

Quantum I (PHYS 3220)

concept questions

Schrödinger Equation

Consider the eigenvalue equation

$$\frac{d^2}{dx^2}[f(x)] = C \cdot f(x)$$

How many of the following give an eigenfunction and corresponding eigenvalue?

I. $f(x) = \sin(kx)$, $C = +k^2$

II. $f(x) = \exp(-x)$, $C = +1$

III. $f(x) = \exp(i k x)$, $C = -k^2$

IV. $f(x) = x^3$, $C = 6$

A) 1 B) 2 C) 3

D) all 4 E) None

Is $\frac{d}{dt} \int_{-\infty}^{\infty} \Psi^*(x, t) \Psi(x, t) dx$

$$= \int_{-\infty}^{\infty} \frac{d}{dt} \left(\Psi^*(x, t) \Psi(x, t) \right) dx \quad ?$$

A) Yes, no problem!

B) There's something not right about this...

Two particles, 1 and 2, are described by plane wave of the form $\exp[i(kx - \omega t)]$. Particle 1 has a smaller wavelength than particle 2: $\lambda_1 < \lambda_2$

Which particle has larger momentum?

- A) particle 1
- B) particle 2
- C) They have the same momentum
- D) It is impossible to answer based on the info given.

$\Psi_1(x, t)$ and $\Psi_2(x, t)$ are two solutions of the time-dependent SE.

Is $\Psi_{\text{sum}}(x, t) = a \cdot \Psi_1(x, t) + b \cdot \Psi_2(x, t)$ also a solution of the TDSE?

- A) Yes, always
- B) No, never
- C) Depends on $\Psi_1(x, t)$ and $\Psi_2(x, t)$
- D) Depends on a and b

$\Psi_1(x, t)$ and $\Psi_2(x, t)$ are two
NORMALIZED solutions of the time-
dependent SE.

Is $\Psi_{\text{sum}}(x, t) = a \cdot \Psi_1(x, t) + b \cdot \Psi_2(x, t)$ also
a normalized solution of the TDSE?

- A) Yes, always
- B) No, never
- C) Depends on $\Psi_1(x, t)$ and $\Psi_2(x, t)$
- D) Depends on a and b

Which expression below would be the QM equation for $\langle KE \rangle$?

A)
$$\int_{-\infty}^{\infty} \frac{-\hbar^2 k^2}{2m} (\Psi^*(x, t) \Psi(x, t)) dx$$

B)
$$\int_{-\infty}^{\infty} \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} (\Psi^*(x, t) \Psi(x, t)) dx$$

C)
$$\int_{-\infty}^{\infty} \frac{-\hbar^2}{2m} \Psi^*(x, t) \frac{\partial^2}{\partial x^2} (\Psi(x, t)) dx$$

D) None of these! E) More than one!

After assuming a product form solution

$\Psi(x,t) = \psi(x) \cdot \phi(t)$, the TDSE becomes

$$i\hbar \frac{1}{\phi} \frac{\partial \phi}{\partial t} = \frac{-\hbar^2}{2m} \frac{1}{\psi} \frac{\partial^2 \psi}{\partial x^2} + V = E$$

If the potential energy function V in the Schrödinger Equation is a function of time, as well as x [$V = V(x,t)$] would separation of variables still work; that is, would there still be solutions to the SE of the form

$$\Psi(x,t) = \psi(x) \cdot \phi(t)?$$

A) Yes, always

B) No, never

C) Depends on the functional dependence of V on x and t

$\Psi_1(x, t)$ and $\Psi_2(x, t)$ are two solutions of the time-dependent SE.

Is $\Psi_{\text{sum}}(x, t) = a \cdot \Psi_1(x, t) + b \cdot \Psi_2(x, t)$ also a solution of the TDSE?

A) Yes B) No

C) Depends on $\Psi_1(x, t)$ and $\Psi_2(x, t)$

D) Depends on a and b

Do you know what the momentum operator is?

A) Yes

B) No

Do you plan to attend today's Tutorial (on relating classical to Quantum, and qualitative “sketching” of wave functions)

A) Yes, at 3 pm

B) Yes, at 4 pm

C) Perhaps, more likely at 3

D) Perhaps, more likely at 4

E) No, can't come/not planning on it.

Given $\Psi_n(x, t)$ as one of the eigenstates of $\hat{H}\Psi_n = E_n\Psi_n$,
what is the expectation value of the
Hamiltonian-squared?

$$\langle \hat{H}^2 \rangle = \int \Psi_n^* \hat{H}(\hat{H}\Psi_n) dx = ?$$

- A) E_n
- B) E_n^2
- C) zero
- D) $E_n^2 - E_n$
- E) Something else/it really depends!!

Ψ_1 and Ψ_2 are two energy eigenstates of the Hamiltonian operator.

They are non-degenerate, meaning they have different eigenvalues E_1 and E_2 .

$\hat{H}\Psi_1 = E_1\Psi_1$ and $\hat{H}\Psi_2 = E_2\Psi_2$ and $E_1 \neq E_2$.

Is $\Psi_s = \Psi_1 + \Psi_2$ also an energy eigenstate?

A) Yes, always

B) No, never

C) Possibly yes, depends!

Given $u_n(x) = A \sin(k x) + B \cos(k x)$,
the boundary condition, $u(0) = 0$,
implies what?

A) $A = 0$

B) $B = 0$

C) $k = 0$

D) $k = n \pi, n = 1, 2, 3 \dots$

E) None of these

Given $u_n(x) = A \sin(k x)$
the boundary condition, $u(a) = 0$,
implies what?

- A) $A = 0$
- B) $B = 0$
- C) $k = 0$
- D) $k = n \pi, n = 1, 2, 3 \dots$
- E) None of these

An electron and a neutron have the same speed. Which particle has the shorter wavelength?

- A) The electron
- B) The neutron
- C) They have the same wavelength

All energy eigenstates (stationary states) have the form $\Psi(x, t) = \psi(x) \cdot e^{-i\omega t}$ so that $|\Psi(x, t)|^2 = |\psi(x)|^2$ is time-independent.

Consider the sum of two non-degenerate stationary states :

$$\Psi_{\text{sum}}(x, t) = \Psi_1 + \Psi_2 = \psi_1(x) \cdot e^{-i\omega_1 t} + \psi_2(x) \cdot e^{-i\omega_2 t}$$

Is this wavefunction stationary; that is, is $|\Psi_{\text{sum}}(x, t)|^2$ time-independent?

