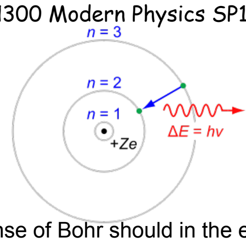
PH300 Modern Physics SP11



"If this nonsense of Bohr should in the end prove to be right, we will quit physics! "
- Otto Stern & Max von Laue

3/3 Day 14:
Questions?
Atomic Models
Magnets/magnetic moments
Stern-Gerlach Experiments

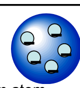
Next Week:
Entanglement
Quantum Cryptography
Quantum Physics & Reality

Last time:
- Balmer formula, atomic spectra
- Bohr model
- de Broglie waves

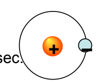
Today:
- Magnets and magnetic fields
- Atomic spin
- Stern-Gerlach experiments

Next Week:
- Reading on Blackboard before class
- Entanglement/EPR "paradox"
- Quantum physics and *reality*

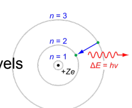
Models of the Atom



- Thomson – "Plum Pudding"
  - Why? Known that negative charges can be removed from atom.
  - Problem: Doesn't match spectral lines
- Rutherford – Solar System
  - Why? Scattering showed a small, hard core.
  - Problem: electrons should spiral into nucleus in ~10^{-11} sec.



- Bohr – fixed energy levels
  - Why? Explains spectral lines.
  - Problem: No reason for fixed energy levels



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deBroglie Waves

- deBroglie (French grad student) suggested: maybe electrons are actually little waves going around the nucleus.
- This seems plausible because…
  - Standing waves have quantized frequencies, might be related to quantized energies.
  - Einstein had shown that light, typically thought of as waves, have particle properties. Might not electrons, typically thought of as particles, have wave properties?

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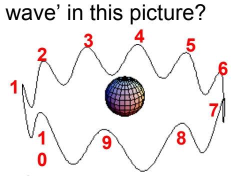
deBroglie Waves

What is n for the 'electron wave' in this picture?



A. 1
B. 5
C. 10
D. 20
E. Cannot determine from picture

Answer: C. 10

n = number of wavelengths.
It is **also** the number of the energy level $E_n = -13.6/n^2$.
So the wave above corresponds to
$E_{10} = -13.6/10^2 = -0.136$eV
(will explain soon)

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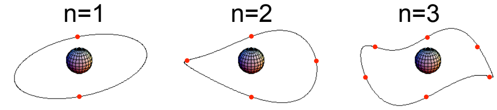
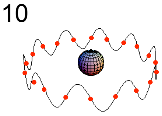
deBroglie Waves

n=1 n=2 n=3



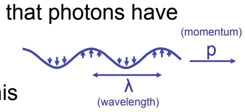
…n=10



• = node = fixed point that doesn't move.

deBroglie Waves

- If electron orbits are standing waves, there is a relationship between orbital radius and wavelength: $2\pi r = n\lambda$
- But what is the wavelength of an electron?!
- For photons, it was known that photons have momentum $E = pc = hc/\lambda$
  ➔ $p = h/\lambda$ ➔ $\boxed{\lambda = h/p}$
- deBroglie proposed that this is also true for massive particles (particles w/mass)!
  $\boxed{\lambda = h/p = \text{“deBroglie wavelength”}}$



deBroglie Waves

- Substituting the deBroglie wavelength ($\lambda = h/p$) into the condition for standing waves ($2\pi r = n\lambda$), gives:
  $$2\pi r = nh/p$$
- Or, rearranging:
  $$pr = nh/2\pi$$
  $$L = n\hbar$$
- deBroglie EXPLAINS quantization of angular momentum, and therefore EXPLAINS quantization of energy!

In the deBroglie picture, the electrons have an intrinsic wavelength associated with them. We have also been told that one wavelength fits around the circumference for the n=1 level of hydrogen, 2 fit around the circumference for n=2, 5 fit for n=5, etc.

Therefore, we expect that the n=5 circumference is 5 times as large as the n=1 circumference.

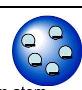
A) True
B) False

False!

From the Bohr model we know that $r_n = n^2 a_o$.

Here the other thing changing with n is the deBroglie wavelength of the electrons, because the electron energy and momentum also change with n.

