PH300 Modern Physics SP11

"I am one of those who think like Nobel, that humanity will draw more good than evil from new discoveries."
– **Marie Curie**

4/12 Day 22:
Questions?
Tunneling
Alpha-decay, radioactivity

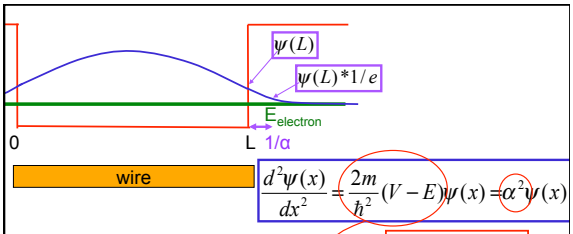
Thursday:
Radioactivity
Scanning Tunneling Microscopes

Recently:
1. Schrödinger equation, free particle
2. Square well potential
3. Quantum tunneling

Today:
1. Tunneling (cont.)
2. Alpha-decay
3. Radioactivity

Thursday:
1. Radioactivity (cont.)
2. Scanning Tunneling Microscopes

2



$$\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2}(V-E)\psi(x) = \alpha^2\psi(x)$$

$$\alpha = \sqrt{\frac{2m}{\hbar^2}(V-E)}$$

How far does wave extend into this "classically forbidden" region?

$$\psi(x) = Be^{-\alpha x}$$

α big -> quick decay
α small -> slow decay

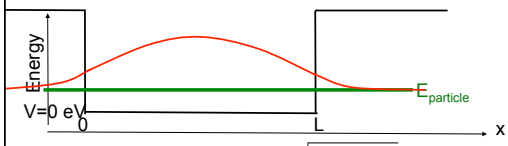
Measure of penetration depth = 1/α
ψ(x) decreases by factor of 1/e

For V-E = 4.7eV, 1/α = 9×10^{-11} meters (very small ~ an atom!!!)

$$\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2}(V-E)\psi(x)$$

Inside well (E>V):

Outside well (E<V):



$$\psi(x) = Be^{-\alpha x} \qquad \alpha = \sqrt{\frac{2m}{\hbar^2}(V-E)}$$

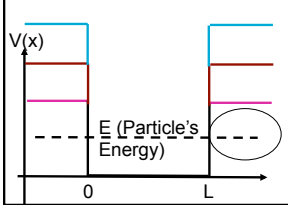
What changes could increase how far wave penetrates into classically forbidden region?
(And so affect chance of tunneling into adjacent wire)

Thinking about α and penetration distance

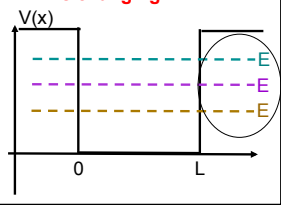
Under what circumstances would you have largest penetration?
Order each of the following case from smallest to largest.

$$\psi(x) = Be^{-\alpha x} \qquad \alpha = \sqrt{\frac{2m}{\hbar^2}(V-E)}$$

Potential curve is changing.



Energy of the particle is changing



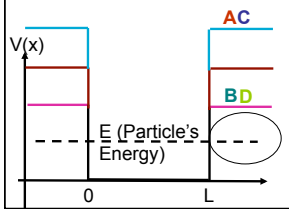
Thinking about α and penetration distance

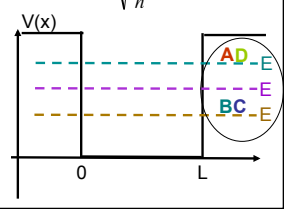
Under what circumstances would you have largest penetration?
Order each of the following case from smallest to largest.

CQ: Which cases correspond to the smallest penetration?

$$\psi(x) = Be^{-\alpha x}$$

$$\alpha = \sqrt{\frac{2m}{\hbar^2}(V-E)}$$

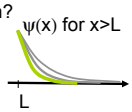




Thinking more about what α means:

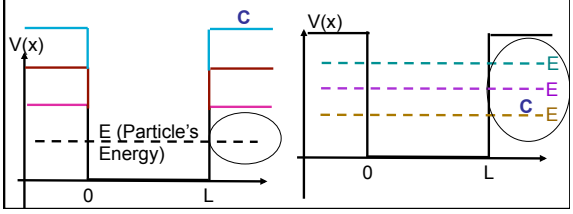
CQ: Which cases correspond to the smallest penetration?