A) Yes, always

B) No, never

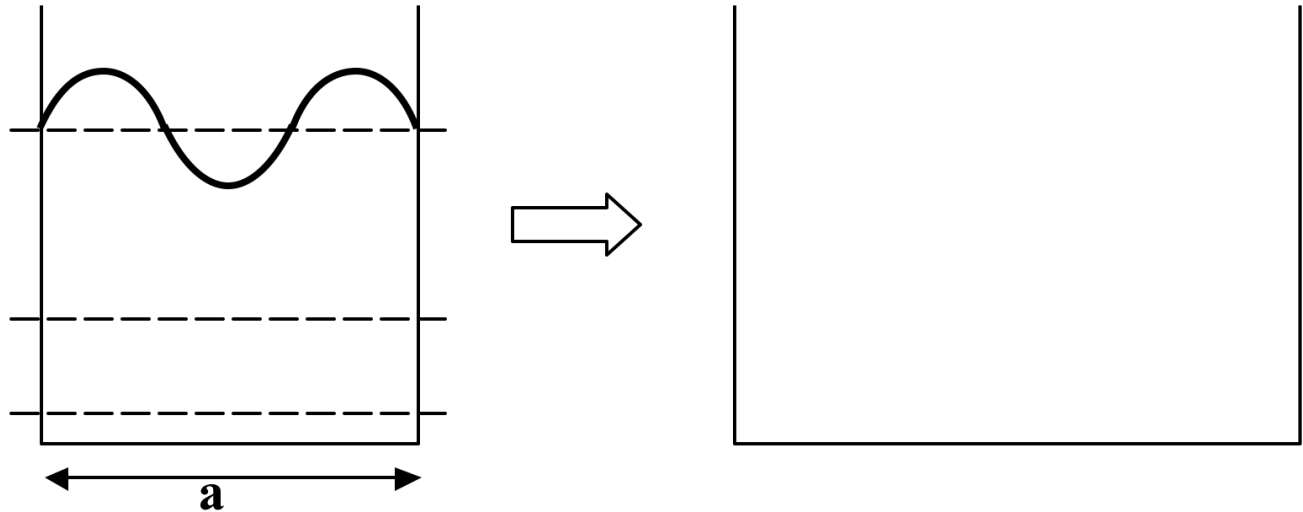
C) Depends on the Ψ s and the ω s.

How does the energy E_1 of the ground state ($n=1$) of an infinite square well of width a compare with the energy of the ground state of a well with a larger width?

The larger well has ...

- A) lower energy
- B) higher energy
- C) the same energy
- D) Need more information

How does the energy E of the $n=3$ state of an infinite square well of width a compare with the energy of the $n=3$ state of a well with a larger width? *The larger well has ...*



- A) lower energy
- B) higher energy
- C) the same energy

In an infinite square well, the lowest two stationary states are $u_1(x)$ and $u_2(x)$.

At time $t=0$, the state of a particle in this square well is $\Psi(x, t = 0) = \frac{1}{\sqrt{2}} (u_1(x) + u_2(x))$.

Is this particle in a stationary state?

- A) Yes, Ψ is a stationary state.
- B) No, Ψ is not a stationary state at any time
- C) No, Ψ is a stationary state only when $t=0$
- D) Not enough information.

The solutions to the energy eigenvalue equation are either $\psi_n(x)$ or

$$\Psi_n(x, t) = \psi_n \cdot \exp(-iE_n t / \hbar)$$

The ψ_n s are orthonormal: $\int \psi_m^* \psi_n dx = \delta_{mn}$

Are the Ψ_n s orthonormal? $\int \Psi_m^* \Psi_n dx = \delta_{mn}$

A) Yes B) No C) Depends on the E_n s

At $t=0$, could the wavefunction for an electron in an infinite square well of width a ($0 < x < a$) be $\Psi(x,0) = A \sin^2(\pi x / a)$, where A is a suitable normalization constant?
(Assume it is zero *outside* the region $0 < x < a$)

A) Sure

B) No, it's not *physical*

The energy eigenstates, u_n , form an orthonormal set, meaning

$$\int u_m^*(x) u_n(x) dx = \delta_{mn}$$

What is $\int u_m^*(x) \left(\sum_n c_n u_n(x) \right) dx = ?$

A) $\sum_n c_n$

B) $c_n c_m$

C) c_m

D) c_n

E) None of these

Given a particle in a box (size a), with

$$\Psi(x, t = 0) = \sqrt{\frac{2}{a}} \left(\sqrt{\frac{1}{3}} \sin(\pi x / a) + \sqrt{\frac{2}{3}} \sin(3\pi x / a) \right)$$

What is the probability of measuring E_1 ? How about measuring E_2 ?

- A) $\text{Prob}(E_1) = 1/3$, $\text{Prob}(E_2) = 2/3$
- B) $P(E_1) = \text{Sqrt}[1/3]$, $P(E_2) = \text{Sqrt}[2/3]$
- C) $P(E_1) = 1/3$, $P(E_2) = 0$
- D) $P(E_1) = \text{Sqrt}[1/3]$, $P(E_2) = 0$
- E) None of these is correct

$$\Psi(x, t = 0) = \sqrt{\frac{2}{a}} \left(\sqrt{\frac{1}{3}} \sin(\pi x / a) + \sqrt{\frac{2}{3}} \sin(3\pi x / a) \right)$$

Assuming your system is isolated,
does the probability of measuring E_1
depend on the *time* (t) of the
measurement?

A) Yes

B) No

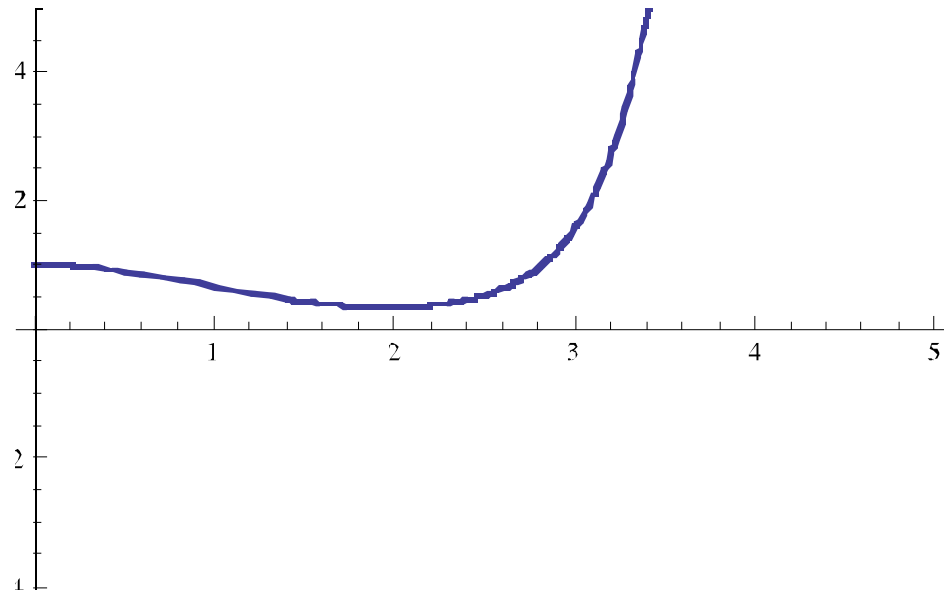
Consider a classical particle with energy E moving in a potential $V(x)$. What is true about a region where $E < V(x)$?