Models of the Atom



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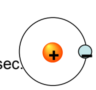

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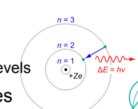
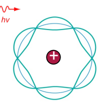
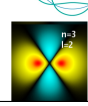


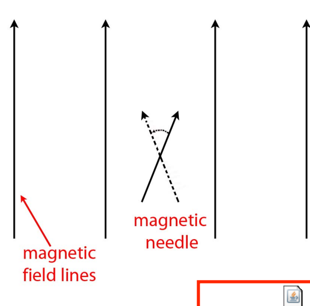
- deBroglie – electron standing waves
  - Why? Explains fixed energy levels
  - Problem: still only works for Hydrogen.

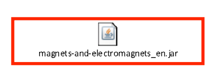


- Schrodinger – quantum wave functions
  - Why? Explains everything!
  - Problem: None (except that it's abstract)



Compass Needle in a <u>Uniform</u> Magnetic Field





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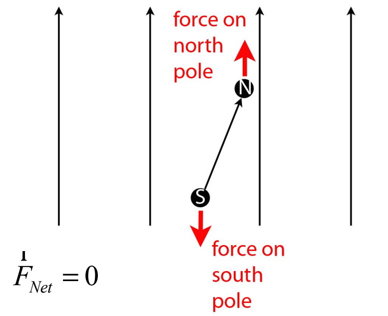
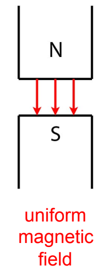
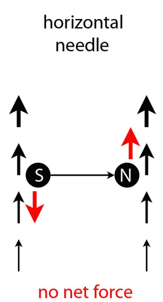


Uniform vs. Non-Uniform Magnetic Fields

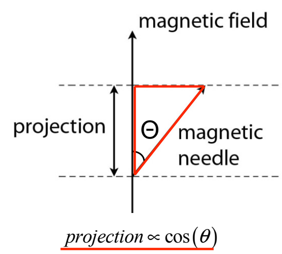


Compass Needle in a Non-Uniform Magnetic Field

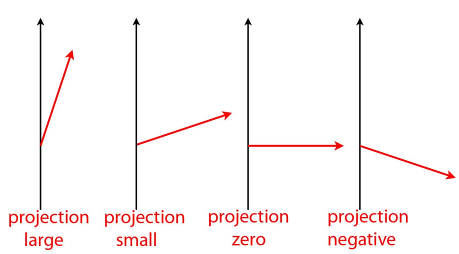


Compass Needle in a Non-Uniform Magnetic Field

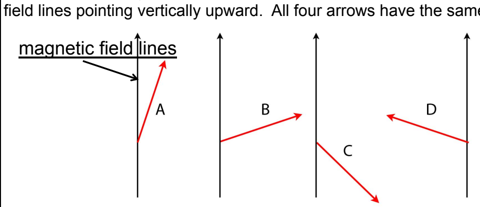
It turns out the net force is proportional to the ***projection*** of the magnetic needle onto the magnetic field.



For a magnetic needle, all types of projections are allowed:



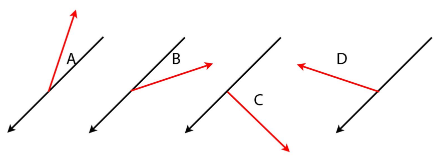
Below are four magnetic arrows (**A**, **B**, **C**, and **D**), shown relative to magnetic field lines pointing vertically upward. All four arrows have the same length.



Rank the arrows from highest value of the projection onto the vertical axis to the lowest. If two projections are equal, then say so.

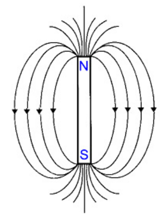
A) A > B > C > D
B) A > B > D > C
C) A > B = D > C
D) A > C > B = D
E) None of the above.

Below are four magnetic arrows (**A**, **B**, **C**, and **D**), shown relative to magnetic field lines pointing left & downward. All four arrows have the same length.

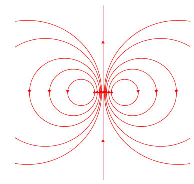


Rank the arrows from highest value to lowest of the projection onto the shown axis. If two projections are equal, then say so.

A) A > B = D > C
B) D > C > B = A
C) C > D = B > A
D) A = B > C > D
E) None of the above.