- Bigger difference btwn V and E ➔
- Larger α
- Faster decay
- Smaller penetration

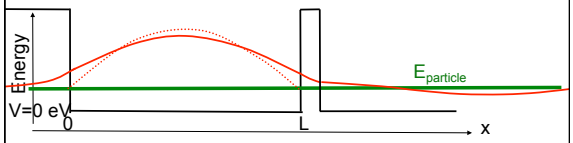


$$\alpha = \sqrt{\frac{2m}{\hbar^2}(V-E)}$$

$$\psi(x) = Be^{-\alpha x}$$



So the thinner and shorter the barrier, the easier it is to tunnel …



…and electron can escape.

Application: Alpha-Decay, Scanning tunneling microscope

Radioactive decay

Nucleus is unstable → emits a particle

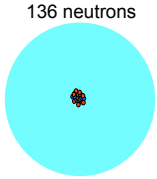
Typically found for large atoms with lots of protons and neutrons.

Alpha Decay → Nucleus emits an alpha particle

An alpha particle is 2 neutrons and 2 protons.

Radon-222
86 protons,
136 neutrons

- Proton (**positive charge**)
- Neutron (**no charge**)



Nucleus has lots of protons and lots of neutrons.

Two forces acting in nucleus:

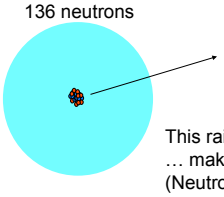
- Coulomb force .. Protons really close together, so very big repulsion from coulomb force
- Nuclear force (attraction between nuclear particles is very strong if very close together) … called the STRONG Force.

Radioactive decay

- Proton (**positive charge**)
- Neutron (**no charge**)

Radon-222
86 protons,
136 neutrons

In alpha-decay, an alpha-particle is emitted from the nucleus.



This raises the ratio of neutrons to protons … makes for a more stable atom. (Neutrons are neutral.. no coulomb repulsion, but nuclear force attraction)

How does this happen?

Starting point *always* to look at potential energy curve for particle

Nucleus (Z protons, Bunch o' neutrons) = New nucleus (Z-2 protons, Bunch o' neutrons) + Alpha particle (2 protons, 2 neutrons)

Look at this system… as the distance between the alpha particle and the nucleus changes.

As we bring α-particle closer, what happens to potential energy?

(Z-2)

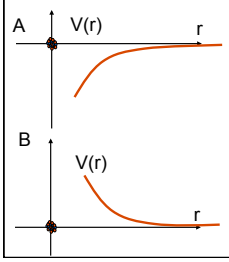
V=0
At a great distance

How does this happen?

Starting point *always* to look at potential energy curve for particle

As we bring α-particle closer, what happens to potential energy?

V=0 At a great distance



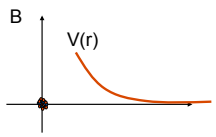
D. Something else

How does this happen…

Starting point *always* to look at potential energy curve for particle

As we bring α-particle closer, what happens to potential energy?

V=0 At a great distance



$$V(r) = \frac{kq_1q_2}{r} = \frac{k(Z-2)(e)(2e)}{r}$$

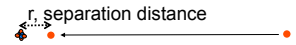
Takes energy to push α-particle towards the nucleus, so potential energy must increase.

What if proton starts out going towards nitrogen nucleus with a lot of speed (lots of kinetic energy)?

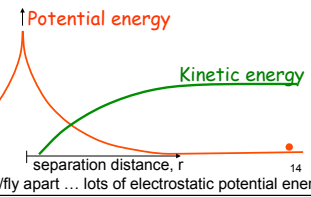
Starts with lots of kinetic energy →

Repelling force from nucleus slows down proton →

Proton's kinetic energy converted into electrostatic potential energy as it gets closer to nucleus



Potential energy curves- represent energy to bring particles together.