- A) The particle can never be there
- B) The particle can be there, but it is trapped
- C) The particle can be there, and can escape to infinity
- D) Depends on the details of $V(x)$

Consider a QUANTUM particle with energy E moving in a potential $V(x)$. What is true about a region where $E < V(x)$?

- A) Ψ must be zero there
- B) Ψ can be nonzero there, and oscillates
- C) Ψ can be nonzero there, and goes to zero with increasing x
- D) Ψ can be nonzero there, and goes to zero or gets larger with increasing x
- E) Something else

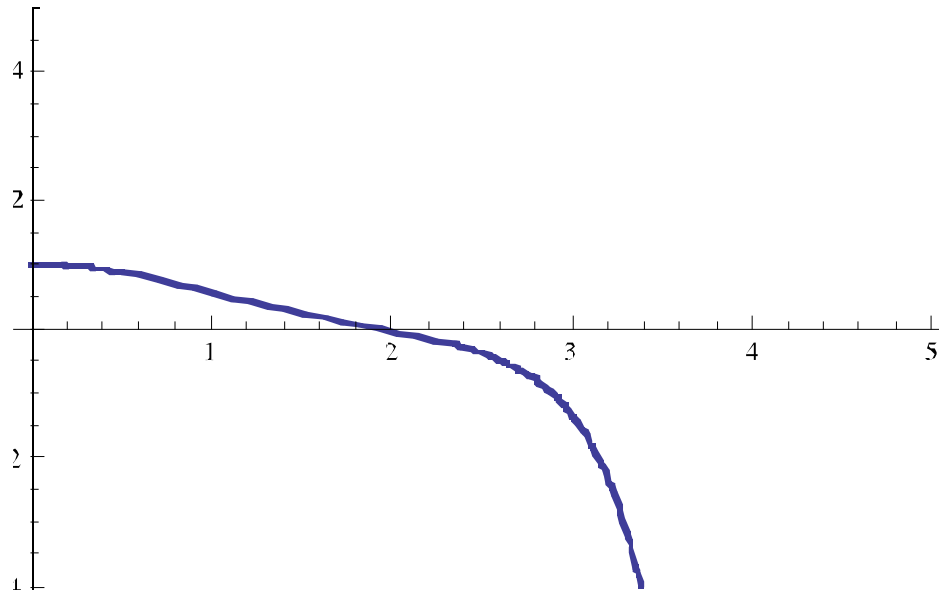
Mathematica has numerically solved the TISE, with $V(x) = \frac{1}{2}m\omega^2 x^2$, starting from $u(0)=1$, with an assumed energy E .



What should be our next try?

- A) Make E a little bigger
- B) Make E a little smaller
- C) Make $u(0)$ a little bigger
- D) Make $u(0)$ a little smaller
- E) None of these/more than one/something else...

Mathematica has numerically solved the TISE, with $V(x) = \frac{1}{2}m\omega^2 x^2$, starting from $u(0)=1$, with an assumed energy E .



What should be our next try?

- A) Make E a little bigger B) Make E a little smaller
- C) Make ω a little bigger D) Make ω a little smaller
- E) None of these/more than one/something else...

What is the behaviour of $u(x) = A x e^{-ax^2}$ as x goes to infinity?

- A) $u(\infty)$ blows up.
- B) $u(\infty)$ goes to a nonzero constant
- C) $u(\infty)$ goes to 0, but $u(x)$ is not normalizable
- D) $u(\infty)$ goes to 0, and $u(x)$ is normalizable
- E) Can't decide without knowing A and a .

In QM, what is $\hat{p}f(x)$
(where \hat{p} is the momentum operator)

- A) 0, for all functions $f(x)$
- B) $\hbar k f(x)$
- C) df / dx
- D) $-i\hbar df / dx$
- E) None of these/something else

$$[x,p]=i\hbar.$$

Is $[p,x]=i\hbar$?

Is $ip+cx = cx+ip$?

A) Yes

and yes

B) Yes

and no

C) No

and yes

D) No

and no

$$\hat{a}_+ \hat{a}_- = \frac{1}{\hbar\omega} \hat{H} - \frac{1}{2}$$

What is $\hat{a}_- \hat{a}_+$

Given $\hat{a}_+ \hat{a}_- = \frac{1}{\hbar\omega} \hat{H} - \frac{1}{2}$

and $\hat{a}_- \hat{a}_+ = \frac{1}{\hbar\omega} \hat{H} + \frac{1}{2}$

What is $[\hat{a}_+, \hat{a}_-]$?

A) Give me a break, it's a bloody mess

B) 0

C) 1

D) -1

E) It's simple, just not given above.

If $\hat{H}(\hat{a}_+ u_n) = (E_n + \hbar\omega)(\hat{a}_+ u_n)$

What can you say about $\hat{a}_+ u_n$

A) Nothing much

B) It is a stationary state (an “energy eigenfunction”) and must be equal (or proportional) to the state u_n

C) It is a stationary state, but is NOT proportional to the state u_n

If there is a "bottom rung", u_0 , with $\hat{H}u_0 = E_0u_0$ (where E_0 is the lowest physically possible energy):

What is \hat{a}_-u_n ?

A) $(E_0 - \hbar\omega)u_0$

B) 0

C) Undefined

D) *Both* **A** and **B**

E) None of these!

Plane waves Fourier Transforms, momentum space

Given $u(x,t=0) = Ae^{ikx} + Be^{-ikx}$
what is the time dependent wave
function, $\Psi(x,t)$?

A) $Ae^{ikx} e^{+i\hbar k^2 t / 2m} + Be^{-ikx} e^{-i\hbar k^2 t / 2m}$

B) $Ae^{ikx} e^{-i\hbar k^2 t / 2m} + Be^{-ikx} e^{+i\hbar k^2 t / 2m}$

C) $Ae^{ikx} e^{-i\hbar k^2 t / 2m} + Be^{-ikx} e^{-i\hbar k^2 t / 2m}$

D) $Ae^{ikx} e^{+i\hbar k^2 t / 2m} + Be^{-ikx} e^{+i\hbar k^2 t / 2m}$

E) Something else/not enough info

What is $\hat{p}\Psi_k(x,t)$, if $\Psi_k(x,t) = e^{ikx} e^{-iEt/\hbar}$?

