

PH300 Spring 2011

Homework 10

Total Points: 30

1. (1 Point) Each week you should review both your answers and the solutions for the previous week's homework to make sure that you understand all the questions and how to answer them correctly. You will receive credit for reviewing your old homework, which will be returned to you every Tuesday. Please review your homework and the solutions from last week and let me know that it was graded correctly. If it was not, state here which problems were incorrectly graded, and then contact me (via email or before/after class).

2. (2 Points) Your first homework question this week is to submit one homework correction from the previous week's homework. Select one problem for which you had the wrong answer, and then:

1. Identify the question number you are correcting.
2. State (copy) your original wrong answer
3. Explain where your original reasoning was incorrect, the correct reasoning for the problem, and how it leads to the right answer.

If you got all the answers correct, Great!!! Then state which was your favorite or most useful homework problem and why.

3-6. (8 Points Total) Many books only present the time-independent form of the Schrödinger equation. This is misleading, because even for cases where you can use the time-independent formulation, there is STILL A TIME DEPENDENT term needed in the full expression for the wave function. In these problems, we will work from the time-dependent Schrödinger equation and see how, for many cases, we can solve for the time-dependence and the position-dependence of the wave function separately.

3. (2 Points) Starting with Schrödinger's equation:

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x,t) + V(x,t)\Psi(x,t) = i\hbar \frac{\partial}{\partial t} \Psi(x,t)$$

Show that, if you re-express the total wave function as a function of position – lower case $\psi(x)$ – times a function of time – lower case $\phi(t)$:

$$\Psi(x,t) = \psi(x)\phi(t)$$

...you can rearrange Schrödinger's equation so that all of the terms involving position are on one side and all of the terms involving time are on the other side (This is called *separation of variables*: we are separating x and t):

$$-\frac{\hbar^2}{2m} \frac{1}{\psi(x)} \frac{d^2}{dx^2} \psi(x) + V(x) = i\hbar \frac{1}{\varphi(t)} \frac{d}{dt} \varphi(t)$$

4. (1 Point) Explain why you could not separate the variables in this way if the potential energy function was changing with time.

5. (1 Point) Explain why the only possible solution for this equation is for both sides to be equal to a constant (We'll call it C). That is:

$$-\frac{\hbar^2}{2m} \frac{1}{\psi(x)} \frac{d^2}{dx^2} \psi(x) + V(x) = i\hbar \frac{1}{\varphi(t)} \frac{d}{dt} \varphi(t) = C$$

...such that we end up with 2 expressions:

$$-\frac{\hbar^2}{2m} \frac{1}{\psi(x)} \frac{d^2}{dx^2} \psi(x) + V(x) = C \quad \& \quad i\hbar \frac{1}{\varphi(t)} \frac{d}{dt} \varphi(t) = C$$

(HINT: It's very similar to the logic that says that $x = t^2$, is not a possible solution for $m \cdot d^2x/dt^2 = -kx$)

6. (4 Points) To find $\varphi(t)$, we must now solve the time equation:

$$i\hbar \frac{1}{\varphi(t)} \frac{d}{dt} \varphi(t) = C$$

1. Show that $\varphi(t) = e^{-iCt/\hbar}$ is a solution
2. Express this solution also in terms of sines and cosines.
3. What are the angular frequency (ω) and the frequency (f) for this solution?
4. Given that the relationship between energy and frequency is the same for matter particles and for photons, what is the relationship between C and the energy (E) of the electron (or matter particle) in this case? What is $\varphi(t)$ in terms of E ?

7. (1 Point) In class we solved Schrodinger's equation to get the wave functions for an electron trapped in an infinite 1-dimensional square well potential. This potential is a reasonable approximation of the potential energy for an electron in an isolated segment of wire. If the wire is length L , what wavelengths are allowed for the wave function of the electron?

8. (2 Points) Why does this restriction on the wavelengths lead to a quantization of the possible energies an electron can have in the wire? Show how to determine these quantized energies from the quantized wavelengths (include the final express for the quantized energies)? (Be sure to explain your reasoning)

9. (1 Point) What is the smallest possible energy (in femto-eV (10^{-15} eV)) – the ground state - for an electron in a 0.001 m (1 mm) wire?

10. (1 Point) How much energy (in eV) does an electron at room temperature (300 K) have? Recall from an earlier homework that the average thermal energy of an electron is given approximately by $3k_B T/2$ where k_B is Boltzmann's constant.

11. (1 Point) What would the length of a wire (in nm (10^{-9} m)) need to be so that the thermal energy of electrons at room temperature matches the energy difference between the ground state energy level and the first excited state?

12-19. For the next 8 questions, consider the electron energy levels in the wire: $E = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$

12. (0.5 Points) (True/False) As the wire gets longer, the ground state energy level gets lower.

13. (0.5 Points) (True/False) As the wire gets longer, the energy differences between the energy levels get larger.

14. (0.5 Points) (True/False) As the wire gets longer, the wavelength of the electron in the lowest energy state gets longer.

15. (0.5 Points) (True/False) The energy of level 5 ($n=5$) is 25 times the energy of the ground state energy.

16. (0.5 Points) (True/False) For a wire that is 1 foot long, there will be many, many (much more than a million) allowed energy levels between the ground state energy & the energy level corresponding to the ground state energy plus the thermal energy of the electrons at room temperature.

17. (0.5 Points) (True/False) If you halve the length of the wire, the energy of the ground state increases by a factor of 4.

18. (0.5 Points) (True/False) If you double the length of the wire, the energy spacing between levels $n=1$ and $n=2$ will decrease and be equal to 0.5 times what it was.