Magnetic field lines of a bar magnet

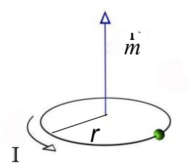


Magnetic field lines of a current loop

magnets-and-electromagnets_en.jar

Magnetic Moment of a Current Loop

$$\vec{m} = I \cdot \vec{a}$$

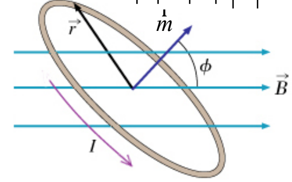


$$\vec{a} = \pi r^2 \cdot \hat{a}$$

$\hat{a}$ points in the direction perpendicular to the plane of the current loop, in the direction given by the ***right-hand rule***.

Torque on a Current Loop in a Magnetic Field

$$\vec{\tau} = \vec{m} \times \vec{B} \qquad |\vec{\tau}| = |\vec{m}| \cdot |\vec{B}| \sin(\phi)$$

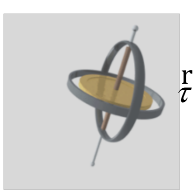


What effect does this torque have on the current loop?

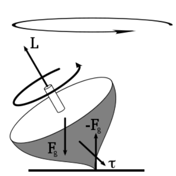
The magnetic moment vector ***precesses*** about the magnetic field lines, but the angle φ remains constant.

$$\phi = \text{constant}$$

Precession of a Gyroscope in a Gravitational Field:



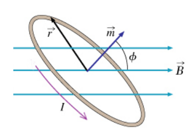
$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

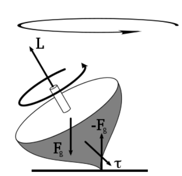


The gyroscope can only spin about the symmetry axis

The torque acts at ***right angles*** to the angular momentum vector, so it changes the ***direction*** of the angular momentum vector, but ***not its magnitude***.

What corresponds to what?

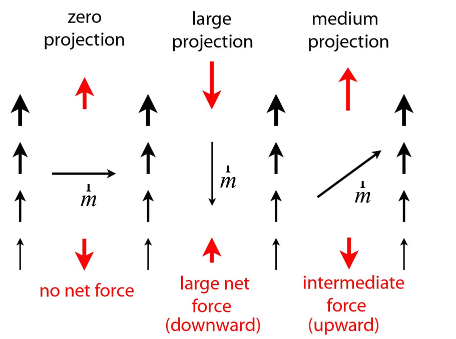




Take-Home Message:

A) The torque does not cause the magnetic moment vector of the current loop to flip directions.
B) Since ϕ = constant, the projection of the magnetic moment vector onto the direction of the B-field does not change while the current loop is interacting with the B-field.

Magnetic Moment in a Non-Uniform Magnetic Field



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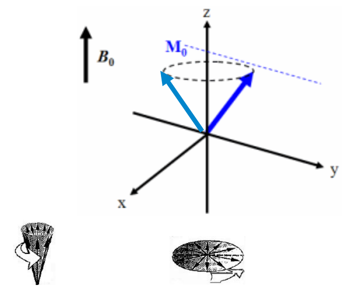
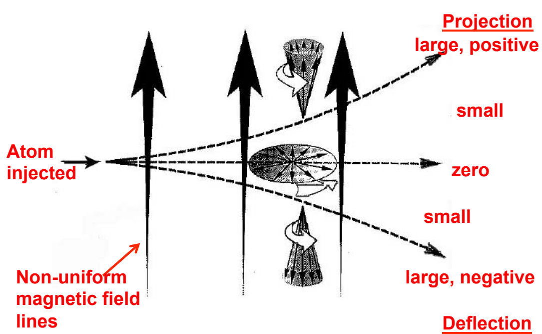
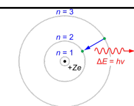


Magnetic Moment in a Non-Uniform Magnetic Field



Remember the Bohr Model:



- We know how magnetic moments (current loops) behave in the presence of a magnetic field.
- Orbital angular momentum does contribute to the magnetic moment of an atom – even in modern theories
- We know a neutral atom has a magnetic moment when it is deflected by a non-uniform magnetic field.
- Measuring this deflection (when the B-field is well known) is a measurement of the projection of the atom's magnetic moment onto the axis of the magnetic field.