What is $\hat{p}\Psi_{-k}(x,t)$, if $\Psi_{-k}(x,t) = e^{-ikx} e^{-iEt/\hbar}$?

$$\text{A) } \hat{p}\Psi_k = \hat{p}\Psi_{-k} = 0$$

$$\begin{aligned} \text{B) } \hat{p}\Psi_k &= k\Psi_k, \\ \hat{p}\Psi_{-k} &= -k\Psi_{-k} \end{aligned}$$

$$\begin{aligned} \text{C) } \hat{p}\Psi_k &= +\hbar k\Psi_k, \\ \hat{p}\Psi_{-k} &= +\hbar k\Psi_{-k} \end{aligned}$$

$$\begin{aligned} \text{D) } \hat{p}\Psi_k &= +\hbar k\Psi_k, \\ \hat{p}\Psi_{-k} &= -\hbar k\Psi_{-k} \end{aligned}$$

$$\begin{aligned} \text{E) } \hat{p}\Psi_k &= -\hbar k\Psi_{-k}, \\ \hat{p}\Psi_{-k} &= +\hbar k\Psi_k \end{aligned}$$

Do you plan to attend today' s Tutorial (on energy and energy measurements – review material for the upcoming midterm)

- A) Yes, at 3 pm
- B) Yes, at 4 pm
- C) Perhaps, more likely at 3
- D) Perhaps, more likely at 4
- E) No, can' t come/not planning on it.

Given the result: $\langle p \rangle = \hbar k \int |\Psi|^2 dx$
(and what you know about expectation values and σ 's) what will σ_p be?

A) 0

B) $\hbar k$

C) $\hbar/2$

D) $(\hbar k)^2 / 2m$

E) Depends/not enough information.

Consider the wave function

$$\Psi(x, t) = A \exp[i(kx - \omega t)] \quad (k > 0)$$

What does the probability density $|\Psi(x, t)|^2$ look like?

- A) Constant in x and t .
- B) Like a traveling wave, traveling to the right.
- C) Like a standing wave.
- D) None of these

Given $\Psi(x,0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \varphi(k) e^{+ikx}$, what is $\Psi(x,t)$?

A) $\Psi(x,t) = e^{-iEt/\hbar} \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \varphi(k) e^{+ikx} \right)$

B) $\Psi(x,t) = \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \varphi(k) e^{+ikx} e^{-i\hbar k^2 t/2m} \right)$

C) $\Psi(x,t) = \sum_n \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \varphi(k) e^{+ikx} e^{-iE_n t/\hbar}$

D) Something else!

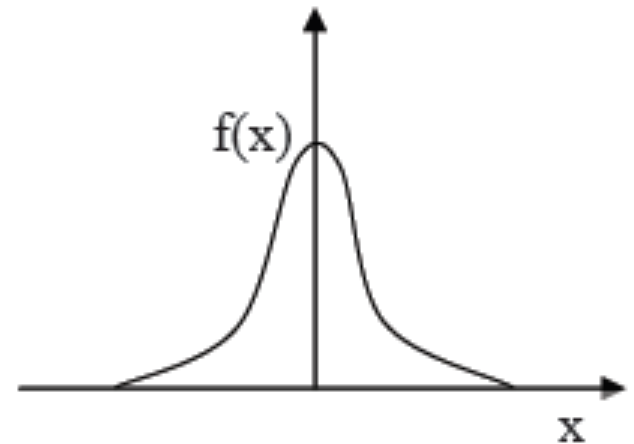
Consider the function $f(x) = e^{-x^2/b}$

What can you say about the integral

$$\int_{-\infty}^{\infty} f(x) e^{ikx} dx?$$

It is ...

- A) zero
- B) non-zero and pure real
- C) non-zero and pure imaginary
- D) non-zero and complex

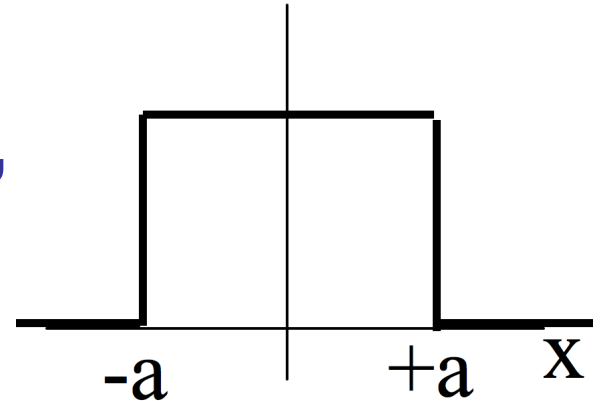


If $\psi(x)$ is given in the picture,
it's easy enough to evaluate

$$\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$$

The answer is

$$\phi(k) = \frac{1}{\sqrt{\pi a}} \frac{\sin ka}{k}$$



Describe (or sketch) $\phi(k)$ if "a" is very SMALL

How about if "a" is very LARGE?

Given the Gaussian function

$$\Psi(x) = Ae^{-x^2/(2a^2)}$$

we can *compute* the Fourier transform,

$$\phi(k) = \tilde{A}e^{-k^2a^2/2}$$

If the width of $\Psi(x)$ is increased, what can you say about the width of $\phi(k)$?

A) Increases also B) Decreases

C) Unaffected in this case

D) Depends on many things!!

What is the Fourier transform of a delta function centered at x_0 , i.e. what is

$$\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \delta(x - x_0) e^{-ikx} dx$$

A) $\frac{1}{\sqrt{2\pi}}$

B) $\frac{1}{\sqrt{2\pi}} \delta(k)$

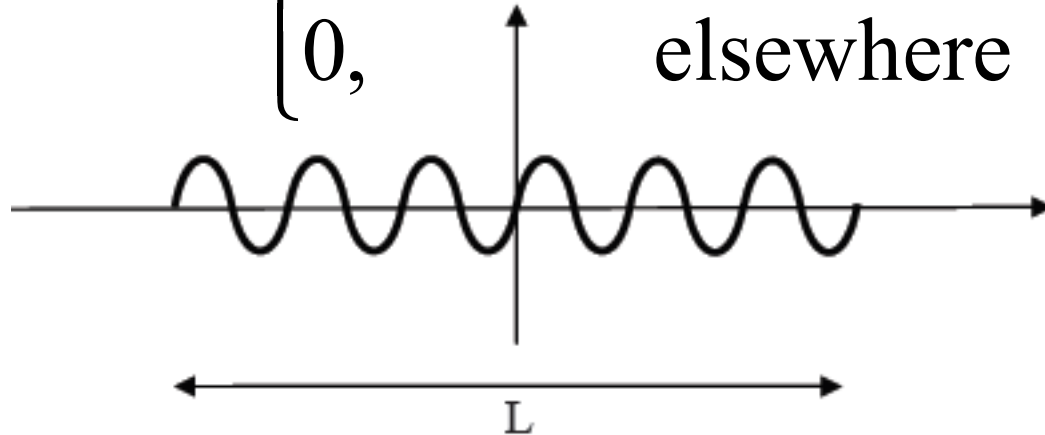
C) $\frac{1}{\sqrt{2\pi}} e^{-ikx}$

D) $\frac{1}{\sqrt{2\pi}} e^{-ikx_0}$

E) Something very different... (or, I don't remember enough about delta functions)

Consider the function $f(x)$ which is a sin wave of length L .