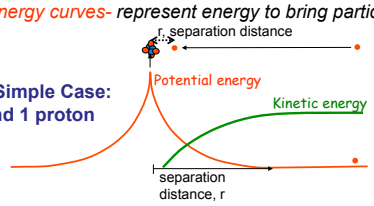


Gravity energy analogy:

If at center, want to roll down hill/fly apart … lots of electrostatic potential energy

14

Potential energy curves- represent energy to bring particles together.



Consider Simple Case: Oxygen and 1 proton

Potential energy at separation distance of r = $\frac{\text{k x charge of 7 nucl. protons x charge of single proton}}{\text{(separation distance)}}$

Charge of 1 proton = 1.6×10^{-19} Coulombs; Charge of 7 protons=11.2×10^{-19} C

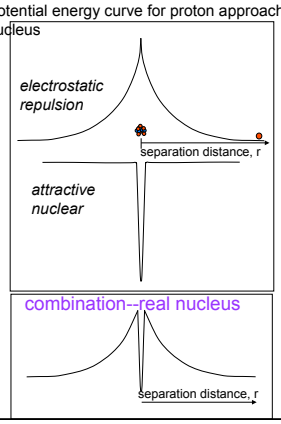
So at 10^{-15} m away (~ radius of nucleus),

Potential energy = $\frac{8.99 \times 10^9 \text{ N m}^2/\text{C}^2 \times 11.2 \times 10^{-19} \text{ C} \times 1.6 \times 10^{-19} \text{ C}}{(10^{-15} \text{ m})}$

= 1.61×10^{-12} Joules = 10 million electron Volts.

15

Potential energy curve for proton approaching nucleus



Energy scale gigantic compared to chemical energy.

Why? Simple coulomb's law.

$F = \frac{k \text{ (charge of \#1)(charge of \#2)}}{r^2}$

Chemistry- forces between electrons and protons on distance scale of atomic size (> 10^{-10} m).

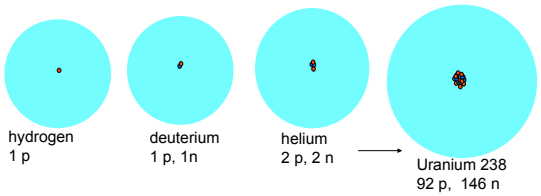
Nuclear forces- forces between protons on distance scale 10-100,000 times smaller. 10,000 times closer means forces **100,000,000 times bigger** because of $1/r^2$. Lots more potential energy stored!!!

16

Each element has different number of protons.

Atom ingredients:

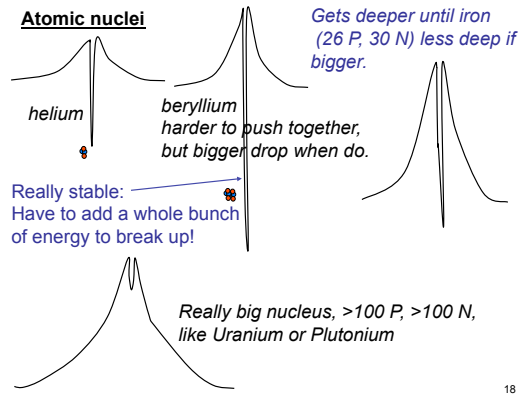
- Proton (**positive charge**) – charge = 1.6×10^{-19} Coulombs, mass = 1.66×10^{-27} kg.
- Neutron (**no charge**) – no charge, mass = 1.66×10^{-27} kg.
- Electron (**negative charge**) – charge = -1.6×10^{-19} Coulombs, mass = 9.10×10^{-31} kg



17

Atomic nuclei

Gets deeper until iron (26 P, 30 N) less deep if bigger.