Remember the Bohr Model:

- We know how magnetic moments (current loops) behave in the presence of a magnetic field.
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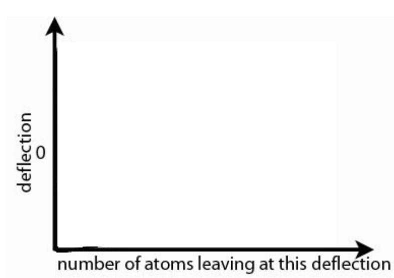
Classical Expectations(?)

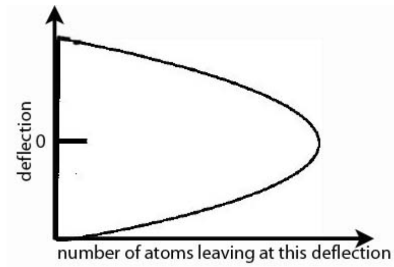


Key Ideas/Concepts

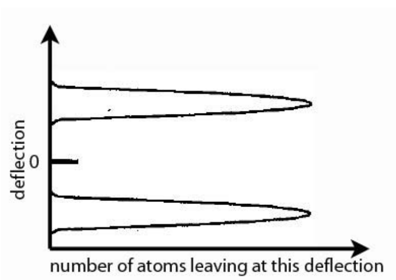
Classically, we expect:

1. Magnetic moments are deflected in a non-uniform magnetic field
2. Magnetic moments precess in the presence of a magnetic field, and the projection of the magnetic moment along the field axis remains constant.
3. If the magnetic moment vectors are randomly oriented, then we should see a broadening of the particle beam

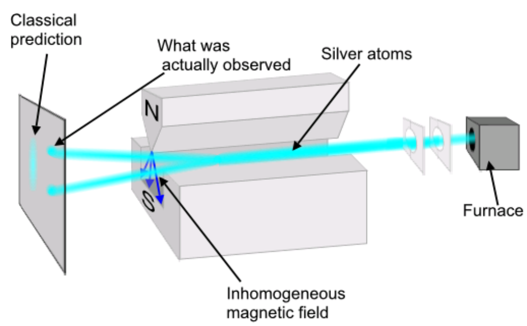
Classical Expectations



Observation:

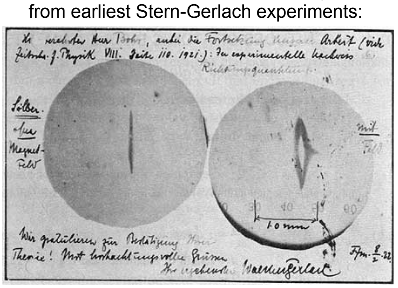


Stern-Gerlach Experiment with Silver Atoms (1922)*



*Nobel Prize for Stern in 1943

Postcard from Gerlach to Bohr showing results from earliest Stern-Gerlach experiments:



"…attached is the continuation of our work: the experimental proof of directional quantization. Silver without magnetic field/with magnetic field

"Stern and Gerlach: How a Bad Cigar Helped Reorient Atomic Physics" – Available on Blackboard

"Directional Quantization"?

Quantized means discrete – the quantity comes in discrete values instead of continuous.

What kinds of quantization have we seen so far?
- Quantization of photon energy
- Quantization of atomic energy levels
- Quantization of angular momentum

Classically, we would expect that the magnetic moment vector could be pointing in any direction when we measure it.

By ***directional quantization***, they are referring to how the projection of the magnetic moment vector for the silver atoms onto any axis only comes in two discrete values [$+m_B$ or $-m_B$], and not in a continuous spectrum.

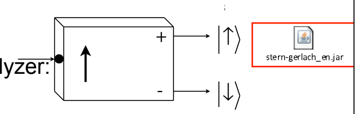
$$m_B = \left(\frac{e}{2m_e}\right)\left(\frac{h}{2\pi}\right) = 9.27\times 10^{-24}\text{ joule/tesla}$$

In the future…

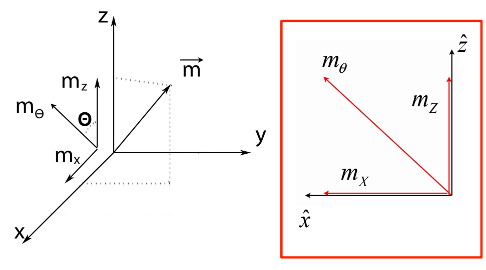
Some atoms have more than two possible projections of the magnetic moment vector along a given axis, but we will only deal with "two-state" systems when talking about atomic spin.