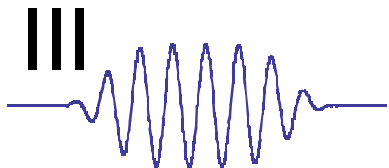
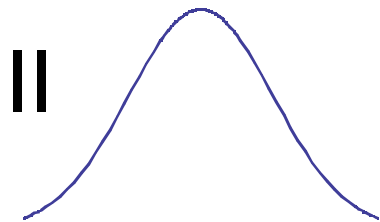
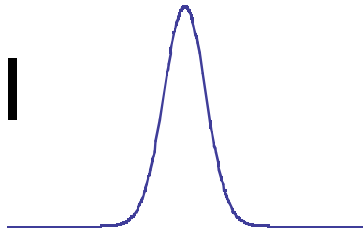
$$f(x) = \begin{cases} \sin(kx), & -\frac{L}{2} < x < +\frac{L}{2} \\ 0, & \text{elsewhere} \end{cases}$$



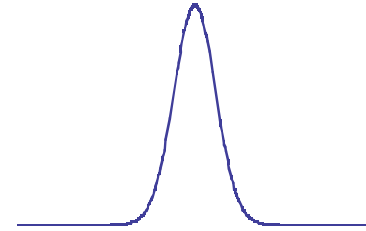
Which statement is closest to the truth?

- A) $f(x)$ has a single well-defined wavelength
- B) $f(x)$ is made up of a range of wavelengths

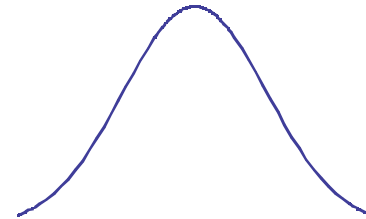
Match $f(x)$ to the magnitude of its Fourier transform $|\phi(k)|$:



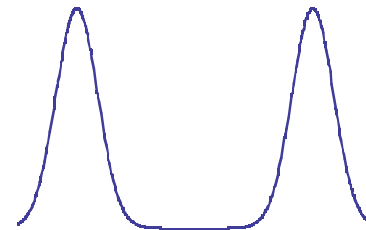
A



B



C



D



Compared to $\Psi(x,0)$, $\Phi(p)$ contains:

- A) more information.
- B) less information.
- C) the same information, you can exchange them.
- D) cannot be determined/depends.

How do you write a Hamiltonian \hat{H} with an arbitrary time-independent potential $V(x)$ in momentum basis?

A) $-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial p^2} + V(p)$

C) $\frac{p^2}{2m} + V(p)$

B) $-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial p^2} + V\left(i\hbar \frac{\partial}{\partial p}\right)$

D) $\frac{p^2}{2m} + V\left(i\hbar \frac{\partial}{\partial p}\right)$

E) something else

As time goes on, the Gaussian wave packet will:

- A) Keep its width
- B) Get narrower
- C) Spread out
- D) Depends on the details of the Gaussian

What is the wavelength λ of the function

$$f(x) = \cos\left(\frac{\pi}{a}x\right) ?$$

A) a

B) $2a$

C) $\frac{1}{2}a$

D) a/π

E) $a/(2\pi)$

What is $\int_{-\infty}^{\infty} dx \delta(x - x_0)$ where $\delta(x)$ is the

Dirac delta function?

A) Zero

B) x_0

C) 1

D) ∞

E) Pass: I don't know what the delta function is.

What is $f(x_1, x_2) = \int dx \delta(x - x_1) \delta(x - x_2)$?

A) zero

B) 1

C) 2

D) $\delta(0)$

E) $\delta(x_2 - x_1)$

True (A) or False (B):

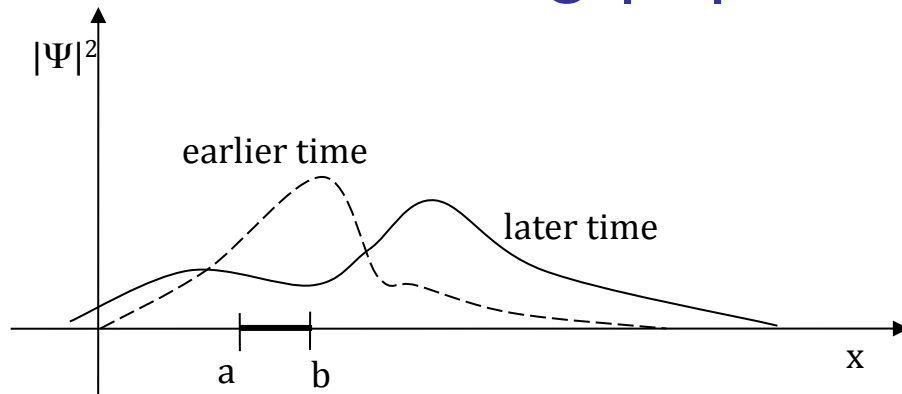
If $\Psi(x, t)$ can be written as

$$\Psi(x, t) = \frac{1}{\sqrt{2\pi}} \int dk \Phi(k, t) e^{ikx}$$

then $\Psi(x, t)$ represents the state of a free particle ($V = 0$).

Stepwise constant V ,
scattering

The probability density $|\Psi|^2$ decreases with time in a region bounded by $x = a$ and $x = b$. Four possible pairs of probability currents J_a and J_b on the left and right ends of the region are indicated. *How many of the pairs could produce the decreasing $|\Psi|^2$ in this region?*



- I $J_a \rightarrow \rightarrow J_b$
- II $\rightarrow \rightarrow$
- III $\leftarrow \rightarrow$
- IV $\rightarrow \leftarrow$

- A) Only one of the pairs
- B) two of the pairs
- C) three
- D) all four
- E) None

Consider a plane wave state

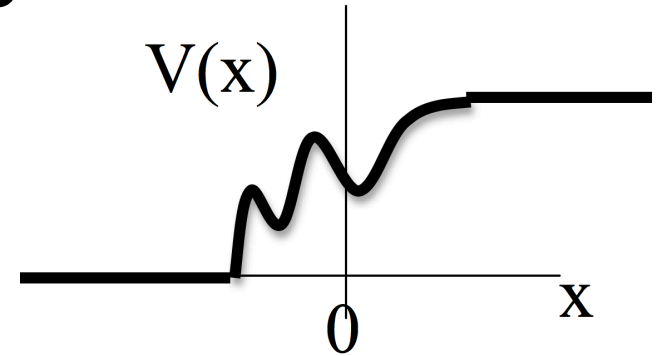
$\Psi(x,t) = A \exp[i(kx - \omega t)]$, with $k > 0$.

