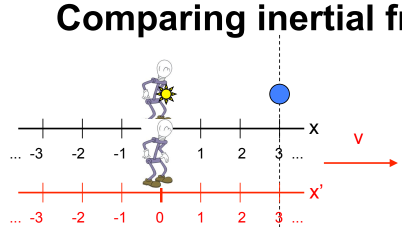


An observer and a ball are at rest in reference frame S. At t = 0, the observer in S flashes a light pulse to be received at x = 3 m.

At Δt = 10 ns, the light is received. Observer S measures a distance Δx = 3 m, so the speed of light in frame S is:

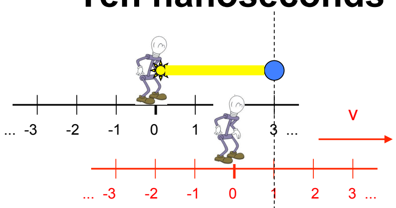
$$c = \frac{\Delta x}{\Delta t} = \frac{3m}{10ns} = 0.3m/ns$$

Comparing inertial frames



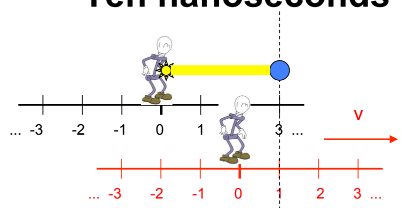
S' is moving with respect to S at v = 0.2 m/ns.
At t = 0, observer in S flashes a light pulse to be received at x = 3 m.

Ten nanoseconds later



S' is moving with respect to S at v = 0.2 m/ns.
At Δt = 10 ns, the light is received. In Galilean relativity, how far does the observer in S' think the light has traveled?
a) 3 m b) 2 m c) 1 m d) 0 m

Ten nanoseconds later



S' is moving with respect to S at v = 0.2 m/ns.
At Δt=10 ns, the light is received. In Galilean relativity, (Δt=Δt') the observer in S' would therefore measure the speed of light as

$$c = \frac{\Delta x'}{\Delta t'} = \frac{1m}{10ns} = 0.1m/ns$$ Uh-oh!

Uh-oh!

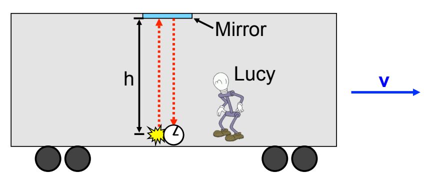
If we are to believe Einstein's postulate, then:

In frame S $\quad c = \dfrac{\Delta x}{\Delta t}$

In frame S' $\quad c = \dfrac{\Delta x'}{\Delta t'}$

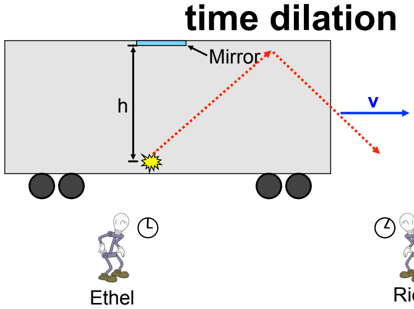
Conclusion: Since we accepted Einstein's postulate of relativity (*'c' is the same in all inertial frames*) and we found that $\Delta x \neq \Delta x'$, we conclude that $\Delta t \neq \Delta t'$. I.e., time passes at different rates in the two frames of reference!!

Another argument for time dilation



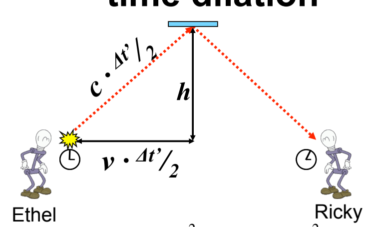
Lucy measures the time interval: $\Delta t = 2h/c$
(Not a big surprise!)

Another argument for time dilation



Note: This experiment requires two observers.

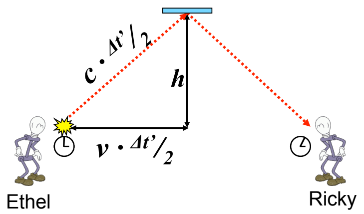
Another argument for time dilation



$$h^2 + \left(v \cdot \Delta t'/2\right)^2 = \left(c \cdot \Delta t'/2\right)^2$$

$$(\Delta t')^2\left(c^2 - v^2\right) = (2h)^2 \qquad (\Delta t')^2 = \left(\frac{2h}{c}\right)^2\left(\frac{1}{1-(v/c)^2}\right)$$

Another argument for time dilation



Ethel and Ricky measure the time interval:

$$\Delta t' = \frac{2h}{c}\gamma, \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

But Lucy measured *Δt = 2h/c* !!

