PH300 Spring 2011

Homework 02

Total Points: 30

1. (1 Point) Each week you should review both your answers and the solutions for the previous week's homework to make sure that you understand all the questions and how to answer them correctly. Each week you will receive credit for reviewing your old homework, which will be returned to you every Tuesday. Please review your homework and the solutions from last week and let me know that it was graded correctly. If it was not, state here which problems were incorrectly graded, and then contact me (via email or before/after class).

2. (2 Points) Your first homework question this week is to submit one homework correction from the previous week's homework. Select one problem for which you had the wrong answer, and then:

1. Identify the question number you are correcting.

2. State (copy) your original wrong answer

3. Explain where your original reasoning was incorrect, the correct reasoning for the problem, and how it leads to the right answer.

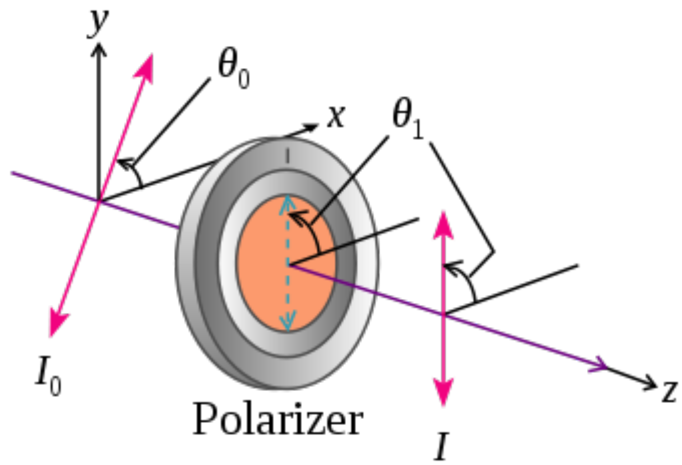
If you got all the answers correct, Great!!! Then state which was your favorite or most useful homework problem and why.

3**. (2 Points)** Explain why accelerating charges generate light but charges that are stationary or moving at a constant velocity do not.

4. (1 Point) It is the thermal motion of charged particles at the sun's surface that produces the electromagnetic radiation emitted by the sun.

To generate a blue light at 400 nm, at what frequency would a charged particle have to be vibrating back and forth? ($1\ nm = 10^{-9}\,m$)

5. (2 Points) For an ideal polarizing filter, the transmitted intensity for a beam of linearly polarized light is given by an equation known as Malus' Law:



$$I = I_0 \cos^2(\theta_i)$$

θ_i is the relative angle between the polarization of the incoming beam (θ_0) and the transmission axis of the filter (θ_1):

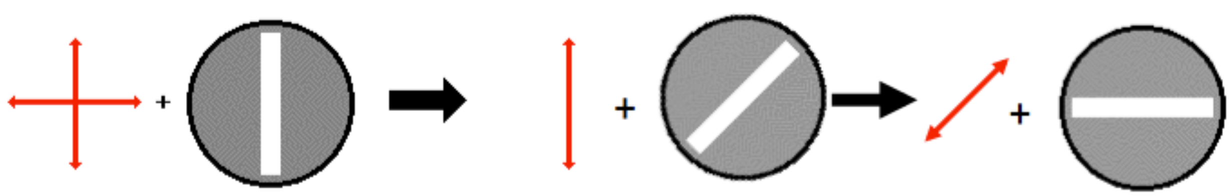
$$\theta_i = \theta_1 - \theta_0$$

I_0 is the intensity of the incoming beam, and I is the intensity of the transmitted beam.

One way of producing a beam of linearly polarized light is by passing a beam of unpolarized light through a single polarizer. Unpolarized light can be thought of as an equal mixture of all types of linear polarizations; since the average of $\cos^2(\theta)$ is equal to 1/2, the transmitted light will have half the intensity of the incoming beam, and will have the same linear polarization as the transmission axis of the filter. This means that 100% of that transmitted light would be transmitted by a second filter oriented along the same axis:

$$\theta_0 = \theta_1 \quad \rightarrow \quad \theta_i = \theta_1 - \theta_0 = 0 \quad \rightarrow \quad \cos^2(\theta_i) = 1 \quad \rightarrow \quad I = I_0$$

If a beam of unpolarized light passes first through a vertical filter, and then a horizontal filter, 0% of the beam will be transmitted ($\theta_i = 90^0$). However, if a filter oriented at 45^0 from the vertical is placed between the vertical and horizontal filters, light will be transmitted. Use Malus' Law to calculate the fraction of the incoming beam of unpolarized light that would be transmitted in this situation:



6. (2 Points) A double-slit experiment is performed with a coherent (single frequency/wavelength) beam of laser light.