What is the probability current $J(x,t)$?

- A) 0 current everywhere
- B) constant positive (right-going) current (constant in x and t)
- C) constant negative (left-going) current (constant in x and t)
- D) current that oscillates with position x :
left-right-left-right-...but constant in time
- E) current that is constant in position at any one time,
but oscillates in time:
left-zero-right-zero-left-zero-right-...

A stream of particles enters steadily from the far left.

The general solution to the TISE is given below:



$$u(x)(\text{far left, } x \ll 0) = Ae^{+ikx} + Be^{-ikx}$$

$$u(x)(\text{far right, } x \gg 0) = Ce^{+ik'x} + De^{-ik'x}$$

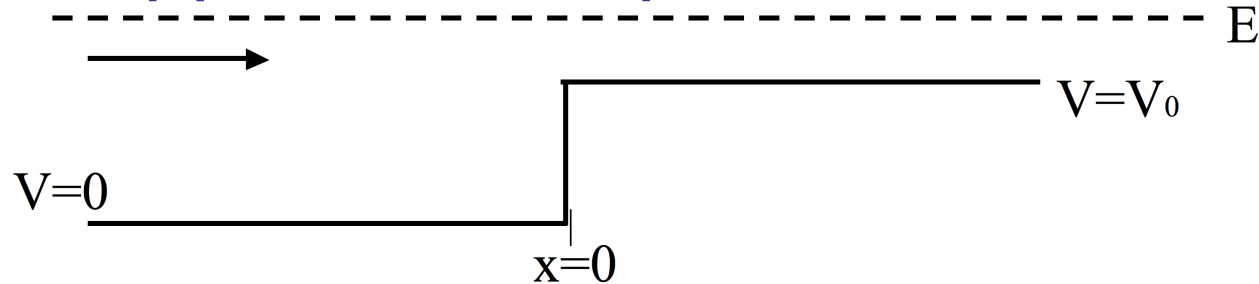
To represent the physics of “entering from the left”,

which coefficient (A-D) must vanish?

Enter E if you think *none*, or *more than one*, should be zero!

A **classical** particle of energy E approaches a potential energy step of height V_0 ($E > V_0$).

What happens to the particle?



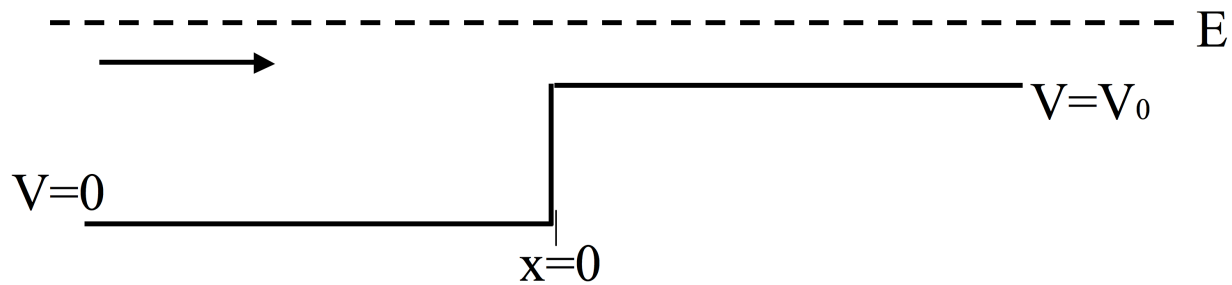
- A) Always moves to the right at constant speed.
- B) Always moves to the right, but its speed is different for $x > 0$ and $x < 0$.
- C) Hits the barrier and reflects
- D) Hits the barrier and has a chance of reflecting, but might also continue on.
- E) None of these, or MORE than one of these!

$$TISE : -\frac{\hbar^2}{2m} \frac{\partial^2 u(x)}{\partial x^2} + V(x)u(x) = Eu(x)$$

A quantum stationary state is called $u(x)$.

What can we say in general about $u'(x)$?

- A) $u'(x)$ must be continuous everywhere.
- B) $u'(x)$ must be continuous everywhere, except at places where $V(x)$ goes to infinity.
- C) $u'(x)$ only needs to be continuous everywhere that $V(x)$ is continuous.
- D) There are no universal/general constraints on $u'(x)$, it is only $u(x)$ which must be continuous!
- E) What's a quantum stationary state?

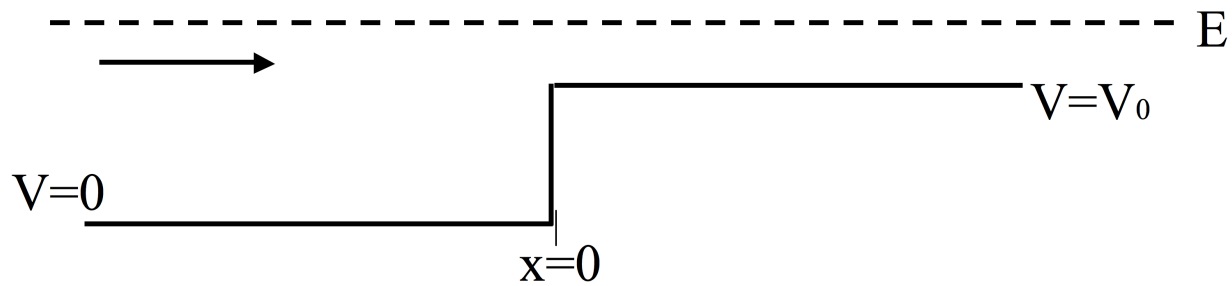


R and T for a “step up” potential ($E > V_0$) are :

$$R = \frac{(k - k')^2}{(k + k')^2}, \quad T = \frac{4k k'}{(k + k')^2}$$

What can you say in the limit $E \gg V_0$?

- A) $k \gg k'$, $R \rightarrow 0$, $T \rightarrow 1$
- B) $k \gg k'$, $R \rightarrow 1$, $T \rightarrow 0$
- C) $k \approx k'$, $R \rightarrow 0$, $T \rightarrow 1$
- D) $k \approx k'$, $R \rightarrow 1$, $T \rightarrow 0$
- E) None of these, it's something else!

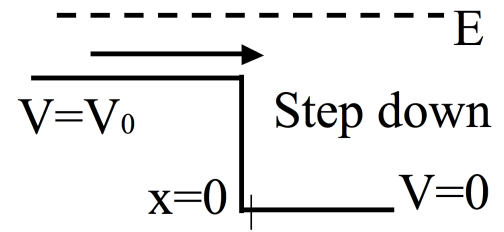
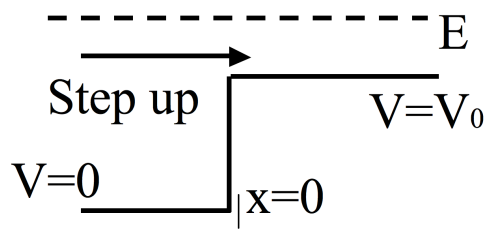


R and T for a “step up” potential ($E > V_0$) are :

$$R = \frac{(k - k')^2}{(k + k')^2}, \quad T = \frac{4k k'}{(k + k')^2}$$

What if E approaches V_0 (from above) ?