Time dilation in moving frames

Lucy measures: Δt

Ethel and Ricky: $\Delta t' = \gamma \Delta t$, with

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

The γ-factor can take on what values?

A) $-\infty < \gamma < \infty$ B) $0 < \gamma \leq 1$ C) $0 \leq \gamma < \infty$

D) $1 \leq \gamma < \infty$ E) Something else…

Time dilation in moving frames

Lucy measures: Δt

Ethel and Ricky: $\Delta t' = \gamma \Delta t$, with

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

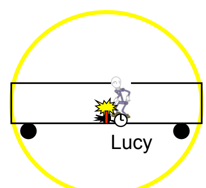
$$\Delta t' = \gamma \Delta t \geq \Delta t$$

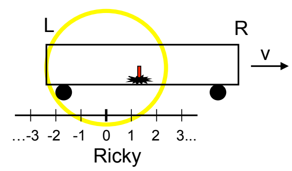
For Lucy time seems to run slower!

(Lucy is moving relative to Ethel and Ricky)

What we found so far:

Simultaneity of two events depends on the choice of the reference frame



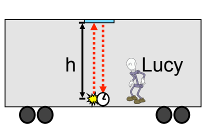


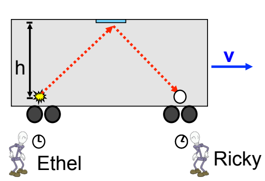
Lucy concludes:
Light hits both ends at the same time.

Ricky concludes:
Light hits left side first.

What we found so far:

Time Dilation: Two observers (moving relative to each other) can measure different durations between two events.





Lucy measures:

$$\Delta t = {}^{2h}/_{c}$$

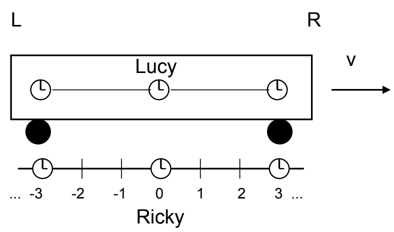
Here: $\Delta t = \Delta\tau$ is the **proper time**

Ethel and Ricky:

$$\Delta t' = \gamma\, {}^{2h}/_{c} \quad \text{with} \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

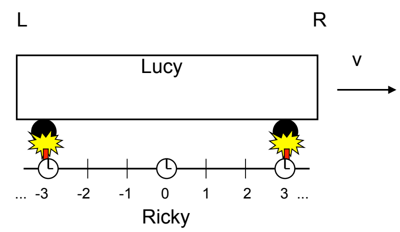
Are your clocks really synchronized?

(I know mine are!)



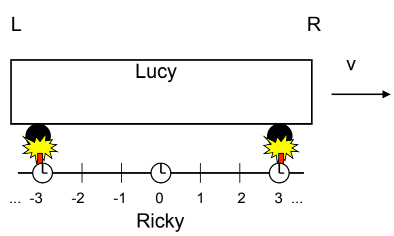
Now Lucy and Ricky each have a set of clocks. Lucy's are synchronized in her frame (the train), while Ricky's are synchronized in his frame (the tracks).

How do the clocks of one frame read in another frame?



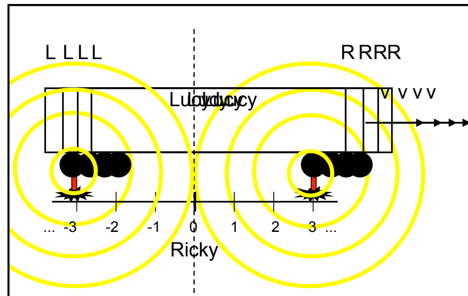
At 3 o'clock in Ricky's frame, two firecrackers go off to announce the time. It so happens that these firecrackers are at the left and right ends of the train, in Ricky's frame.

Event 1: firecracker 1 explodes at 3:00
Event 2: firecracker 2 explodes at 3:00



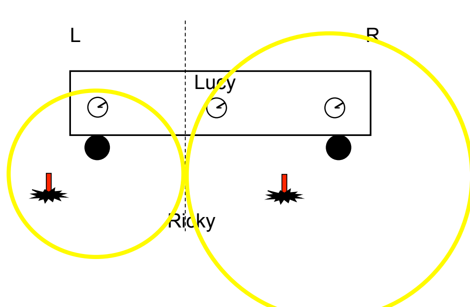
Event 1: firecracker 1 explodes at 3:00
Event 2: firecracker 2 explodes at 3:00

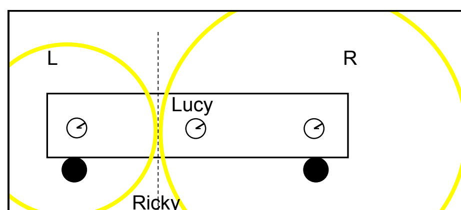
When Lucy sees each flash from the two firecrackers, she sees that Ricky's clocks both read 3:00 when each of the firecrackers went off.