18

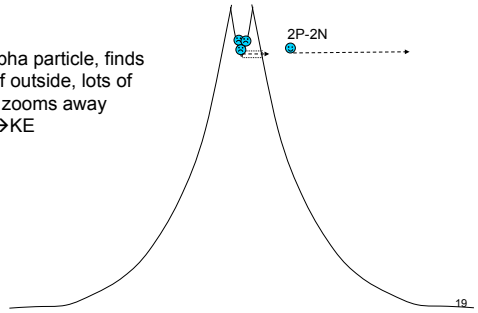
Nuclear decay- one kind of nucleus changes into another.

alpha decay, beta decay, induced fission

Alpha decay: α-particle = 2p and 2n.

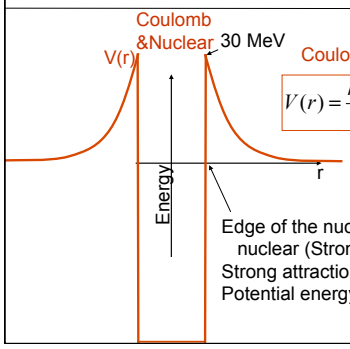
Most radioactivity is this type (e.g. radon).

If alpha particle, finds itself outside, lots of PE, zooms away PE→KE



Starting point *always* to look at potential energy curve for particle

Bring alpha-particle closer

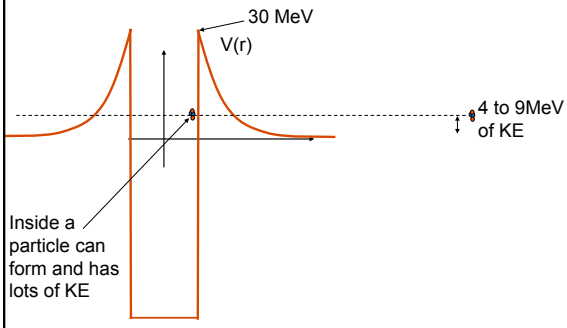


Coulomb force dominates

$$V(r) = \frac{kq_1q_2}{r} = \frac{k(Z-2)(e)(2e)}{r}$$

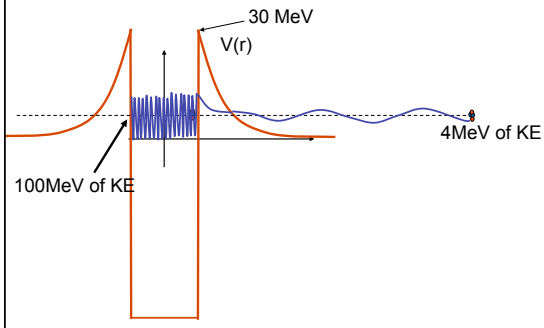
Edge of the nucleus (~8x10^{-15} m), nuclear (Strong) force starts acting. Strong attraction between nucleons. Potential energy drops dramatically.

Observations show Alpha-particles exit with a range of energies

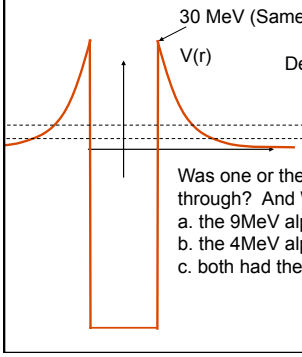


How does this happen?

Starting point *always* to look at potential energy curve for particle



Observe α-particles from **<u>different isotopes</u> (same protons, different neutrons)**, exit with different amounts of energy.



Decay constant: $\alpha = \sqrt{\frac{2m}{\hbar^2}(V-E)}$

Was one or the other more likely to tunnel through? And WHY???

a. the 9MeV alpha particle was more likely

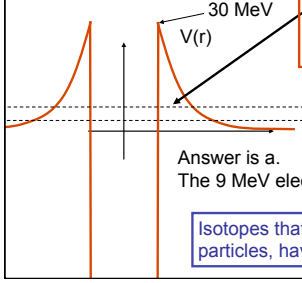
b. the 4MeV alpha particle was more likely

c. both had the same probability of tunneling

Observe α-particles from **<u>different isotopes</u> (same protons, different neutrons)**, exit with different amounts of energy.

$$\alpha = \sqrt{\frac{2m}{\hbar^2}(V-E)}$$

1. Less distance to tunnel.
2. Decay constant always smaller
3. Wave function doesn't decay as much before reaches other side … more probable!