- A) $k \gg k'$, $R \rightarrow 0$, $T \rightarrow 1$
- B) $k \gg k'$, $R \rightarrow 1$, $T \rightarrow 0$
- C) $k \approx k'$, $R \rightarrow 0$, $T \rightarrow 1$
- D) $k \approx k'$, $R \rightarrow 1$, $T \rightarrow 0$
- E) None of these, it's something else!



The Reflection and Transmission formulas for the “step up” potential (with $E > V_0$) are :

$$R = \frac{(k - k')^2}{(k + k')^2}, \quad T = \frac{4k k'}{(k + k')^2}$$

What are R and T for the “step down” potential?

- A) Exactly the same formulas.
- B) Same, but replace k' with $-k'$
- C) Same, but swap R and T
- D) It's a very different case, formulas won't be related in any simple way

The formula for probability current is:

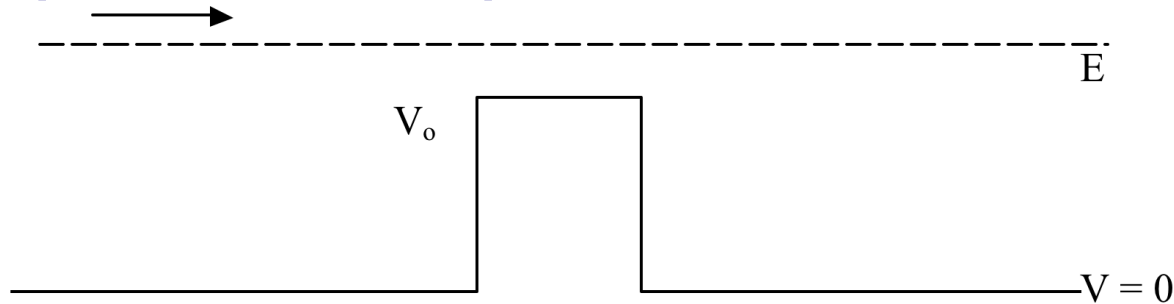
$$J = \frac{\hbar}{m} \text{Im} \left(\Psi^* (x, t) \frac{\partial}{\partial x} \Psi(x, t) \right)$$

The wave function for a particle (energy E) in a classically forbidden region is: $\Psi(x, t) = C e^{-\kappa x} e^{-iEt/\hbar}$

What can you say about J in this region?

- A) J is positive (current is to the right)
- B) J is negative (current is to the left)
- C) J is imaginary (or complex)
- D) J is zero
- E) You can't say anything about J (it depends)

A **classical** particle of energy E approaches a potential energy barrier of height V_0 ($E > V_0$).
What happens to the particle?



- A) Move to the right, but its speed is different *over* the barrier than elsewhere.
- B) Moves to the right at constant speed.
- C) Hits the barrier and reflects, moving to the left after the collision.
- D) Hits the barrier and has a chance of reflecting, but might also continue on.
- E) None of these, or MORE than one of these!

The solution to the TISE in the “tunneling” situation is:

$$\Psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} & x < 0 \\ Ce^{Kx} + De^{-Kx} & 0 < x < L \\ Fe^{ikx} & L < x \end{cases}$$

$$k = \sqrt{\frac{2mE}{\hbar^2}},$$

$$K = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

What is the transmission coefficient T?

- A) $|F/A|^2$
- B) $|C/A|^2$
- C) $(\kappa/k) |F/A|^2$
- D) $(\kappa/k) |C/A|^2$
- E) Something else!

Do you plan to attend today's Tutorial (on qualitative features of wave functions)

A) Yes, at 3 pm

B) Yes, at 4 pm

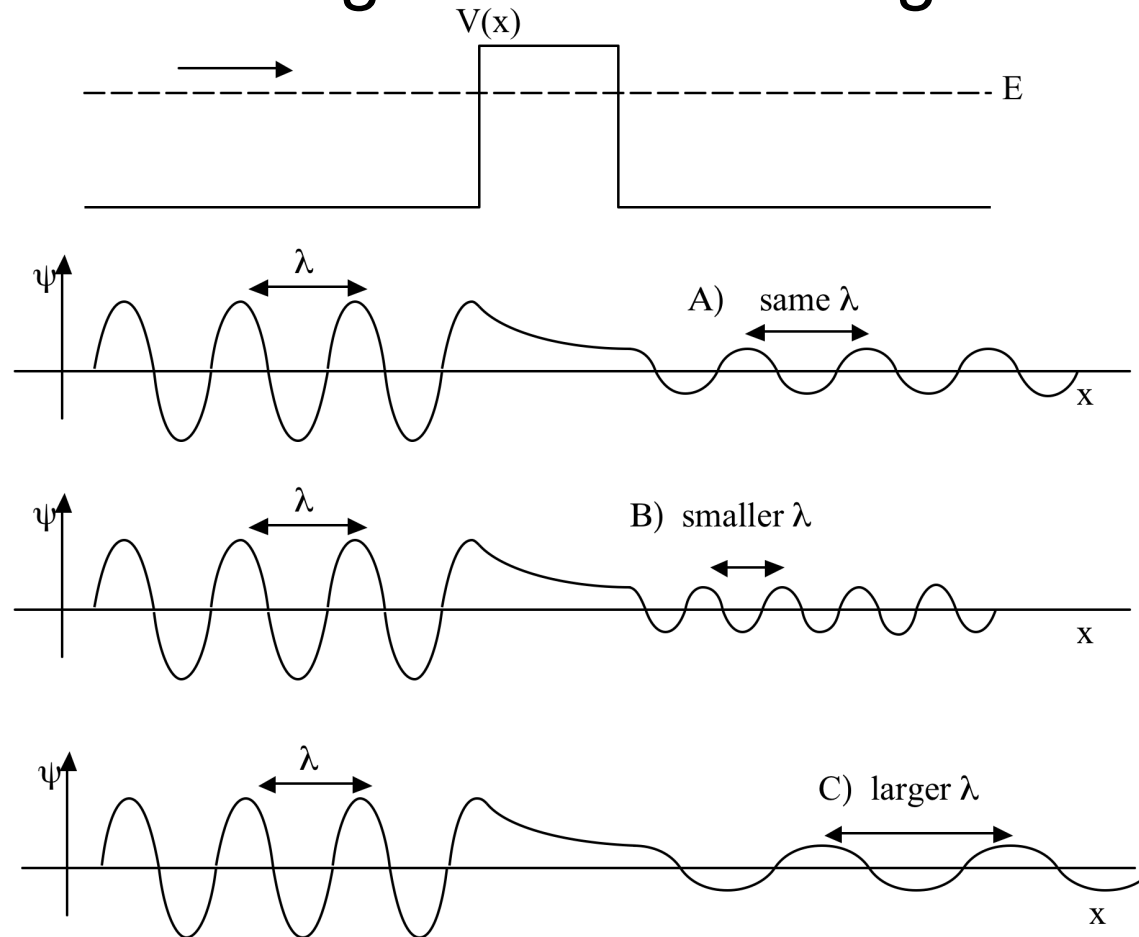
C) Perhaps, more likely at 3

D) Perhaps, more likely at 4

E) No, can't come/not planning on it.