Sometime later, the wavefronts meet. The meeting point is halfway between the firecrackers in Ricky's frame, but is somewhere toward the left in the train car, in Lucy's frame.
Event 3: two light pulses meet, shortly after 3:00.

The situation as seen by Lucy

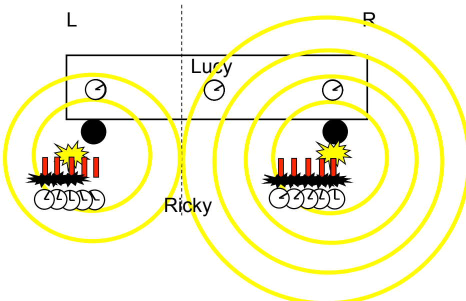




In *Lucy's frame*, light left first from the right end of the car. The light pulses both show clocks reading 3:00 in Ricky's frame. According to Lucy's reference frame, which of the following is true:

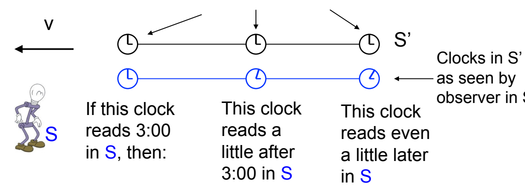
A) Ricky's clock on the left reads a later time than Ricky's clock on the right.
B) Ricky's clock on the right reads a later time than Ricky's clock on the left.
C) Both of Ricky's clocks read the same time.

In Lucy's frame:



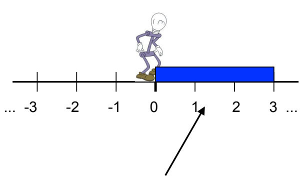
Important conclusion

Clocks in S' (synchronized in S')
moving to the left with respect to S



Even though the clocks in S' are synchronized (in S') the observer in S sees each clock showing a different time!!

Length of an object



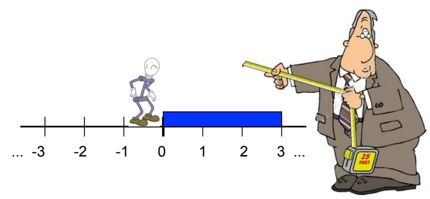
This length, measured in the stick's rest frame, is its proper length.

This stick is 3m long. I measure both ends at *the same time* in my frame of reference.

"Same time" or not doesn't actually matter here, because the stick isn't going anywhere.

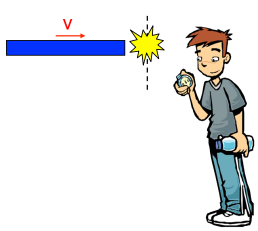
'Proper length'

Proper length: Length of object measured in the frame where it is at rest (use a ruler)

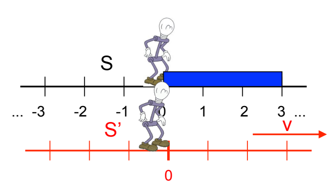


Remember 'proper time'

Proper time: Time interval $\Delta\tau = t_2 - t_1$ between two events measured in the frame where the two events occur at the same spatial coordinate, i.e. a time interval that can be **measured with one clock**.



Length of an object

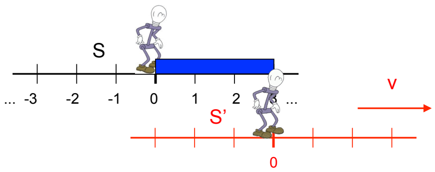


Observer in S measures the proper length L of the blue object.

Another observer comes whizzing by at speed v. This observer measures the length of the stick, *and keeps track of time*.

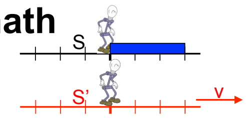
Event 1 – Origin of S' passes left end of stick.

Length of an object



Event 1 – Origin of S' passes left end of stick.
Event 2 – Origin of S' passes right end of stick.

A little math



In frame S: (rest frame of the stick)
- length of stick = L (this is the proper length)
- time between events = Δt
- speed of frame S' is v = L/Δt

In frame S':
- length of stick = L' (this is what we're looking for)
- time between events = $\Delta t'$ $= \Delta\tau$
- speed of frame S is -v = -L'/$\Delta t'$

Follow the proper time!