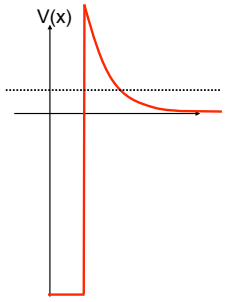


Answer is a.

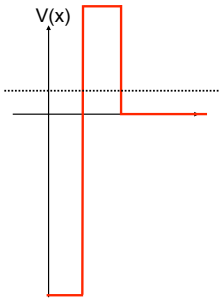
The 9 MeV electron more probable…

Isotopes that emit higher energy alpha particles, have shorter lifetimes!!!

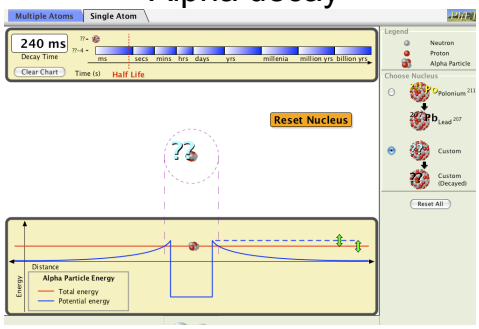
Solving Schrodinger equation for this potential energy is hard!



Square barrier is much easier… and get almost the same answer!

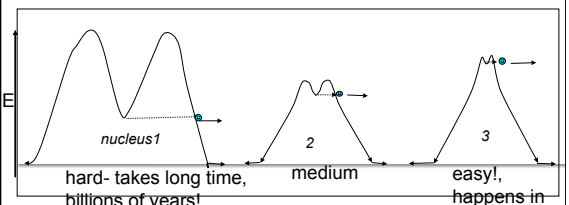


Alpha decay



26

tunneling difficulty = width x depth of tunnel



hard- takes long time, billions of years!

medium

easy!, happens in millionths of a second!

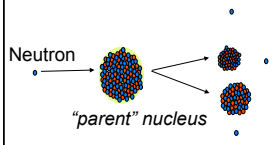
How much energy released?
a. 1 most, 2 second, 3 least
b. 2 most, 1, 3 least
c. 3 most, 2, 1 least
d. 3 most, 1, 2 least

Energy is difference from bottom of crater to outside.
3 is most, 2 second, 1 is least

27

Radioactive decay-fission

Neutron Induced fission- key to atomic bombs



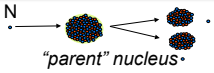
- *two smaller nuclei*
- *few extra free neutrons*
- ***LOTS OF ENERGY!!***
- *(+sometimes other bad stuff)*

"daughter" nuclei – come out in *excited* nuclear energy state
…. Give off gamma rays as drop to lower energy.

Jumps down in energy …
Gives off gamma ray…
VERY HIGH ENERGY PHOTON

28

Neutron Induced fission - key to atomic bombs



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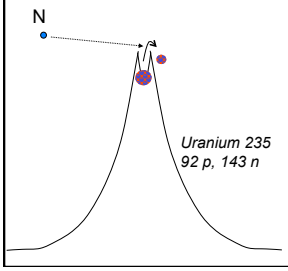
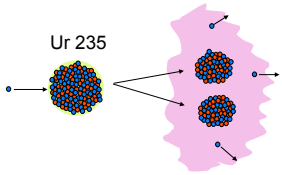


Neutron absorbed →
Excites U235 nucleus up above potential barrier →
Splits into two smaller nuclei…
which zoom apart due to electrostatic repulsion!