A plane wave, incident from the left, tunnels through the potential barrier shown. The TISE solution has ...

- A) the same wavelength on both sides of the barrier
- B) a smaller wavelength after tunneling
- C) a larger wavelength after tunneling

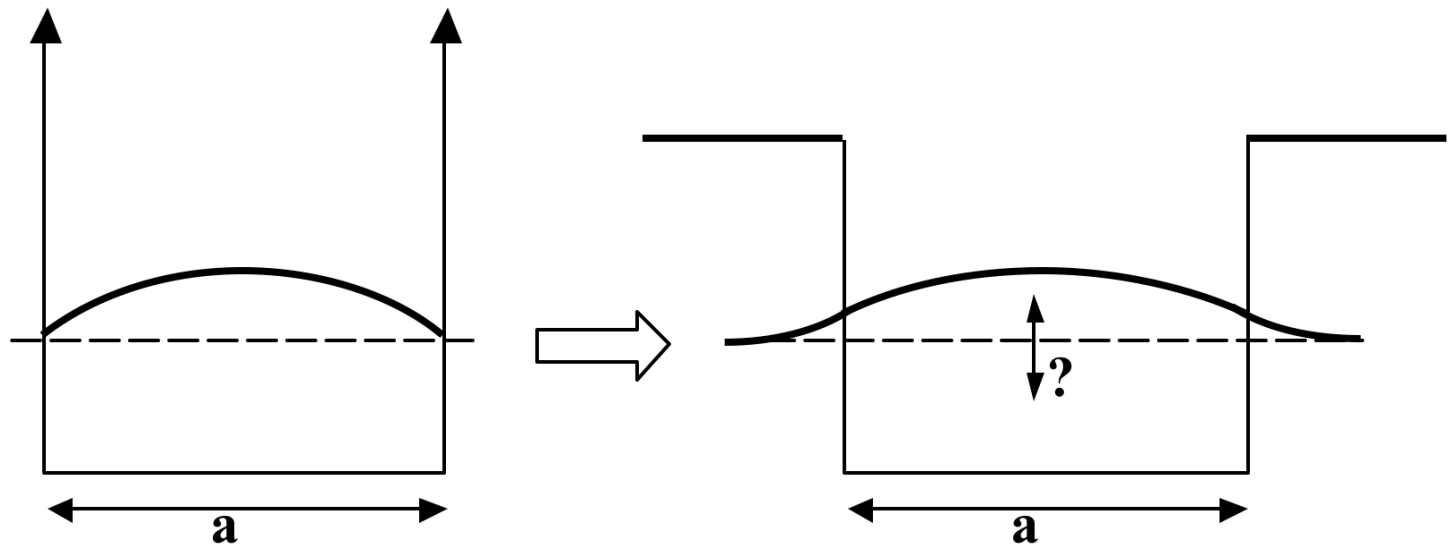


In the classically forbidden region of the square barrier, the wavefunction is $u(x) = Ce^{kx} + De^{-kx}$

When particles are incident from the left, the probability current, J , in this region must be...

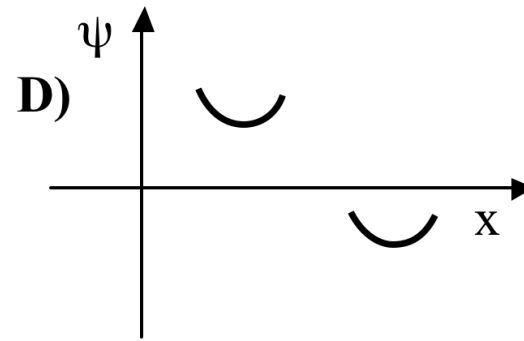
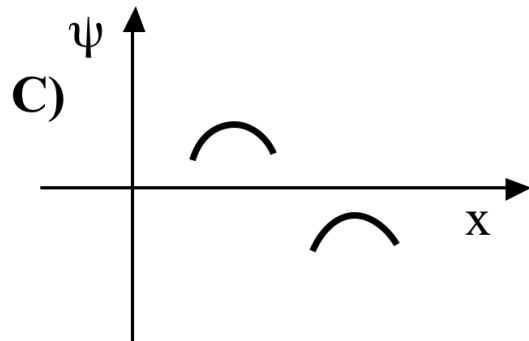
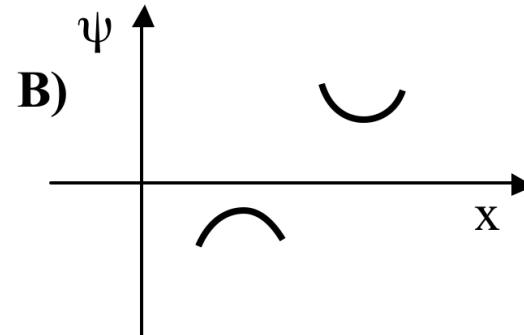
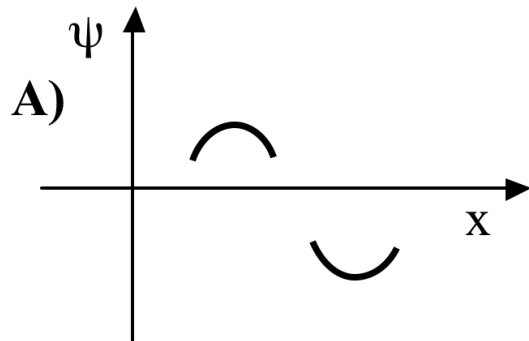
- A) zero
- B) Equal to $J(\text{incident})$
- C) Oscillating between 0 and $J(\text{incident})$ with time
- D) Varying in space
- E) Something else!

Compared to the infinite square with the same width a , *the ground state energy of a finite square well is...*



- A) the same
- B) higher
- C) lower

Which of the graphs correctly shows parts of the wavefunction ψ that satisfies $\psi'' = -k_2^2 \psi$?



Which of the graphs correctly shows parts of the wavefunction ψ that satisfies $\psi'' = +k_2\psi$?