Q: a) $\Delta t = \Delta t'$ b) $\Delta t = \gamma \Delta t'$ c) $\Delta t' = \gamma \Delta t$

A little math

Speeds are the same (both refer to the relative speed).
And so

$$|v| = \frac{L'}{\Delta t'} = \frac{L}{\Delta t} = \frac{L}{\gamma \Delta t'}$$

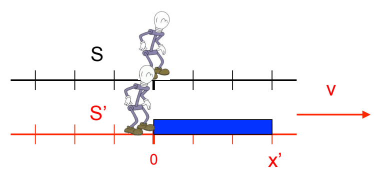
$$L' = \frac{L}{\gamma}$$

Length in moving frame

Length in stick's rest frame (proper length)

Length contraction is a consequence of time dilation (and vice-versa).

The Lorentz transformation

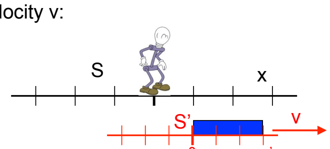


A stick is at rest in S'. Its endpoints are the events (x,t) = (0,0) and (x',0) in S'. S' is moving to the right with respect to frame S.

Event 1 – left of stick passes origin of S. Its coordinates are (0,0) in S and (0,0) in S'.

Lorentz transformation

An observer at rest in frame S sees a stick flying past him with velocity v:

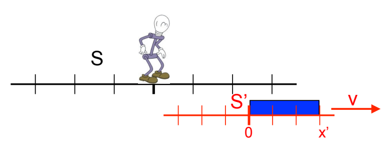


As viewed from S, the stick's length is x'/γ. Time t passes. According to S, where is the *right* end of the stick? (Assume the *left* end of the stick was at the origin of S at time t=0.)

A) $x = \gamma v t$ B) $x = vt + x'/\gamma$ C) $x = -vt + x'/\gamma$

D) $x = vt - x'/\gamma$ E) Something else…

The Lorentz transformation



$x = vt + x'/\gamma$ This relates the spatial coordinates of an event in one frame to its coordinates in the other.

Algebra $x' = \gamma(x - vt)$

Transformations

If S' is moving with speed v in the positive x direction relative to S, then the coordinates of the same event in the two frames are related by:

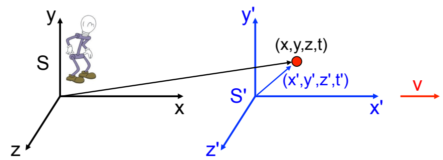
Galilean transformation (classical)	Lorentz transformation (relativistic)
$x' = x - vt$	$x' = \gamma(x - vt)$
$y' = y$	$y' = y$
$z' = z$	$z' = z$
$t' = t$	$t' = \gamma(t - \frac{v}{c^2}x)$

Note: This assumes (0,0,0,0) is the same event in both frames.

⚠ A note of caution: ⚠

The way the Lorentz and Galileo transformations are presented here assumes the following:

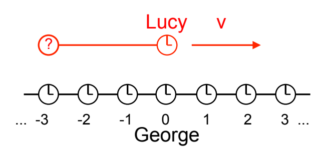
An observer in S would like to express an event (x,y,z,t) (in his frame S) with the coordinates of the frame S', i.e. he wants to find the corresponding event (x',y',z',t') in S'. The frame S' is moving along the x-axes of the frame S with the velocity v (measured relative to S) and we assume that the origins of both frames overlap at the time t=0.



Transformations

If S' is moving with speed v in the positive x direction relative to S, then the coordinates of the same event in the two frames are related by:

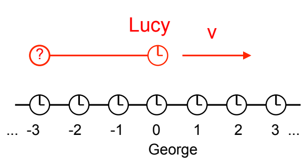
Galilean transformation (classical)	Lorentz transformation (relativistic)
$\Delta x' = \Delta x - v\Delta t$	$\Delta x' = \gamma(\Delta x - v\Delta t)$
$\Delta y' = \Delta y$	$\Delta y' = \Delta y$
$\Delta z' = \Delta z$	$\Delta z' = \Delta z$
$\Delta t' = \Delta t$	$\Delta t' = \gamma(\Delta t - \frac{v}{c^2}\Delta x)$



$$x' = \gamma(x - vt)$$
$$t' = \gamma(t - \frac{v}{c^2}x)$$

George has a set of synchronized clocks in reference frame S, as shown. Lucy is moving to the right past George, and has (naturally) her own set of synchronized clocks. Lucy passes George at the event (0,0) in both frames. An observer in George's frame checks the clock marked '?'. Compared to George's clocks, this one reads

A) a slightly earlier time B) a slightly later time C) same time



$$x' = \gamma(x - vt)$$
$$t' = \gamma(t - \frac{v}{c^2}x)$$

The event has coordinates (x = -3, t = 0) for George.
In Lucy's frame, where the ? clock is, the time t' is

$$t' = \gamma(0 - \frac{v}{c^2}(-3)) = \frac{3\gamma v}{c^2}$$, a positive quantity.

'?' = slightly later time