19. (0.5 Points) (True/False) The quantization of energy levels and the fact there is a gap between allowed levels only become large and important (relative to the typical thermal energies important in describing the behavior of the electron) when the length of the wire becomes submicroscopic as it does in nanotechnology.

20-28. (4.5 Points Total) We saw that Bohr was able to accurately predict the energy levels for the hydrogen atoms. So far we've used Schrodinger's equation to predict the energy levels for an electron in a wire (approximated by an infinite square well potential).

For which of these two physical situations are the following statements true:

20. (0.5 Points) The spacing between the allowed energies decreases for higher energy levels:

Bohr Hydrogen Atom	Infinite Square Well	Both	Neither
---------------------------	-----------------------------	-------------	----------------

21. (0.5 Points) The quantized energies go as $1/n^2$:

Bohr Hydrogen Atom	Infinite Square Well	Both	Neither
---------------------------	-----------------------------	-------------	----------------

22. (0.5 Points) The quantized energies go as n :

Bohr Hydrogen Atom	Infinite Square Well	Both	Neither
---------------------------	-----------------------------	-------------	----------------

23. (0.5 Points) The quantized energies go as n^2 :

Bohr Hydrogen Atom	Infinite Square Well	Both	Neither
---------------------------	-----------------------------	-------------	----------------

24. (0.5 Points) The quantized energies depend on length:

Bohr Hydrogen Atom	Infinite Square Well	Both	Neither
---------------------------	-----------------------------	-------------	----------------

25. (0.5 Points) Quantization of energy levels results from boundary conditions:

Bohr Hydrogen Atom	Infinite Square Well	Both	Neither
---------------------------	-----------------------------	-------------	----------------

26. (0.5 Points) There are an infinite number of allowed energy levels:

Bohr Hydrogen Atom	Infinite Square Well	Both	Neither
---------------------------	-----------------------------	-------------	----------------

27. (0.55 Points) The kinetic energy of the electron increases with increasing n:

Bohr Hydrogen Atom Infinite Square Well Both Neither

28. (0.5 Points) The potential energy of the electron increases with increasing n:

Bohr Hydrogen Atom Infinite Square Well Both Neither

29. (2 Points) The **correspondence principle** states that as the scale of things becomes larger, the behavior predicted by quantum mechanics must match the behavior predicted/observed in classical mechanics. So in the case of something in a potential well, explain what happens in terms of the quantum behavior as you make the well wider or you make the particle more massive that makes it look more and more classical in its behavior.

30. (0.5 Points) Suppose that in the infinite square well problem, you set the potential energy at the bottom of the well to be equal to some constant V_0 , rather than zero. Which of the following is the correct Schrodinger Equation inside the well?

- A. $-\frac{\hbar^2 \partial^2 \Psi(x,t)}{2m \partial x^2} = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$
- B. $-\frac{\hbar^2 \partial^2 \Psi(x,t)}{2m \partial x^2} + V_0 = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$
- C. $-\frac{\hbar^2 \partial^2 \Psi(x,t)}{2m \partial x^2} - V_0 = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$
- D. $-\frac{\hbar^2 \partial^2 \Psi(x,t)}{2m \partial x^2} + V_0 \Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$
- E. $-\frac{\hbar^2 \partial^2 \Psi(x,t)}{2m \partial x^2} - V_0 \Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$

31. (0.5 Points) Which of the following are the correct energy levels for the infinite square well of width L with potential energy equal to V_0 at the bottom of the well?

- A. $n^2\pi^2\hbar^2/2mL^2$
- B. $n^2\pi^2\hbar^2/2mL^2 + V_0$
- C. $n^2\pi^2\hbar^2/2mL^2 - V_0$
- D. $V_0 - n^2\pi^2\hbar^2/2mL^2$
- E. $n^2\pi^2\hbar^2/2mL^2V_0$
- F. $V_0n^2\pi^2\hbar^2/2mL^2$
- G. None of the above – the potential energy has to be zero at the bottom of an infinite square well.

32. (0.5 Points) Which of the following are possible solutions to the 1D Schrodinger equation

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$$

for the case where $V(x,t) = 0$ and the particle has a positive total energy? (Choose all that apply)

- A. $\Psi(x,t) = \exp(ikx)\exp(-iEt/\hbar)$ (k and E are + real constants)
- B. $\Psi(x,t) = A \exp(ikx - i\omega t) + B \exp(-ikx - i\omega t)$ (A , B , k , and ω are + real constants)
- C. $\Psi(x,t) = A \exp(\alpha x)\exp(-iEt/\hbar) + B \exp(-\alpha x)\exp(-iEt/\hbar)$ (A , B , α , and E are + real constants)
- D. $\Psi(x,t) = (A \sin(kx) + B \cos(kx))\exp(-i\omega t)$ (A , B , k , and ω are + real constants)
- E. $\Psi(x,t) = \exp(-\alpha x)\exp(-i\omega t)$ (α and ω are + real constants)
- F. $\Psi(x,t) = (L \exp(iUx) + T \cos(Qx))\exp(-iGt)$ (L , U , T , Q , and G are + real constants)

33. (1 Point) You have 3 wires of equal length: a copper wire, an aluminum wire, and a nickel wire. The wires are long enough that in each case the energy levels are spaced close together such that they are essentially continuous. In each wire, an electron has a small amount of kinetic energy (say 0.5eV). In the regions outside of either end of the wire, the solutions to the Schrodinger equation gives you a real exponential. For which scenario is α , the decay constant, smallest?

34. (1 Point) An electron is bound in a potential well. The wave function of the electron is $\Psi(x,t) = \psi(x)e^{-i\omega t}$ where $\psi(x)$ is shown below and is a real number. Which plot of $V(x)$ vs. x below could represent the potential well in which this electron is bound?