29

Neutron induced fission

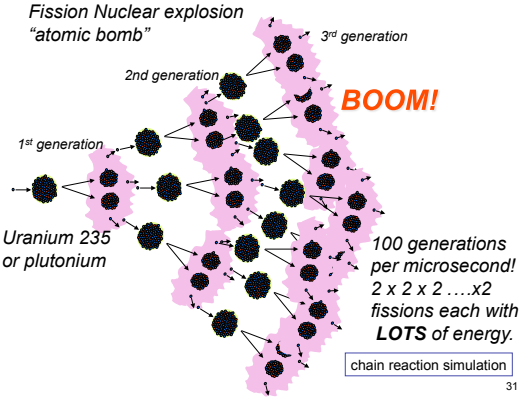


If don't get swallowed the 3 free neutrons can induce more fission.

Chain reaction: neutrons making fission that makes more neutrons, that makes more fission that makes more neutrons, that makes more fission that makes more neutrons, that makes more fission that makes more neutrons, that makes more fission that makes more neutrons, that makes more fission that makes more neutrons, that makes

30

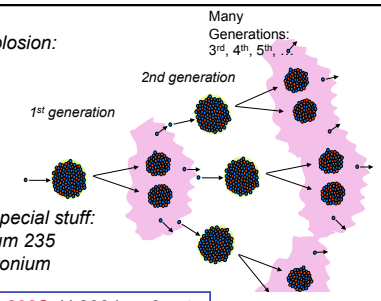
Fission Nuclear explosion
"atomic bomb"



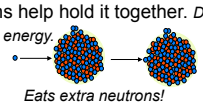
chain reaction simulation

31

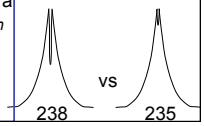
CHAIN REACTION
Fission Nuclear explosion:
"atomic bomb"



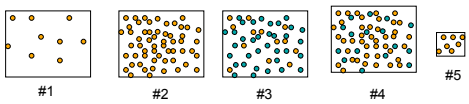
Why U 235 & not U 238? U 238 has 3 extra neutrons help hold it together. *Deeper crater in potential energy.*



Eats extra neutrons!



U235 and U238 atoms are placed into a container, which are likely to result in a chain reaction (resulting in explosion) when a free neutron triggers fission of one of the U235:



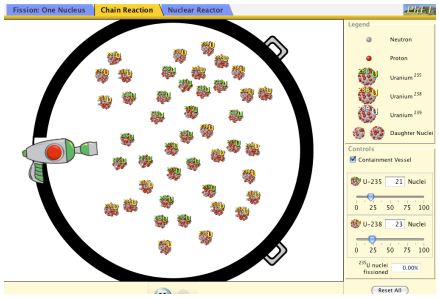
a. #2 only
b. #1, #2, and #5
c. #2 and #4
d. #2, #3, and #4
e. #2, #4, and #5.

Lots of uranium in the ground… why doesn't just blow up?

Correct answer is c. (#2 and #4) Analysis:
#1 is too sparse .. most neutrons will leave box before hitting another U235.
#2 is good.. Pure U235, densely packed, large package
#3 has too many U238's… more U238's than 235's. Free neutrons more likely to be absorbed by 238's than to hit and fission another 235.
#4 is OK … More U235's than 238's, still densely packed, large package
#5 is too small of package … neutrons likely to escape package before hitting another U235.

33

A useful simulation

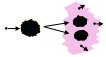


http://phet.colorado.edu/simulations/sims.php?sim=Nuclear_Fission

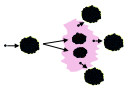
34

Recipe for fission bomb.

1. Find neutron induced fissionable material that produces bunch of extra free neutrons when fissions.
2. Sift it well to remove all the other material that will harmlessly swallow up the extra neutrons. (THE HARDEST STEP.)



3. Assemble "supercritical mass", really fast!. Need enough stuff that the neutrons run into other nuclei rather than just harmlessly leaving sample.

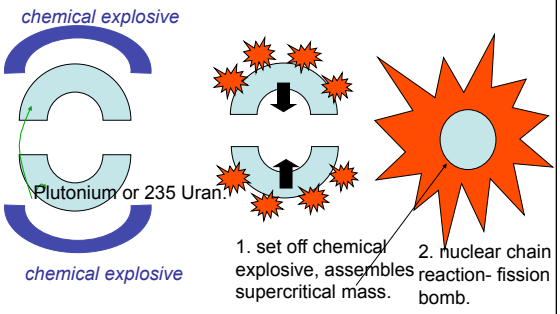


If your mass tends to melt with a small fizzle you are not assembling fast enough to be supercritical. Put together faster.