A) Suppose that the two slits are separated by a distance $D = 0.1\ mm$; the distance (H) between the center of the pattern and the ***second*** ($m = 2$) bright region is $1.0\ cm$; and the distance (L) between the screen and the slits is $1.0\ m$. Use the results of this experiment to determine the wavelength of the light.

B) Suppose that the frequency of the laser light is doubled. Determine the new spacing between the center of the pattern and the ***first*** fringe maximum.

Introduction to the final problem:

It will be to your benefit to read the example below more than once to understand what you are doing fully. Basically this problem is for you to start from what Maxwell's equations say about time dependence of electric fields, and go through to solve what the oscillating electric field looks like inside a microwave oven or a laser cavity. If you can do that, you can in principle solve pretty much any problem involving electromagnetic waves. The following is an example that illustrates all the math and physics steps you need to follow, and the solution has been broken down into the different steps that are required. This problem also serves as a "mini-course" covering the differential equations you will need later in the semester. You should read all the way through this problem, including the worked example at the beginning, before you start trying to solve it. Doing so will help you see how the pieces fit together and make solving it easier.

This problem deals with the basic properties of light. Skipping a hundred years or so of experiment and argument, it is now well established that light, and all electromagnetic radiation, are coupled oscillating electric and magnetic fields. In this course we will concentrate primarily on the electric part because that is what exerts a force on the electron, which is the primary mechanism by which light interacts with atoms. It was well established in the 19th century that light behaves as a classical wave, with the most compelling experiment being that the interference pattern observed when light passed through a double slit was identical to that observed with sound waves or water waves as they pass through a double slit. However, to get a microscopic view of the electric field in a light wave we must use the fact that the fields obey Maxwell's equations, and then calculate from these equations what the electric field does as a function of space and time. Once we know that, we can then use that information to start understanding how that electric field interacts with matter by exerting forces on electrons. With a bit of algebra, one can show that Maxwell's equations describing electric and magnetic fields say that the vector field E is given by:

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

…which is a highly condensed form of a long equation. Taking just the 1-dimensional case, which is a lot simpler and all we will need to worry about in this class, the equation becomes:

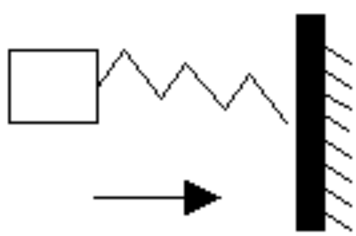
$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} \qquad \text{(EM wave equation.)}$$

This just says that the second derivative of the electric field with respect to x is equal to a constant times the second derivative of E with respect to time. If you are not familiar with the symbol, ∂, it means a "partial derivative". Just think of it as like the derivative symbol d, but it is used when there is a quantity like E that is a function of more than one variable. Here E is a function of both position and time.

The problem is now to solve this equation to see what E(x,t) is. Later in the course we will need to solve other equations that are similar to this one, so you should remember the steps involved in the solution so you can use them again. If you have already studied differential equations, you can do this problem quickly. However, we are going to assume you have not had differential equations, and so we will first show you how to solve a similar problem involving something you are familiar with, a mass attached to a spring. Then you will need to apply the same process to solve the equation for the electric field of a light wave.

<u>Example problem</u> - illustrating the steps required in solving a differential equation.

To find the position as a function of time [x(t)] of a mass [m] attached to a spring of known spring constant k, after it is pulled back 1 cm and then released.



As you should remember from earlier physics courses, the force on the mass due to the spring is $F = -kx$, but Newton's laws also tell us that $F = ma$, so we have $-kx = ma$, and plugging in for $a = \frac{\partial^2 x}{\partial t^2}$, we have that $-kx = m\frac{\partial^2 x}{\partial t^2}$.

This is the differential equation governing the position of the mass, x(t). There are basically two ways to solve differential equations like this. 1) Go ask a computer the answer, and 2) Guess the solution and check that it works. In this course, we will use the second approach most of the time (it's not a math class, after all). To solve this, we first try to guess a suitable algebraic function for x that satisfies this equation.

A) Guessing and testing solutions to the differential equation

Let's try $x = At^2$ (where A is an arbitrary constant to be determined) as a trial solution.

Plugging into the above equation and taking the derivatives, we get $-k(At^2) = m(2A)$.

This equation makes no sense, because on the left side of the equation we have the variable t, and on the right we only have constants. That means that $x = At^2$ does not work as a solution.