When we say a particle is measured as "***spin up***" (or ***down***) along a given axis, we are saying that we measured the projection of the magnetic moment along that axis ($+m_B$ or $-m_B$), and observed it exiting the ***plus-channel*** (***minus-channel***) of a Stern-Gerlach type apparatus.

A simplified Stern-Gerlach analyzer:

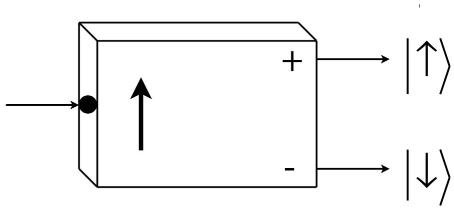


Notation for projection values:



Head-on view

A simplified Stern-Gerlach analyzer:

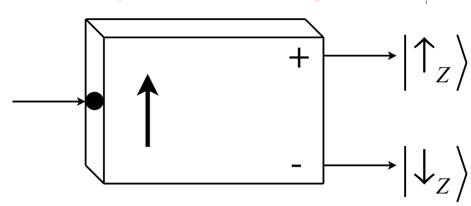


However the analyzer is oriented, we always measure one of two values: ***spin-up*** ($+m_B$) or ***spin-down*** ($-m_B$).

$$m_B = 9.27\times10^{-24} \text{ joule/tesla}$$

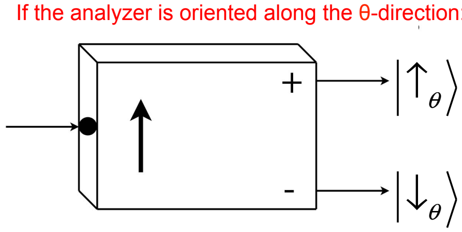
A simplified Stern-Gerlach analyzer:

If the analyzer is oriented along the +z-direction:

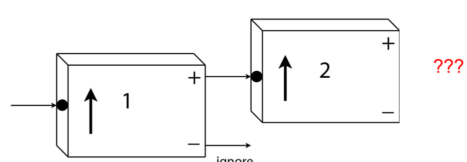


A simplified Stern-Gerlach analyzer:

If the analyzer is oriented along the θ-direction:



Repeated spin measurements:



Ignore the atoms exiting from the ***minus-channel*** of **Analyzer 1**, and feed the atoms exiting from the ***plus-channel*** into **Analyzer 2**.

All of the atoms also leave from the ***plus-channel*** of **Analyzer 2**.

State Preparation

The atoms leaving the plus-channel of vertical **Analyzer 1** were all in the state $|\uparrow_Z\rangle$

We confirmed this when all these atoms also leave the plus-channel of vertical **Analyzer 2**. We can repeat this as often as we like…

All of these atoms were in the ***definite state*** $|\uparrow_Z\rangle$

Bracket Notation

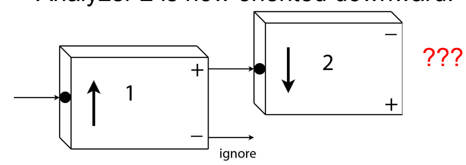
We'll place ***any information*** we have about a ***quantum state*** inside a bracket: $|\Psi\rangle$ Ψ = "Psi" is a generic symbol for a quantum state

For the atoms leaving the plus-channel of **Analyzer 1** we write:

$|\Psi\rangle = |\uparrow_Z\rangle$ **OR** $|m_Z = +m_B\rangle$

$m_B = 9.27\times10^{-24}$ joule/tesla

Analyzer 2 is now oriented downward:

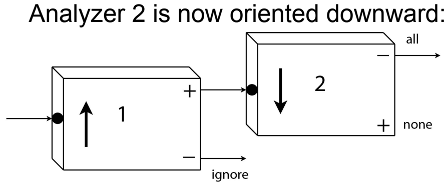


Ignore the atoms exiting from the ***minus-channel*** of **Analyzer 1**, and feed the atoms exiting from the ***plus-channel*** into **Analyzer 2**.

What happens when these atoms enter **Analyzer 2**?

A) They all exit from the plus-channel.
B) They all exit from the minus-channel.

Analyzer 2 is now oriented downward:



All the atoms leave from the ***minus-channel*** of **Analyzer 2**.

Remember, with **Analyzer 2** we are now measuring $m_{(-Z)}$

An atom in the state $|\uparrow_Z\rangle$ is also in the state $|\downarrow_{(-Z)}\rangle$