4. Let sit for 1 millionth of a second- will bake itself!

35

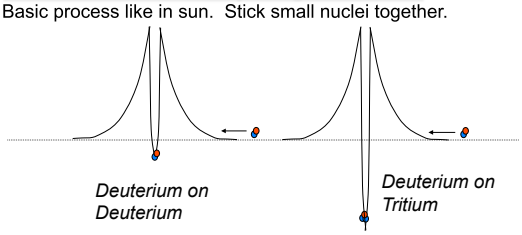
Fission bomb (basic picture)



36

Fusion bomb or "hydrogen bomb"

Basic process like in sun. Stick small nuclei together.



Which will release more energy during fusion?
a. Deuterium combining with deuterium
b. Deuterium combining with tritium

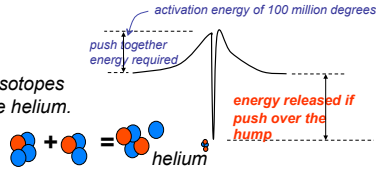
Answer is b. Incoming deuterium particle has comparatively more potential energy!

37

Fusion bomb or "hydrogen bomb"

Basic process like in sun. Stick small nuclei together.

Stick hydrogen isotopes together to make helium.



Simple if can push hard enough- just use fission bomb!
More energy per atom than fission. Can use LOTS of hydrogen.

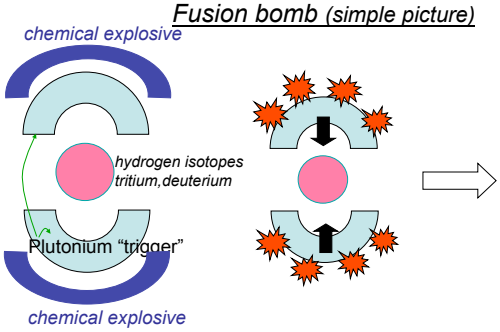
End up with GIGANTIC bombs

1000 times bigger than first fission bombs

http://www.youtube.com/watch?v=WwlNPhn64TA

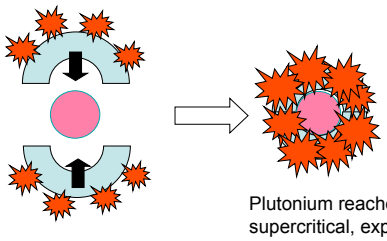
38

Fusion bomb (simple picture)



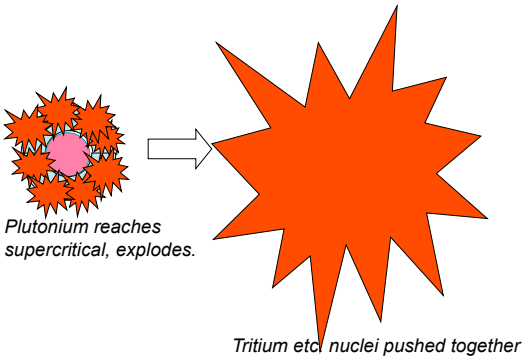
Shaped plutonium and assembled bombs at Rocky Flats.

39



Plutonium reaches supercritical, explodes. *(fission bomb)*

40



Plutonium reaches supercritical, explodes.

Tritium etc nuclei pushed together and combine to form helium.

41

Energy:

1 fission of Uranium 235 releases:
~10^{-11} Joules of energy

1 fusion event of 2 hydrogen atoms:
~10^{-13} Joules of energy

Burning 1 molecule of TNT releases:
~10^{-18} Joules of energy

1 green photon:
~10^{-19} Joules of energy

Dropping 1 quart of water 4 inches ~ 1J of energy
Useful exercise… compare this volume of TNT, H2, and U235

42

US Nuclear weapons