Let's try another guess, $x = A\sin(Bt)$. Plugging this in we get $-kA\sin(Bt)$ on the left and $-AB^2\sin(Bt)$ on the right, and so $-kA\sin(Bt) = -mB^2A\sin(Bt)$.

Does this equation make sense? The A's and $\sin(Bt)'s$ divide out, leaving $k = mB^2$. Since these are all constants, if $B^2 = k/m$, this equation works. That means that $x = A\sin(Bt)$ is a solution to this differential equation, but only if we constrain B, such that $B^2 = k/m$.

This is a solution, but it is not the only solution, and hence may not describe the position of the mass as a function of time for all situations. We might guess another possible solution would be $x = D\cos(Gt)$. You should plug in to check that this also works as a solution as long as $G^2 = k/m$.

B) Interpreting the behavior of solutions (finding the period and frequency).

This solution says that the value of x increases and decreases periodically as a function of time. The time it takes to repeat is the period, and in this time, the quantity $G \cdot t$ must change by 2π radians. So the period is $T = 2\pi / G$ seconds/cycle. The frequency is how many times the position (x) oscillates through a full cycle per second, and so this is just 1 second/(the period). As a check to your understanding, you should show for yourself that the frequency of oscillation is $f = G/2\pi$.

C) Developing a general solution.

We see that $x = A\sin\left(\sqrt{k/m} \cdot t\right)$ is a solution, but so is $x = D\cos\left(\sqrt{k/m} \cdot t\right)$ as well. How are you supposed to know which solution to use? The most general possible solution is the sum (superposition) of these two: $x = A\sin\left(\sqrt{k/m} \cdot t\right) + D\cos\left(\sqrt{k/m} \cdot t\right)$.

You can check for yourself: if you have two functions that work as a solution to a linear differential equation, when you plug them in their sum will also work as a solution. It may seem like this could go on forever with you just guessing an infinite number of solutions that would work, but the theory of differential equations tells us things are not so bad. If we only have two derivatives in the differential equation (second-order), there are only two independent solutions, so once we have guessed both sine and cosine and see that they both work, their sum is the most general solution we can have.

D) Going from a general to a specific solution (applying boundary conditions).

The next step is to figure out what those arbitrary constants A and D are. If you look at the form of the solution, you realize that A and D will describe where the mass will be and how fast it will be moving at any particular time. So these constants are determined by what we call the "boundary conditions" for the solution, namely what value the solution is at some particular time and/or place. This value is set by the physical situation that is specified. If we are given no other information about the position of this mass, there is no way we can figure out what these constants must be.

However, if we are given more information, for example the mass is pulled to stretch the spring a distance of 1 cm and it is released with zero velocity at t=0. The constants A and D must be the correct values to describe this situation. So at t=0, the solution must be $x(t=0)=-1\ cm$ and $v(t=0)\ =\ 0$, and we apply these specific conditions to our general solution:

$$x = A\sin\left(\sqrt{k/m}\cdot t\right) + D\cos\left(\sqrt{k/m}\cdot t\right)$$

To apply the first boundary condition, $x(t=0)=-1\ cm$, we just plug in t=0 and $x=-1\ cm$:

$$-1\ cm = A\sin(0) + D\cos(0).$$

If you think about this a little, you should be able to convince yourself that the only way to satisfy this condition is for $D=-1\ cm$. The value of A could be any value, and we would still have the correct position at that time. To find A, we need to apply the second boundary condition, $v(t=0)\ =\ 0$. To do this, we need to first take the time derivative of the general solution to get an equation for v:

$$v = \sqrt{k/m}\cdot A\cos\left(\sqrt{k/m}\cdot t\right) - \sqrt{k/m}\cdot D\sin\left(\sqrt{k/m}\cdot t\right)$$

Then we plug in t=0 and v=0:

$$0 = \sqrt{k/m}\cdot A\cos(0) - \sqrt{k/m}\cdot D\sin(0)$$

You should be able to convince yourself that the only way to satisfy this condition is for A=0. Finally, we have a solution to our differential equation that describes the position of our mass at any subsequent time: $x = -1cm\cdot\cos\left(\sqrt{k/m}\cdot t\right)$.

For slightly more complex differential equations, such as you will be encountering later on, there will be more than two constants, and you will need to know additional information (more “boundary conditions”) about the system in order to figure out what the constants must be for any particular physical situation.

To review this example, we figured out the behavior of the mass by:

1) Finding the differential equation that governs the mass by applying the physics that describes it in this situation (Newton’s laws).

2) Finding a general solution to the differential equation by guessing trial solutions that have particular functional forms of the variables in the problem, and plugging them into the differential equation to see if they make sense. They make sense if there are some possible values for the unknown constants that might work, and you do not have variables equal to constants. To know you have the correct solution, you must guess a number of independent solutions and the most general solution is the sum of these. (You need 2 if you have 2 derivatives in your equation).

3) Finding the specific solution to the differential equation by using the boundary conditions that are given for the situation, to determine the values that the unknown constant must have.

(IMPORTANT: You will want to keep in mind the following when answering this problem)

1) Identifying the physical principles or "key ideas" (expressed in words) that apply to the problem and your strategy for approaching the problem.

2) Explaining (in words) the reasoning that goes along with the equations/math you are doing.

3) Showing the details of your solution. **(equations/math)**

4) Clarity of your explanation.

You now need to follow the same procedure as shown in this example to figure out what the electric field E is as a function of x and t in a microwave oven or a laser cavity. Although the solution has been broken up into a set of steps for you to follow, keep in mind these are just pieces to the puzzle, and you should keep in mind how they all fit together. In future homework problems I will expect you to go through all the steps on your own.

Problem 7. (20 Points Total – see individual parts for each point value) In the example above, we studied the wave equation for the position x of a mass as a function of time t. Now we will study the wave equation for the electric field E as a function of x and t. We are starting with the fact that the in 1-dimension E is described by the equation:

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

7A. (2 Points) At any fixed point along x, the function $D\cos(Fx + Gt)$ **i**ncreases and decreases as a function of time. The period is how long it takes to go through one complete cycle at any given x (say x=0, for example). What is the period and frequency of the oscillation in terms of the unknown constants in the solution? (Remember <u>always</u> to explain your reasoning in words).

7B. (2 Points) A wave repeats as a function of position as well as time. At a fixed time, the wavelength is how much x must change to get one full cycle of the wave. What is the wavelength in terms of the unknown constants? Explain your reasoning.

7C. (4 Points) Sketch what this solution will look like as a function of x at t = 0, t = 0.2π/G, and t = 0.4π/G. Use this sketch to determine which direction this wave is traveling. Explain what is meant by the speed of the wave and how this speed is related to the constants in the solution.

7D. (2 Points) Find a second functional form for $E(x,t)$ that also works as a solution.

(**<u>Hint:</u>** You want a wave traveling in the opposite direction for simplicity! Just picking Sine as in the worked example is a <u>much harder</u> approach to this and will give you problems later on).

Show that your functional form works and find any constraints there may be on the unknown constants in the solution. The general solution is then the sum of this function and whatever function you found previously. Note that you cannot assume that the amplitudes of the two functions are the same. (Why not?)

For working with the general solution, you may find the following trig identities useful when simplifying your solution to the boundary conditions:

$$\cos(s+t) = \cos(s)\cos(t) - \sin(s)\sin(t)$$
$$\sin(s+t) = \sin(s)\cos(t) + \cos(s)\sin(t)$$
$$\cos(s-t) = \cos(s)\cos(t) + \sin(s)\sin(t)$$

$\sin(s-t) = \sin(s)\cos(t) - \cos(s)\sin(t)$

7E. (6 Points) Now we will consider the effect of the boundary conditions on the oscillating electric field in a 0.59 m wide microwave oven where the metal walls require that the electric field is always zero at the position of the wall. This boundary condition applies since metals have a great number of electrons that are free to move about the metal in response to any applied E-field. The electrons in the metal will configure themselves so that the total E-field in the metal will vanish. This implies that the waves describing the electric field inside the cavity must have nodes at either ends of the cavity. So, $E(x=0)=0$ and $E(x=0.59m)=0$ for all values of t. What values can the constants in the general solution have and still satisfy these boundary conditions? What constraint does this place on the possible wavelengths in the x direction that can be inside the microwave oven? Sketch out what the field in the oven will look like for a few possible wavelengths that would work. (If you need a hint, this problem is mathematically quite similar to the problem of a wave on a violin string, with the boundary condition being that the string is held so it cannot move at either end. The solutions to the respective differential equations have corresponding similarities, and you should end up with a standing wave.)

7F. (4 Points) Microwave ovens operate at a frequency of very close to 2.54 GHz. Use this to determine the wavelength of your wave. Make a sketch, with explanation and scale showing what the electric field will look like as a function of space at several different times (e.g. t=0, t=0.25π, t=0.5π, t=0.75π, and t=π) in the oven. Is your picture consistent with your boundary conditions? What on your sketch represents the wavelength? The period? In a microwave oven the oscillating electric field causes the molecules in the food to oscillate back and forth and this motion is turned into heat, thereby heating up the food. How does your sketch of the field explain why there are particular locations in a microwave oven where the food does not heat as well as at other locations?